Y. J. ZHANG, J. Y. PAN, S. S. WANG, L. B. WU, D. X. GAO

ISSN 0543-5846
METABK 63(3-4) 395-398 (2024)
UDC – UDK 621.74.04:621.41: 539.417-531.78.51/621.72=111

FIXED-TIME FUZZY ADAPTIVE OUTPUT FEEDBACK CONTROL BASED ON STEEL STRUCTURE ROBOTIC ARM

Received – Primljeno: 2024-01-12
Accepted – Prihvaćeno: 2024-03-15
Original Scientific Paper – Izvorni znanstveni rad

This paper proposes a fixed-time fuzzy adaptive output feedback control scheme for the robotic arm model (RAM) of steel structures. Firstly, the process of transforming RAM into a nonlinear system is elaborated. Secondly, a fuzzy observer is designed to approximate the nonlinear function and estimate the observed state of the system. Subsequently, a fixed-time adaptive controller is constructed, ensuring the system's stability within a fixed time, with the convergence time unaffected by the initial state. Finally, the effectiveness of the strategy is verified through simulation examples.

Key words: steel structure, robotic arm, fuzzy control, output feedback, fixed-time

INTRODUCTION

Steel structure robotic arm find extensive applications in the industry, and their flexibility and efficiency make them indispensable in many industrial sectors [1]. They are widely used in manufacturing for the assembly, handling, and processing of products. Capable of executing high-precision tasks, they enhance production efficiency, reduce manual labor, and ensure consistency in product quality [2].

However, due to their nonlinear, time-varying, and uncertain characteristics, achieving excellent control performance with a single control method is challenging [3]. To ensure superior system performance, applying adaptive control strategies to such systems is a viable solution. Adaptive control strategies find extensive applications across various fields, aiming to enhance system performance and stability by continuously monitoring and adjusting to real-time dynamic characteristics, accommodating changes in system parameters and external disturbances. To address the impact of parameter uncertainties and unobservable states on control effectiveness, numerous scholars have proposed effective adaptive control strategies, including neural networks or fuzzy output feedback approaches [4-6]. These methodologies have proven to be valuable in mitigating the challenges posed by uncertain and unpredictable system dynamics, contributing to improved control outcomes.

The control schemes designed in the aforementioned studies have been discussed under the assumption of infinite time. However, in practical applications of engineering systems, the convergence time required for the system to achieve stability is also a crucial consideration for ensuring robustness [7,8]. This paper addresses these issues by proposing a fixed-time fuzzy output feedback control strategy for RAM.

STEEL STRUCTURE ROBOTIC ARM MODEL

Considering the steel structure single-link robotic arm system shown in Figure 1, its dynamic equations can be described as follows:

$$M\ddot{q} + \frac{1}{2} mgl \sin(q) = \tau \tag{1}$$

where q is the angle position, $\dot{q}$ is the angular velocity, $\ddot{q}$ is the angular acceleration, $g = 9{,}8\ m/s^2$ is the acceleration due to gravity, M is the inertia, l is the length of the link, m is the mass of the link, and τ is the control force.

Setting $x_1 = q$, $x_2 = \dot{q}$ and $u = \tau$, then the above equation can be rewritten in the following state-space form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{1}{M}u - \dfrac{mgl\sin(x_1)}{2M} \\ y = x_1 \end{cases} \tag{2}$$

where $x = [x_1, x_2]^T$ is state vector, u is control input, y is output of the system.

Assumption 1: For the above system (2), there are unknown positive constants $\overline{d}$ and $\dot{\overline{d}}$, such that $y_d \le \overline{d}$ and $\dot{y}_d \le \dot{\overline{d}}$.

Lemma 1: For the system (2), if there exists a positive definite and radial unbounded function $V(x)$ satisfying $\delta_1 \|x\| \le V(x) \le \delta_2 \|x\|$, such that

Science and Technology Special Plan Project of China Minmetals Group Co., Ltd. (2021ZXA06).

Y. J. Zhang (e-mail: 1997zyj@163.com), J. Y. Pan (1838719677@qq.com), L. B. Wu (e-mail: beyondwlb@163.com), D. X. Gao (e-mail: 1074501593@qq.com), School of Computer Science and Software Engineering, University of Science and Technology Liaoning, China.
S. S. Wang (e-mail: 969704626@qq.com), China 22MCC Group Co. LTD.

