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Fault diagnosis and fault-tolerant control design for neutral time delay system

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ABSTRACT
This paper presents a new approach of fault-tolerant control (FTC) for the transmission line as a neutral variable time-delay system. The main goal of this work guarantees faulty neutral variable time delay system stabilization using the state feedback control design based on Lyapunov function and the Linear Matrix Inequality resolution. The use of the FTC method is to achieve actuator and sensor fault compensation. This method is based on two steps. The first one is the synthesis of a nominal control, which remains to maintain the closed-loop system stability. The second step is based on adding a new control law to the nominal one to compensate the fault effect on system behaviour and maintain the desired performance in the closed loop system. Then, a conception of an adaptive observer is used to detect and estimate the fault. Finally, the developed approach is applied for the transmission line. The given results are presented to prove the effectiveness of this approach.

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KEYWORDS
Transmission line; neutral variable time delay; fault additive control; adaptive observer

1. Introduction

Time delay, as a source of instability and poor performance, often appears in many dynamic systems. Neutral systems, as a special type of time delay systems appear in many dynamic systems: chemical systems, nuclear reactors and transmission line systems [1–14].

In this work, we are interested in the transmission line as a neutral variable time-delay system. The presence of variable time delay induces poor performance and instability in control systems. In addition, the fault can occur during the system operation that can be a source of degradation of its performance. Therefore, it is necessary to introduce a fault-tolerant control (FTC) strategy to maintain the system behaviour.

Benjemaa et al. [1], interested in the FTC of transmission line system with variable delay and actuator fault based on robust approach and PIM method, proved that the robust control is able to reduce the fault effect but it fails to make system output converged to desired one. The PIM method was used. It was found that the error between the closed loop system output and the nominal one minimized.

This method requires a reference model to modify it and design a control law for the fault-free system such that the closed-loop behaviour follows the reference model.

This work compensates the fault effect on system behaviour applied for two types of faults, sensor and actuator fault using a new type of FTC (additive control).

This FTC is designed by adding a new control law to the nominal one. The results of FTC application for transmission line system are compared with results given in [1].

The simulation results show that the actuator fault compensation is better than the methods previously used, because it allows to minimize the fault effect on system behaviour more than the two other approaches. In addition, this approach is not used only for actuator fault compensation but also for sensor fault.

This paper is organized as follows: Section 2 presents the problems statement where the transmission line is modelled as a neutral variable time delay system. In Section 3, the main results of the proposed method; a synthesis of FTC law for the case of sensor fault and actuator fault, and a conception of an adaptive observer are given.

Section 4 gives the simulation results of control application for neutral variable time delay transmission line system. The last part is the conclusion.

Notation: *Consider the following notations: R^n and $R^{n\times m}$ are respectively the n-dimensional Euclidean space and the space of all real matrices, the transpose is denoted by the superscript "T" and LMI denote Linear Matrix Inequality. I is the identity matrix of appropriate dimension and "$*$" is used to denote the transposed elements in the symmetric position.*

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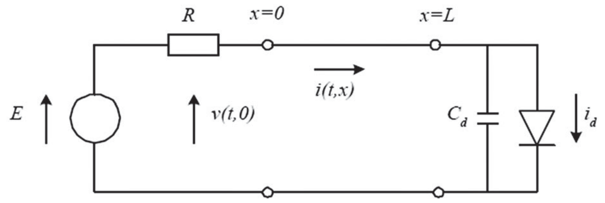

Figure 1. Transmission line circuit.

2. PROBLEMS STATEMENT

Consider the transmission line shown in Figure 1, at one end ($x = 0$) there is a constant voltage source E and at the other end ($x = l$), a capacitor is connected in parallel with a tunnel diode. The current $i(.,.)$ and the voltage $v(.,.)$ are functions of t and x, which satisfy the telegraph equation, a partial derivative equation of the hyperbolic type:

$$L\frac{\partial i}{\partial t} + \frac{\partial v}{\partial x} = 0, \quad C'\frac{\partial v}{\partial t} + \frac{\partial i}{\partial x} = 0, \qquad t \succ 0, \quad 0 \le x \le l \tag{1}$$

where l is the length of the line, L and C' are the inductance and capacitance of the conductor per unit length, R is the resistance at the input and C_d is the capacitance in parallel with the tunnel diode.

The conditions cited in [1] give the following neutral linear system:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Psi}(\Psi) - D\frac{\mathrm{d}\Gamma}{\mathrm{d}\Psi}\left(\Psi - r'\right) = R\Gamma(\Psi) + S\Gamma\left(\Psi - r'\right) \tag{2}$$

where

$$R = \frac{m - \sqrt{\frac{C'}{L}}}{C_d}, \quad S = -\frac{\left(\sqrt{\frac{L}{C'}} - R_0\right)\left(m + \sqrt{\frac{C'}{L}}\right)}{\left(\sqrt{\frac{L}{C'}} + R_0\right)C_d}$$

$$D = \frac{\left(\sqrt{\frac{L}{C'}} - R_0\right)}{\left(\sqrt{\frac{L}{C'}} + R_0\right)}, \quad r' = 2l\sqrt{LC'}$$

$$\Psi = t - bl, \quad b = \sqrt{LC'}$$

m and R_0 are respectively a positive constant and an equilibrium resistance.