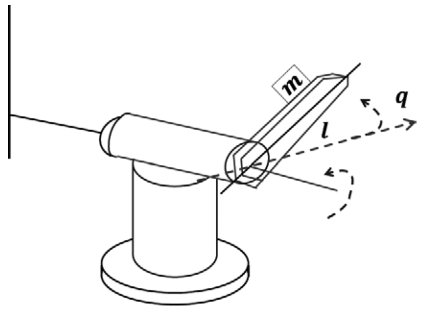

Figure 1 Schematic Diagram of RAM

$$\begin{aligned} &\dot{V}(x) \le -aV^p(x) - bV^q(x) + \Delta, t \ge 0 \\ &\Delta < min\{(1-\varsigma)a, (1-\varsigma)b\}\ (0<\varsigma<1) \end{aligned} \tag{3}$$

where δ_1 and δ_2 are k_∞ functions. $a > 0$, $b > 0$, $0 < p < 1$, $q > 1$, and $\Delta > 0$ are parameters. Then the nonlinear system (3) is actually fixed-time stable, and the boundary of its convergence time T can be expressed as

$$T \le T_{max} = \frac{1}{a(1-p)\varsigma} + \frac{1}{b(q-1)\varsigma} \tag{4}$$

Lemma 2 [4]: The fuzzy logic system can uniformly approximate the continuous nonlinear function $f(x)$ with arbitrary precision on the compact set Ω_x as follows

$$\sup_{x\in\Omega_x} | f(x) - \theta^T\varphi(x) | \le \varepsilon(x) \tag{5}$$

where $\varepsilon(x)$ is approximate error, and satisfy $|\varepsilon(x)| < \varepsilon$. $\varphi(x)$ is fuzzy basis function. Due to this approximation capability, we can assume that the nonlinear terms in (2) can be approximated as

$$f(x\,|\,\hat{\theta}) = \hat{\theta}^T\varphi(x) \tag{6}$$

where $\tilde{\theta} = \theta - \hat{\theta}$, $\hat{\theta}$ is an estimate of θ, and θ is a given ideal parametric regression vector, can be expressed as

$$\theta = \arg\min_{\theta\in\Omega_\theta}\{\sup_{x\in\Omega_x} | f(x\,|\,\hat{\theta}) - f(x) |\} \tag{7}$$

where Ω_θ is a compact set.

FUZZY OBSERVER AND FIXED-TIME CONTROLLER DESIGN

In this section establishes a fuzzy observer. Subsequently, theory with an adaptive mechanism is designed utilizing the Lyapunov stability analysis method.

Fuzzy observer design

Selecting an appropriate K ensures that the matrix A is a strictly Hurwitz matrix. It is assumed that the state x_2 of system (2) are not available for feedback. In this situation, to estimate the state of the system, a fuzzy observer is designed for (2) as

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Ky + B(\dfrac{1}{M}u + f(\hat{x}\,|\,\hat{\theta})) \\ y = C\hat{x} \end{cases} \tag{8}$$

where $A = \begin{bmatrix} -k_1 & 1 \\ -k_2 & 0 \end{bmatrix}$, $K = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $C = [1, 0]$.

Moreover, $\hat{x}$ is the estimate of state vector x, $f(\hat{x}\,|\,\hat{\theta}) = \hat{\theta}^T\varphi(\hat{x})$, and the observer error is defined as $e = x - \hat{x} = [e_1, e_2]^T$. Therefore, there exist positive definite matrices P and Q, such that $A^TP + P^TA = -2Q$.

From , we can obtain

$$\dot{e} = Ae + B\left(\tilde{\theta}^T\varphi(\hat{x}) + \varepsilon\right) \tag{9}$$

where $\varepsilon = f(x) - f(x\,|\,\theta)$.

To ensure the boundedness of the (9), choose Lyapunov function as $V_0 = e^TPe$.

Then, we have

$$\dot{V}_0 = e^T(A^TP + PA^T)e + 2e^TPB(\tilde{\theta}^T\varphi(\hat{x}) + \varepsilon) \tag{10}$$

By using Young's inequality, can calculate

$$2e^TPB\tilde{\theta}^T\varphi(\hat{x}) \le \|P\|^2\|e\|^2 + \tilde{\theta}^T\tilde{\theta} \tag{11}$$

$$2e^TPB\varepsilon \le \|P\|^2\|e\|^2 + \varepsilon^2 \tag{12}$$

Substituting (11), (12) into (10) yields

$$\dot{V}_0 \le -\beta\|e\|^2 + \tilde{\theta}^T\tilde{\theta} + \varepsilon^2 \tag{13}$$

where $\beta = \lambda_{min}(Q) - 2\|P\|^2$.

Fixed-time controller design

In this subsection, we will develop an adaptive fixed-time control scheme for system (2). An adaptive fixed-time controller is designed, the controller design process involves two recursive design steps and stability analysis is conducted on it using Lyapunov stability theory.

Defining error variables $z_1 = x_1 - y_d$, $z_2 = \hat{x}_2 - \alpha_1$, where y_d is the reference signal, and α_1 is the virtual control function.

Step 1: Selecting the Lyapunov function as $V_1 = V_0 + \frac{1}{2}z_1^2$.

Then, the time derivative of V_1 yields

$$\begin{aligned} \dot{V}_1 &= \dot{V}_0 + z_1\dot{z}_1 \\ &= \dot{V}_0 + z_1(\dot{x}_1 - \dot{y}_d) \\ &\le \dot{V}_0 + z_1(\hat{x}_2 - \dot{y}_d) \\ &\le \dot{V}_0 + z_1(z_2 + \alpha_1 - \dot{y}_d) \end{aligned} \tag{14}$$

Choose the virtual control function α_1 as

$$\alpha_1 = -c_1z_1^{2p-1} - \lambda_1z_1^{2q-1} + \dot{y}_d \tag{15}$$

where $c_1 > 0$, $\lambda_1 > 0$, $0 < p < 1$, and $q = \dfrac{2m+n}{2m+1} > 1$ $(m \in N\ and\ n \in N^*)$ are design parameters.

By using (15), can be obtain

$$\begin{aligned} \dot{V}_1 \le &-a\|e\|^2 + \tilde{\theta}^T\tilde{\theta} - c_1z_1^{2p} - \lambda_1z_1^{2q} \\ &+ z_1z_2 + \varepsilon^2 \end{aligned} \tag{16}$$

Step 2: Choose the following Lyapunov function

$$V_2 = V_1 + \frac{1}{2}v_2^2 + \frac{1}{2\Gamma}\tilde{\theta}^{\mathrm{T}}\tilde{\theta} \tag{17}$$

where Γ is a positive design parameter. The time differentiation of (17) can be represented as

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + z_2\dot{z}_2 - \frac{1}{\Gamma}\tilde{\theta}\dot{\hat{\theta}} \\ &\le \dot{V}_1 + z_2\Big(\frac{1}{M}u + \hat{\theta}^{\mathrm{T}}\varphi(\hat{x}) + \tilde{\theta}^{\mathrm{T}}\varphi(\hat{x}) \\ &\quad + \varepsilon - \dot{\alpha}_1\Big) - \frac{l}{\Gamma}\tilde{\theta}\dot{\hat{\theta}} \end{aligned} \tag{18}$$

By utilizing the Young's inequality, have

$$z_2\varepsilon \le \frac{1}{2}z_2^2 + \frac{1}{2}\varepsilon^2 \tag{19}$$

Design the following fixed-time controller u and adaptive law θ:

$$\begin{aligned} u = \frac{1}{M}\Big(&-\frac{1}{2}z_2 - \hat{\theta}^{\mathrm{T}}\varphi(\hat{x}) - c_2 z_2^{2p-1} \\ &-\lambda_2 z_2^{2q-1} + \dot{\alpha}_1 - z_1\Big) \end{aligned} \tag{20}$$

$$\dot{\hat{\theta}} = \Gamma z_2\varphi(\hat{x}) - \sigma\hat{\theta} - \lambda\hat{\theta}^{2q-1},\ \hat{\theta}(0) \ge 0 \tag{21}$$

where c_2, λ_2, σ, and λ are positive design parameters.

STABILITY ANALYSIS

Theorem 1: Consider the nonlinear system (2), under Assumption 1, Lemmas 1-2, if choosing the fuzzy observer (8), the fixed-time controller (20), the virtual control function (15), and the adaptive law (21), such that all signals of the closed-loop system are bounded within fixed time.