The neutral variable delay system can be written as follows:

$$\begin{cases} \dot{x}(t) = A_1x(t) + A_2x(t - s(t)) \\ \qquad + A_3\dot{x}(t - s(t)) + Bu(t) \\ y(t) = Cx(t) \\ x(t) = \phi(t), \quad t \in [-s, 0] \end{cases} \tag{3}$$

where

$$A_1 = R, \quad A_2 = S, \quad A_3 = D,$$
$$s(t) = bl + r' = 3l\sqrt{LC'}.$$

$x(t) \in R^n$, $u(t) \in R^m$ and $y(t) \in R^p$ are respectively the state, the input and the output vector.

A_1, A_2, A_3, B and C are constant matrices. $\phi(t)$ is an initial function.

The variation in delay is the consequence of the variation in line length.

3. Main results

The following lemmas are very useful for the main results of this work.

Lemma 3.1 (Schur complement): *Consider constant matrices $\omega_1, \omega_2, \omega_3$ with $\omega_1 = \omega_1^T$ and $\omega_2 = \omega_2^T \succ 0$ then, $\omega_1 + \omega_3^T\omega_2^{-1}\omega_3 \prec 0$ if:*

$$\begin{bmatrix} \omega_1 & \omega_3^T \\ \omega_3 & -\omega_2 \end{bmatrix} \prec 0 \quad \text{or} \quad \begin{bmatrix} -\omega_2 & \omega_3 \\ \omega_3^T & \omega_1 \end{bmatrix} \prec 0.$$

Lemma 3.2: *Increase Lemma:*

Given scalar X and Y two matrices of appropriate dimensions, the following inequality is always true for any matrix $Q > 0$:

$$XY^T + YX^T \le XQX^T + YQ^{-1}Y^T. \tag{4}$$

3.1. Synthesis of an FTC, sensor fault case

Consider the following neutral variable time delay system (3) with sensor fault:

$$\begin{cases} \dot{x}(t) = A_1x(t) + A_2x(t - s(t)) \\ \qquad + A_3\dot{x}(t - s(t)) + Bu(t) \\ y(t) = Cx(t) + F_sf_s(t) \\ x(t) = \phi(t), \quad t \in [-s, 0] \end{cases} \tag{5}$$

where $f_s(t)$ is the sensor fault and F_s is a constant matrix, and it is assumed that the pair (A_i, C) is observable, $i = 1, 2, 3$.

3.1.1. Synthesis of a nominal control

The nominal control law remains to maintain stability of closed-loop system.

A nominal control in case of sensor fault noted $u_s^{nom}(t)$ which is represented as follows:

$$u_s^{nom}(t) = -Ky(t), \tag{6}$$

where K is the feedback matrix gain. The pole placement method is used to determine K.

3.1.2. A strategy of additive control

For the sensor free fault system (3), the nominal control law remains to maintain stability of the closed-loop system which satisfies Equation (6).

When the fault occurs, an additive control becomes necessary to compensate the fault effect.

For this reason, we suggest adding a new term $u_s^{add}(t)$ to the nominal law to compensate the fault effect on closed loop system behaviour.

The control, to be effectively applied to the neutral variable time delay system can be written as

$$u_s(t) = u_s^{nom}(t) + u_s^{add}(t) . \tag{7}$$

The closed-loop state equation can be rewritten as follows:

$$\begin{aligned}\dot{x}(t) = {} & (A_1 - BKC)\,x(t) + A_2 x(t - s(t)) \\ & + A_3 \dot{x}(t - s(t)) \\ & + B u_s^{add}(t) + BF_s f_s(t) ,\end{aligned} \tag{8}$$

where $u_s^{add}(t)$ satisfies the following equation:

$$B u_s^{add}(t) + BF_s f_s(t) = 0 \tag{9}$$

then

$$u_s^{add}(t) = -F_s f_s(t) \tag{10}$$

or the adaptive observer (21) is trying to estimate the fault such that $\lim_{t\to\infty} \hat{f}_s = f_s$.
where $\hat{f}_s$ is the estimate of sensor fault f_s.

Also, the error fault equation is described as follows:

$$e_f(t) = \hat{f}_s(t) - f_s(t) \tag{11}$$

And, we knew $\lim_{t\to\infty} \hat{f}_s = f_s$, so:

$$e_f(t) = \hat{f}_s(t) - f_s(t) = 0$$

And $\hat{f}_s(t) = f_s(t)$.

Therefore, we can replace the sensor fault by its estimation in the equation (10), which gives

$$u_s^{add}(t) = -F_s \hat{f}_s(t) . \tag{12}$$

3.2. Synthesis of an FTC, actuator fault case

Consider the following neutral variable time delay system (3) with actuator fault:

$$\begin{cases} \dot{x}(t) = A_1 x(t) + A_2 x(t - s(t)) \\ \qquad\qquad + A_3 \dot{x}(t - s(t)) + Bu(t) + F_a f_a(t) \\ y(t) = Cx(t) \\ x(t) = \phi(t) ; \quad t \in [-s, 0] , \end{cases} \tag{13}$$

where $f_a(t)$ is actuator fault and F_a is a constant matrix.