Proof: Substituting (16) and (19)-(21) into (18) and carrying out some computations, one has

$$\begin{aligned} \dot{V}_2 \le &-a\|e\|^2 + \tilde{\theta}^{\mathrm{T}}\tilde{\theta} - \sum_{i=1}^{2}c_i z_i^{2p} - \sum_{i=1}^{2}\lambda_i z_i^{2q} \\ &+ \frac{\sigma}{\Gamma}\tilde{\theta}\hat{\theta} + \frac{\lambda}{\Gamma}\tilde{\theta}\hat{\theta}^{2q-1} + \varepsilon^2 \end{aligned} \tag{22}$$

Utilizing the Young's inequality yields

$$\tilde{\theta}\hat{\theta} = \tilde{\theta}(\theta - \tilde{\theta}) \le -\frac{1}{2}\tilde{\theta}^{\mathrm{T}}\tilde{\theta} + \frac{1}{2}\theta^2 \tag{23}$$

$$\Big(\frac{1}{2\Gamma}\tilde{\theta}^{\mathrm{T}}\tilde{\theta}\Big)^p \le \frac{1}{2\Gamma}\tilde{\theta}^{\mathrm{T}}\tilde{\theta} + (1-p)p^{\frac{p}{1-p}} \tag{24}$$

$$\|e\|^{2p} \le \|e\|^2 + (1-p)p^{\frac{p}{1-p}} \tag{25}$$

$$\|e\|^{2q} \le \|e\|^2 + (1-q)q^{\frac{q}{1-q}} \tag{26}$$

Following the scaling method in [8].

$$\tilde{\theta}\hat{\theta}^{2q-1} \le \frac{2q-1}{2q}(\theta^{2q} - \tilde{\theta}^{2q}) \tag{27}$$

In the light of formulas (23)-(27), have

$$\begin{aligned} \dot{V}_2 \le &-\frac{\beta}{2}\|e\|^{2p} - \frac{\beta}{2}\|e\|^{2q} - \sum_{i=1}^{2}c_i z_i^{2p} \\ &-\sum_{i=1}^{2}\lambda_i z_i^{2q} - (\sigma - 2\Gamma)\Big(\frac{1}{2\Gamma}\tilde{\theta}^{\mathrm{T}}\tilde{\theta}\Big)^p \\ &-2^q\Gamma^{q-1}\lambda\frac{2q-1}{2q}\Big(\frac{1}{2\Gamma}\tilde{\theta}^{\mathrm{T}}\tilde{\theta}\Big)^q + \Delta \\ \le &-aV_2^p - bV_2^q + \Delta \end{aligned} \tag{28}$$

where

$$a = min\{\frac{\beta}{2}, 2^p c_i, \sigma - 2\Gamma, i = 1,2\},$$

$$b = min\{\frac{\beta}{2}, 2^p\lambda_i, 2\Gamma\lambda, i = 1,2\},$$

$$\begin{aligned} \Delta = &\varepsilon^2 + (\sigma - 2\Gamma + \frac{\beta}{2})(1-p)p^{\frac{p}{1-p}} + \frac{\beta}{2}(1-q)q^{\frac{q}{1-q}} \\ &+ \frac{\lambda}{\Gamma}\frac{2q-1}{2q}\theta^{2q} + \frac{\sigma}{\Gamma}\theta^2. \end{aligned}$$

From (28), we can see that we only need to let $\sigma - 2\Gamma > 0$ to conclude that both z_i and θ are bounded. The proof for Theorem 1 is now complete.

Simulation example

In this section, a simulation example will be provided for the RAM system (2). The corresponding system parameters are chosen as m = 10 kg, M = 0,5 kg/m^2,

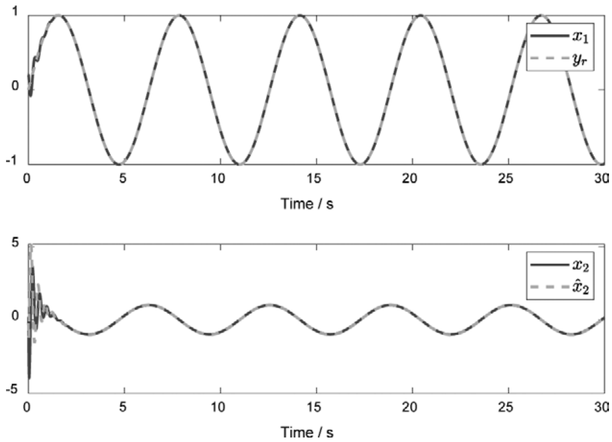

Figure 2 Trajectories of x_i and $\hat{x}_i$ $(i = 1,2)$

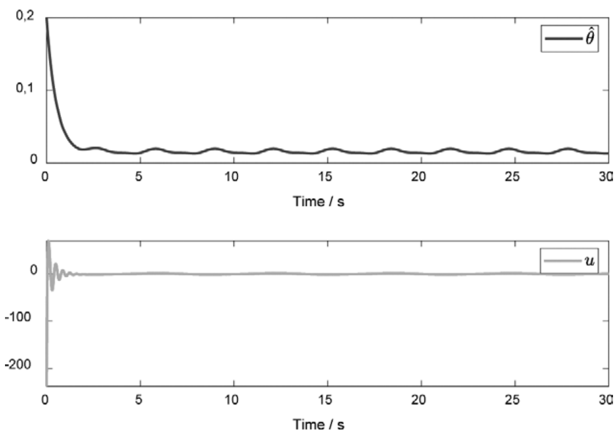

Figure 3 Trajectories of $\hat{\theta}$ and u

$l = 1$ *m*, and the reference signal y_d is selected as $y_d = \sin(t)$. In addition, the fuzzy membership functions are chosen to be Gaussian functions uniformly distributed in the range [–5,5].

The control parameters are selected as $c_1 = 12$, $c_2 = 30$, $k_1 = 25$, $k_2 = 300$, $\lambda_1 = 15$, $\lambda_2 = 25$, $\lambda = 20$, $\Gamma = 2$, $\sigma = 10$, $p = 99/101$, $q = 102/99$, and the initial values are selected as $x_1(0) = 0{,}2$, $x_2(0) = 0{,}2$, $\hat{x}_1(0) = 0{,}2$, $\hat{x}_2(0) = 0{,}2$, $\hat{\theta}(0) = 0{,}2$.

The simulation results are shown in Figures 2 and 3.

CONCLUSIONS

This paper proposes an adaptive fuzzy output feedback fixed-time control scheme for the RAM of steel structures. This scheme can effectively estimate the observed state while ensuring that the system reaches stability within a fixed time. Simulation results indicate that this control strategy not only exhibits excellent tracking performance but also guarantees that signals in the closed-loop system are bounded within fixed time. Therefore, this strategy is of significant importance for improving the overall performance of the RAM and has the potential for practical applications.

Acknowledgments

The work was supported from China 22MCC Group Co. LTD. Special thanks to Shanshan Wang (China 22MCC Group Co. LTD) for her assistance in co-authoring the paper.

REFERENCES

[1] Reddy G R, Eranki V K P. Design and structural analysis of a robotic arm [J]. Blekinge Institute of Technology Karlskrona, Sweden, (2016), 101.

[2] Yin H, Huang S, He M, et al. An overall structure optimization for a light-weight robotic arm [C]//2016 IEEE 11th Conference on Industrial Electronics and Applications (ICIEA). IEEE, 2016: 1765-1770.

[3] Jahnavi K, Sivraj P. Teaching and learning robotic arm model [C]//2017 International Conference on Intelligent Computing, Instrumentation and Control Technologies (ICICICT). IEEE, 2017: 1570-1575.

[4] Liu Y J, Gong M Z, Tong S C, et al. Adaptive fuzzy output feedback control for a class of nonlinear systems with full state constraints [J]. IEEE Transactions on Fuzzy Systems, 5 (2018) 26, 2607-2617.

[5] Li Y X, Tong S C, Yang G H. Observer-based adaptive fuzzy decentralized event-triggered control of interconnected nonlinear system [J]. IEEE transactions on cybernetics, 7 (2019) 50, 3104-3112.

[6] Liu L, Cui Y, Liu Y J, et al. Observer-based adaptive neural output feedback constraint controller design for switched systems under average dwell time [J]. IEEE Transactions on Circuits and Systems I: Regular Papers, 9 (2021) 68, 3901-3912.

[7] Chen M, Wang H, Liu X. Adaptive fuzzy practical fixed-time tracking control of nonlinear systems [J]. IEEE Transactions on Fuzzy Systems, 3 (2019) 29, 664-673.

[8] Wang F, Lai G. Fixed-time control design for nonlinear uncertain systems via adaptive method [J]. Systems & Control Letters, (2020) 140, 104704.

Note: The responsible translators for English language is J. Wang - University of Science and Technology Liaoning, China