3.2.1. Nominal control

The following state feedback controller proposed by (2) is designed to stabilize the closed loop system:

$$u_a^{nom}(t) = -Kx(t) . \tag{14}$$

3.2.2. Additive control

For the case of actuator fault, the proposed control is also written as follows:

$$u_a(t) = u_a^{nom}(t) + u_a^{add}(t) , \tag{15}$$

where $u_a^{add}(t)$ satisfies the following equation:

$$B u_a^{add}(t) + F_a f_a(t) = 0 \tag{16}$$

which gives

$$u_a^{add}(t) = -B^+ F_a f_a(t) \tag{17}$$

where B^+ is the pseudo-inverse of matrix B.

In the same way, we have

$$e_f(t) = \hat{f}_a(t) - f_a(t) \tag{18}$$

$\hat{f}_a(t)$ is the estimate actuator fault, and we knew $\lim_{t\to\infty} \hat{f}_a = f_a$, so

$$\hat{f}_a(t) = f_a(t) . \tag{19}$$

So, we find the following equation:

$$u_a^{add}(t) = -B^+ F_a \hat{f}_a(t) . \tag{20}$$

The FTC law synthesis needs the determination of sensor and actuator fault estimates respectively $\hat{f}_s(t)$ and $\hat{f}_a(t)$. For this reason, an adaptive observer will be used in the following paragraph to detect and estimate these faults.

3.3. Conception of an adaptive observer

The following adaptive observer generates residuals in order to detect and estimate the fault:

3.3.1. Adaptive observer in the case of sensor fault

Consider the adaptive observer in the case of sensor fault as follows:

$$\begin{cases} \dot{\hat{x}}(t) = A_1 \hat{x}(t) + A_2 \hat{x}(t - s(t)) \\ \qquad\qquad + A_3 \dot{\hat{x}}(t - s(t)) + Bu(t) \\ \quad + L\left[y(t) - \hat{y}(t)\right] \\ \quad + H\left[y(t - s(t)) - \hat{y}(t - s(t))\right] \\ \hat{y}(t) = C\hat{x}(t) + F_s \hat{f}_s(t) \\ r(t) = y(t) - \hat{y}(t) . \end{cases} \tag{21}$$

Based on [3], consider the estimate sensor fault as follows:

$$\hat{f}_s(t) = \Gamma V_1 \left[r(t) + \sigma \int r(t)\,\mathrm{d}t \right] , \tag{22}$$

where $\hat{x}(t)$ and $\hat{y}(t)$ represent respectively the estimate state, the output vector and $r(t)$ is the residual vector.

Γ, V_1 are respectively a positive definite and an arbitrary given matrices with appropriate dimension, σ is a given scalar.

A theorem is proposed in (2) to determine the gains of the observer L and H.

Theorem 3.1: *Given scalar $h>0$ and if there exist matrices P_1, Q, W,Z_1, Z_2, Z_3,$G\in R^{n*n}$, and Y_1,$Y_2 \in R^{n*p}$,M_i, $N_i \in R^{n*n}$, $i = 1,.., 4$, such that:*

$$\begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & hM_1 & hN_1 & \phi_{18} & \phi_{19} & \phi_{110} \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & hM_2 & hN_2 & \phi_{28} & \phi_{29} & \phi_{210} \\ * & * & \phi_{33} & A_3^T N_4^T & 0 & hM_3 & hN_3 & 0 & 0 & 0 \\ * & * & * & -Z_1 & -A_3^T P_1 F_s & hM_4 & hN_4 & A_3^T P_1 & hA_3^T P_1 & 0 \\ * & * & * & * & \phi_{55} & 0 & 0 & F_s^T P_1 & hF_s^T P_1 & hF_s^T P_1 \\ * & * & * & * & * & -hZ_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -hZ_3 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -P_1 Z_1^{-1} P_1 & 0 & 0 \\ * & * & * & * & * & * & * & * & -hP_1 Z_2^{-1} P_1 & 0 \\ * & * & * & * & * & * & * & * & * & -hP_1 Z_3^{-1} P_1 \end{bmatrix} \prec 0 \tag{23}$$

where

$$\phi_{11} = P_1 A_1 + A_1^T P_1 - Y_1 C - C^T Y_1^T + Q + M_1 + M_1^T + N_1 + N_1^T$$
$$\phi_{12} = P_1 A_2 - Y_2 C - A_1^T P_1 A_3 + C^T Y_1^T A_3 - M_1 + M_2^T - N_1 (A_3 + I) + N_2^T$$
$$\phi_{13} = M_3^T + N_1 A_3 + N_3^T$$
$$\phi_{14} = M_4^T + N_4^T$$
$$\phi_{15} = -A_1^T P_1 F_s + C^T Y_1^T F_s$$
$$\phi_{18} = A_1^T P_1 - C^T Y_1^T$$
$$\phi_{19} = \phi_{110} = hA_1^T P_1 - hC^T Y_1^T$$
$$\phi_{22} = -A_3^T P_1 A_2 - A_3^T P_1 A_3 + A_2^T Y_2 C + C^T Y_2^T A_3 - Q + W - M_2 - M_2^T - N_2 (A_3 + I) - (A_3 + I)^T N_2^T$$
$$\phi_{23} = -M_3^T + N_2 A_3 - (A_3 + I)^T N_3^T$$
$$\phi_{24} = -M_4^T - (A_3 + I)^T N_4^T$$
$$\phi_{25} = -A_2^T P_1 F_s - A_2^T P_1 F_s + C^T Y_2^T F_s$$
$$\phi_{28} = A_2^T P_1 - C^T Y_2^T$$
$$\phi_{29} = \phi_{210} = hA_3^T P_1 - hC^T Y_2^T$$
$$\phi_{33} = -W + N_3 A_3 + A_3^T N_3^T$$
$$\phi_{55} = -2{F_s}^T P_1 F_s + G$$

then

$$\mathrm{L} = \mathrm{Y}_1 P_1^{-1} \tag{24}$$
$$\mathrm{H} = \mathrm{Y}_2 P_1^{-1}. \tag{25}$$

3.3.2. Adaptive observer in the case of actuator fault

Consider the adaptive observer in the case of actuator fault to detect and estimate the fault as follows:

$$\begin{cases} \dot{\hat{x}}(t) = A_1 \hat{x}(t) + A_2 \hat{x}(t - s(t)) \\ \qquad + A_3 \dot{\hat{x}}(t - s(t)) + Bu(t) \\ \qquad + F_a \hat{f}_a(t) + L\left[y(t) - \hat{y}(t)\right] \\ \qquad + H\left[y(t - s(t)) - \hat{y}(t - s(t))\right] \\ \hat{y}(t) = C\hat{x}(t) \\ r(t) = y(t) - \hat{y}(t) \\ \dot{\hat{f}}_a(t) = -\delta_a F_a r(t), \end{cases} \tag{26}$$

where δ_a is a positive definite matrix, it is the learning rate.

The following theorem is considered to find δ_a:

Theorem 3.2: *Given positive scalar β, if there exist $P>0$, $Q>0$, R, M and V positive matrices such that:*

$$\begin{bmatrix} 2P(A_1 - LC) + C^T Z C + Q & P(A_2 - HC) & PA_3 + (1-\beta) A_1^T R A_3 & -PF_a \\ * & -(1-\beta) Q & (1-\beta) RA_3 & 0 \\ * & * & A_3^T (1-\beta) RA_3 - R & 0 \\ * & * & * & M + \delta_a^2 \end{bmatrix} \prec 0 \tag{27}$$

where

$$Z = V^T V.$$

Proof: The error dynamic is described by

$$\begin{cases} \dot{e}_x(t) = (A_1 - LC) e_x(t) + A_3 \dot{e}_x(t - s(t)) \\ \qquad + (A_2 - HC) e_x(t - s(t)) + F_a e_f(t) \\ e_y(t) = Ce_x(t), \end{cases} \tag{28}$$

where

$$e_x(t) * = \hat{x}(t) - x(t)$$
$$e_y(t) * = \hat{y}(t) - y(t)$$
$$e_f(t) * = \hat{f}_a(t) - f_a(t).$$

The reference model is

$$\begin{cases} \dot{e}_{ref}(t) = (A_1 - LC)\, e_{ref}(t) \\ \qquad\qquad + (A_2 - HC)\, e_{ref}(t - s(t)) \\ \qquad\qquad + A_3 \dot{e}_{ref}(t - s(t)) + F_a e_f(t) \\ r_{ref}(t) = VCe_{ref}(t) \\ e_{ref}(0) = e_{ref_0}; \quad t \leq 0. \end{cases} \tag{29}$$

The Lyapunov functional is chosen as

$$\begin{aligned} & V\left(e_{ref}, t\right) \\ & = e_{ref}^T(t)\, Pe_{ref}(t) + \int_{t-s(t)}^{t} e_{ref}^T(\theta)\, Qe_{ref}(\theta)\, \mathrm{d}\theta \\ & \quad + \int_{t-s(t)}^{t} \dot{e}_{ref}^T(\theta)\, R\dot{e}_{ref}(\theta)\, \mathrm{d}\theta \\ & \quad + \int_{-s(t)}^{0} \left(\int_{t-s(t)}^{t} \dot{e}_{ref}^T(\theta)\, R\dot{e}_{ref}(\theta)\, \mathrm{d}\theta \right) \mathrm{d}\sigma \\ & \quad + e_f^T(t)\, \delta_a^{-1} e_f(t). \end{aligned} \tag{30}$$

The derivative of this functional is

$$\begin{aligned} & \dot{V}(e_{ref}, t) \\ & = \Big[2e_{ref}^T(t)P(A_1 - LC)e_{ref}(t) \\ & \quad + 2e_{ref}^T(t)P(A_2 - HC)e_{ref}(t - s(t)) \\ & \quad + +2e_{ref}^T(t)PA_3\dot{e}_{ref}(t - s(t)) \Big] \\ & \quad - 2e_{ref}^T(t)PF_a e_f(t) + e_{ref}^T(t)Qe_{ref}(t) \\ & \quad - (1 - s(t))e_{ref}^T(t - s(t))Qe_{ref}(t - s(t)) \\ & \quad + 2e_{ref}^T(t)A_1^T(1 - s(t))RA_3\dot{e}_{ref}(t - s(t)) \\ & \quad + 2e_{ref}^T(t - s(t))(1 - s(t))RA_3\dot{e}_{ref}(t - s(t)) \\ & \quad + \dot{e}_{ref}^T(t - s(t))A_3^T(1 - s(t))RA_3\dot{e}_{ref}(t - s(t)) \\ & \quad - \dot{e}_{ref}^T(t - s(t))R\dot{e}_{ref}(t - s(t)) - 2e_f^T(t)FVCe_{ref}(t) \\ & \quad - 2e_f^T(t)\delta_a^{-1}\dot{\hat{f}}_a(t). \end{aligned}$$

The derivative of the estimate actuator fault is given by

$$\dot{\hat{f}}_a = -\delta_a F_a r_{ref}. \tag{31}$$

By using Lemma 3.2, we end up with the following inequality:

$$\begin{aligned} & -2e_f^T(t)\, \delta_a^{-1}\dot{f}_a(t) \leq e_f^T(t)\, Me_f(t) + \dot{f}_a(t) \\ & \quad \leq e_f^T(t)\, Me_f(t) + {f_a}^2 \lambda_{\max}\left(\delta_a^{-1} M^{-1} \delta_a^{-1}\right). \end{aligned} \tag{32}$$

Thus, the derivative of the Lyapunov functional is increased as follows:

$$\begin{aligned} & \dot{V}\left(e_{ref}, t\right) \\ & \leq \Big[2e_{ref}^T(t)\, P(A_1 - LC)\, e_{ref}(t) \\ & \quad + 2e_{ref}^T(t)\, P(A_2 - HC)\, e_{ref}(t - s(t)) \\ & \quad + 2e_{ref}^T(t)\, PA_3\dot{e}_{ref}(t - s(t)) \Big] \\ & \quad - 2e_{ref}^T(t)\, PF_a e_f(t) + e_{ref}^T(t)\, Qe_{ref}(t) \\ & \quad - (1 - \beta)\, e_{ref}^T(t - s(t))\, Qe_{ref}(t - s(t)) \\ & \quad + 2e_{ref}^T(t)\, A_1^T (1 - \beta)\, RA_3\dot{e}_{ref}(t - s(t)) \\ & \quad + 2e_{ref}^T(t - s(t))\, (1 - \beta)\, RA_3\dot{e}_{ref}(t - s(t)) \\ & \quad + \dot{e}_{ref}^T(t - s(t))\, A_3^T (1 - \beta)\, RA_3\dot{e}_{ref}(t - s(t)) \\ & \quad - \dot{e}_{ref}^T(t - s(t))\, R\dot{e}_{ref}(t - s(t)) + e_f^T(t)\, Me_f(t) \\ & \quad + {f_a}^2 \lambda_{\max}\left(\delta_a^{-1} M^{-1} \delta_a^{-1}\right). \end{aligned} \tag{33}$$

Let

$$J = r_{ref}^T(t)\, r_{ref}(t) + \delta_a^2 {f_a}^T(t)\, f_a(t) + \dot{V}\left(e_{ref}, t\right) \prec 0, \tag{34}$$

where

$$r_{ref}^T(t)\, r_{ref}(t) = e_{ref}^T(t)\, C^T ZCe_{ref}(t). \tag{35}$$

Then, the addition between (33) and (35), Equation (34) becomes as follows:

$$\begin{aligned} J & \leq e_{ref}^T(t)C^T ZCe_{ref}(t) + \Big[2e_{ref}^T(t)P(A_1 - LC)e_{ref}(t) \\ & \quad + 2e_{ref}^T(t)(A_2 - HC)e_{ref}(t - s(t)) \\ & \quad + 2e_{ref}^T(t)PA_3\dot{e}_{ref}(t - s(t)) \Big] \\ & \quad - 2e_{ref}^T(t)PF_a e_f(t) + e_{ref}^T(t)Qe_{ref}(t) \\ & \quad - (1 - \beta)e_{ref}^T(t - s(t))Qe_{ref}(t - s(t)) \\ & \quad + 2e_{ref}^T(t)A_1^T(1 - \beta)RA_3\dot{e}_{ref}(t - s(t)) \\ & \quad + 2e_{ref}^T(t - s(t))(1 - \beta)RA_3\dot{e}_{ref}(t - s(t)) \\ & \quad + \dot{e}_{ref}^T(t - s(t))A_3^T(1 - \beta)RA_3\dot{e}_{ref}(t - s(t)) \\ & \quad - \dot{e}_{ref}^T(t - s(t))R\dot{e}_{ref}(t - s(t)) + e_f^T(t)Me_f(t) \\ & \quad + {f_a}^2\lambda_{\max}(\delta_a^{-1}M^{-1}\delta_a^{-1}) + \delta_a^2 {f_a}^T(t)f_a(t) \\ & \leq e_{ref}^T(t)\left[2P(A_1 - LC) + C^T ZC + Q \right] e_{ref}(t) \\ & \quad + 2e_{ref}^T(t)\, P(A_2 - HC)\, e_{ref}(t - s(t)) \\ & \quad + 2e_{ref}^T(t)\left[PA_2 + (1 - \beta)\, A_1^T RA_3 \right] \dot{e}_{ref}(t - s(t)) \\ & \quad - 2e_{ref}^T(t)\, PF_a e_f(t) - (1 - \beta)\, e_{ref}^T(t - l(t)) \\ & \quad\; Qe_{ref}(t - s(t)) \\ & \quad + 2e_{ref}^T(t - s(t))\, (1 - \beta)\, RA_3\dot{e}_{ref}(t - s(t)) \\ & \quad + \dot{e}_{ref}^T(t - s(t)) \left[A_3^T (1 - \beta)\, RA_3 - R \right] \\ & \quad\; \dot{e}_{ref}(t - s(t)) \\ & \quad + e_f^T(t)\left(M + \delta_a^2 \right) e_f(t) + {f_a}^2\lambda_{\max}\left(\delta_a^{-1}M^{-1}\delta_a^{-1}\right) \end{aligned} \tag{36}$$

$$J \leq \Delta^T \Xi \Delta \prec 0, \tag{37}$$

where

$$\Delta = \left[e_{ref}^T(t) \quad e_{ref}^T(t - s(t)) \quad \dot{e}_{ref}^T(t - s(t)) \quad e_f^T(t) \right] \tag{38}$$

and

$$\Xi = \begin{bmatrix} 2P(A_1 - LC) + C^T ZC + Q & P(A_2 - HC) & PA_3 + (1-\beta) A_1^T RA_3 & -PF_a \\ * & -(1-\beta) Q & (1-\beta) RA_3 & 0 \\ * & * & A_3^T (1-\beta) RA_3 - R & 0 \\ * & * & * & M + \delta_a^2 \end{bmatrix} \tag{39}$$

■

4. Simulation results

In this section, the transmission line is considered with the form (3). This system is described by the following matrices:

$$A_1 = -1; \quad A_2 = -0.5; \quad A_3 = 0.5; \quad B = 1$$
$$C = 1; \quad F_a = 1; \quad F_s = 1.$$
$$s(t) = 0.1\sin(10t) + 0.5$$

$$f_a(t) = \begin{cases} 0.2\sin(5t) + 0.5; & 20 \leq t \leq 40 \\ 0; & \text{otherwise.} \end{cases}$$

$$f_s(t) = \begin{cases} 0.5\sin(t) + 0.1; & 30 \leq t \leq 50 \\ 0; & \text{otherwise.} \end{cases}$$

From Theorem 3.1 and 3.2, we obtain respectively

$$L = 0.0248 \quad \text{and} \quad H = -0.2574$$

and

$$\delta_a = 3.8729.$$

We obtained the feedback matrix gains by pole statement as follows:

$$K = 0.5$$

Figures 2 and 3 show that the residual is sensible to the fault (in the case of actuator and sensor fault). Figures 4 and 5 show that the adaptive observer can enhance the performance of fault estimation.

In conclusion, the proposed observer, based on fault tolerant additive control strategy, is considered as a good fault compensator for neutral time-delay systems. The robust and PIM methods in [1] achieve the stability and the performance system in closed loop. But the robust controller, Figure 6, cannot achieve the final value ($\simeq 0.6$). Using the PIM method, Figure 7, the error between the faulty system and the reference one

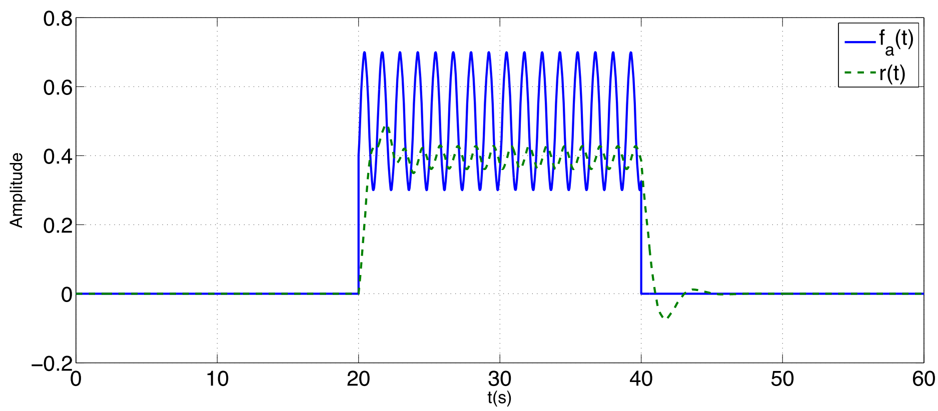

Figure 2. Residual and actuator fault signal.

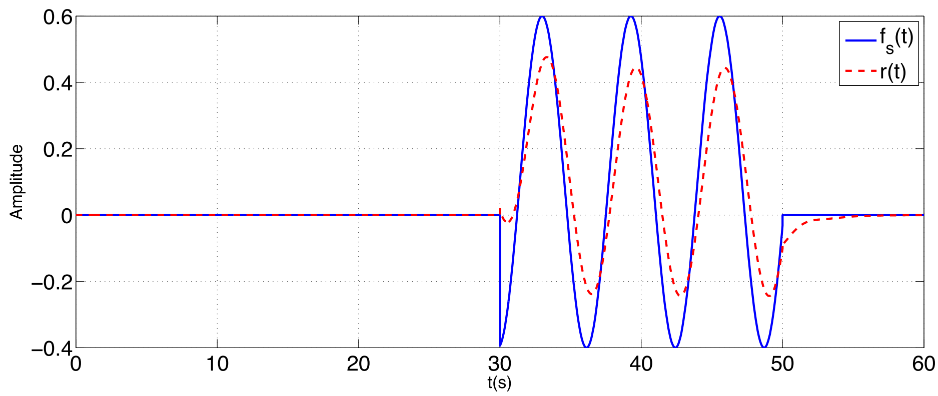

Figure 3. Residual and sensor fault signal.

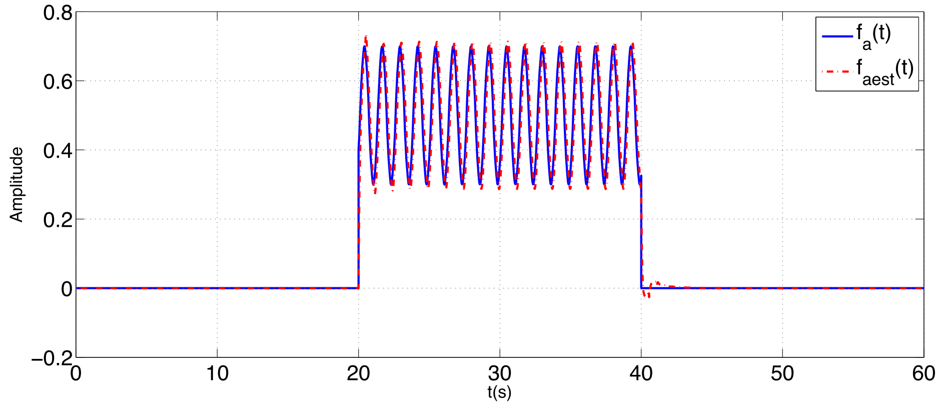

Figure 4. Actuator fault and their estimate signal.

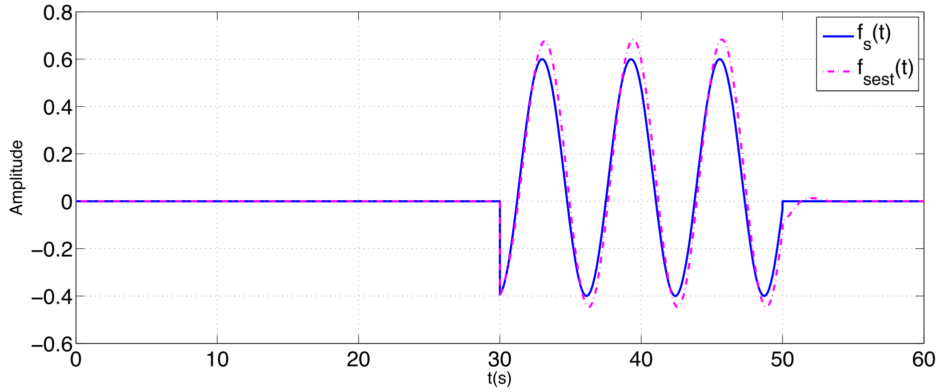

Figure 5. Sensor fault and their estimate signal.

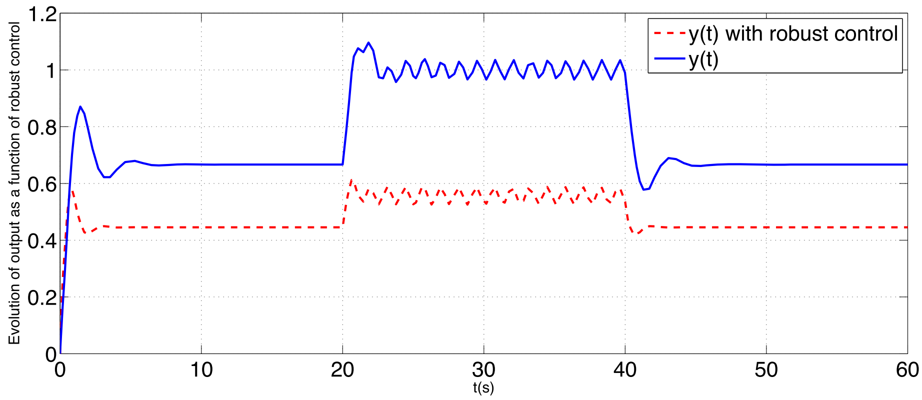

Figure 6. Output signal with robust control law.

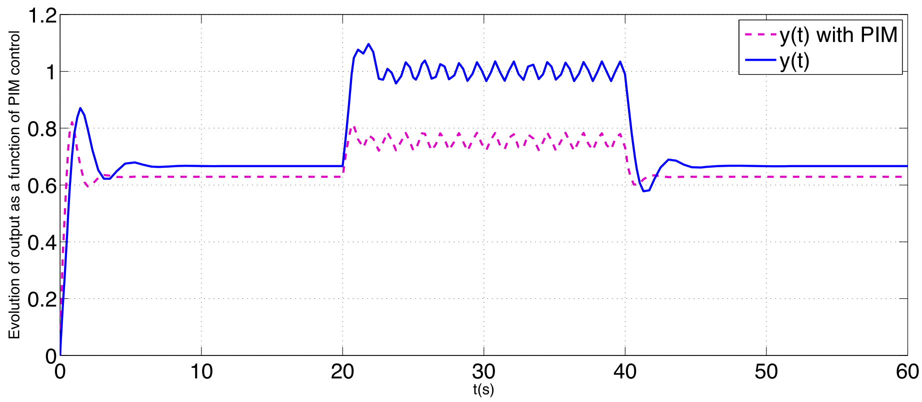

Figure 7. Output signal with PIM control.

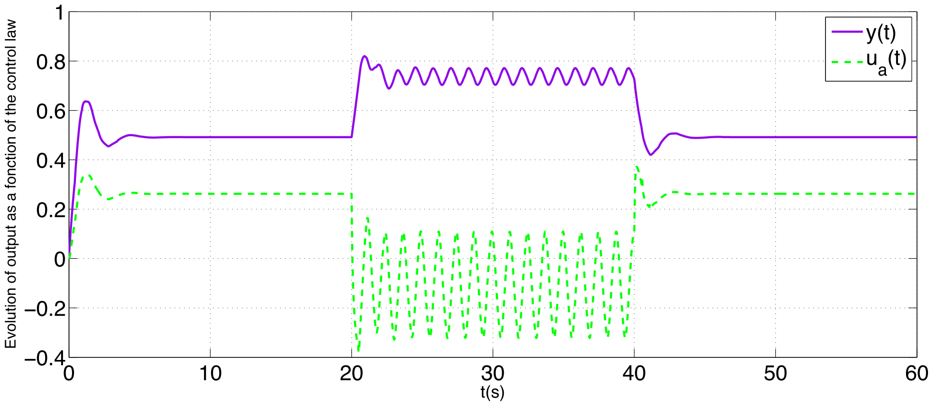

Figure 8. Output signal with $u_a(t)$ control law.

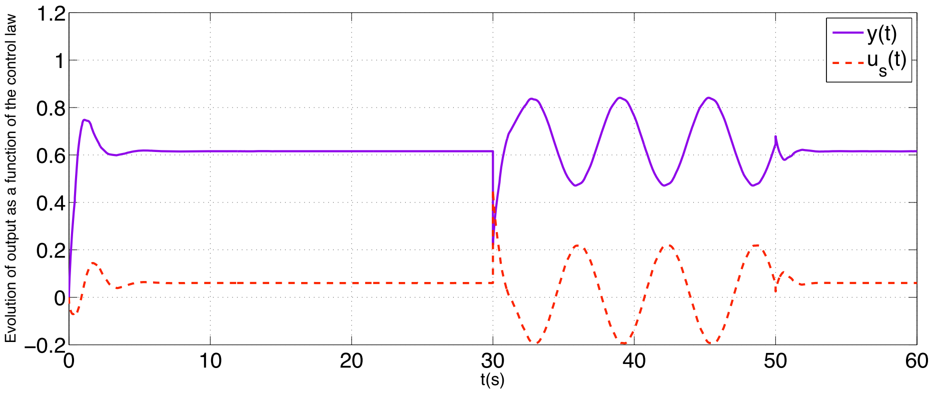

Figure 9. Output signal with $u_s(t)$ control law.

is achieved, so the error can be minimized and the convergence to the final value is obtained.

Unlike the PIM method, the additive control does not require a reference model. It also allows the fault to be detected and estimated online.

The effectiveness of the proposed additive control strategy is thus noted inspite of the presence of an actuator and sensor fault. It emphasizes the ability of this strategy to maintain closed-loop performance (Figures 8 and 9). In addition, it minimizes the fault amplitude better than the PIM method (amplitude in PIM $\simeq 1$) and with additive control in Figure 8, ($0.6 <$ amplitude < 0.8).

The strategy developed, therefore, shows its ability to maintain performance in terms of compensation for the considered fault.

5. Conclusion

In this work, the transmission line is considered as a neutral variable delay system. The FTC is composed of two terms as presented in Section 3.

This type of control is better than the PIM and the robust control. In fact, it is applicable for the sensor fault and it guarantees the desired performance of the neutral variable time-delay case systems.

Simulation results demonstrate the effectiveness of the control strategies developed for the neutral variable delay transmission line system.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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