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Stabilization analysis of a class of nonlinear time delay systems with time-varying full-state constraints

Youguo He[a], Yu Zhou[a], Dapeng Wang[b], Shaogang Liu[b] and Xiuping Wei[c]

[a]Automotive Engineering Research Institute, Jiangsu University, Zhenjiang, People's Republic of China; [b]College of Mechanical and Electrical Engineering, Harbin Engineering University, Harbin, People's Republic of China; [c]Zhenjiang Dongfang Electric Heating Technology Co., Ltd, Zhenjiang, People's Republic of China

ABSTRACT

In this paper, a novel tracking control strategy is proposed to address the problem of stabilization of a class of nonlinear time delay systems with time-varying full-state constraints. The effect of the nonlinear system resulting from the time delays is canceled out with the utilization of the novel iterative procedures optimized by dynamic surface control (DSC) and the appropriate time-varying asymmetric barrier Lyapunov functions (ABLFs) are employed to stem the violation of time-varying states constraints. Finally, it is proved that the proposed control method guarantees the uniformly ultimate boundedness of all the signals in the closed-loop system, meanwhile, the tracking errors converge to a small interval. The effectiveness of the presented control strategy is confirmed by a simulation example provided in this paper.

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Time delay systems; time-varying full-state constraints; dynamic surface control; barrier Lyapunov functions

1. Introduction

Constraints are significant factors leading to instability and undesirable performance existing ineluctably in a wide variety of practical engineering systems. Among the numerous effective research results on constraint-handling approaches promoted by practical requirements, such as reference governors [1,2] model predictive control [3,4] and the set invariance notions [5], the barrier Lyapunov function (BLF) is the leading method to handle the constraint problems. An adaptive controller based on asymmetrical Barrier Lyapunov Functions was first proposed in [6] to handle parametric uncertainties meanwhile deter constraints from being breached. Subsequently, some different forms of novel adaptive controllers based on Integral Barrier Lyapunov Functionals (IBLF) and tangent BLFs (TBLFs) were proposed for nonlinear single-input single-output (SISO) output-constrained systems in [7–9] respectively. A BLF-based backstepping was presented in [10] for strict-feedback systems with partial state constraints to deal with the control design problem. The full state constraint control problem of nonlinear systems in pure-feedback was addressed in [11,12]. Relying on the combination of neural networks (NNs) and BLF, adaptive controllers were designed in [13–15] for full state constrained systems. Meanwhile, time delay, which has an impact that can not be neglected on the performance of control systems and even leads to the deterioration of system stability, is also frequently encountered in many practical engineering systems, such as spacecraft attitude control systems [16], active suspension system, vehicle control system [17,18], microwave oscillators.

The issue of time-delay systems has received considerable critical attention. With regard to the robust stability proof for systems affected by time delay, two theorems are employed by a large proportion of effective methods, one is the Lyapunov–Krasovskii theorem [19–23], the other is the Lyapunov–Razumikhin theorem [24–27], and a great deal of research based on Lyapunov–Krasovskii functionals (LKFs) and Lyapunov–Razumikhin functions (LRFs) has been done on how to deal with time delay problems. In [28], LRF was combined with an adaptive stability control scheme designed by backstepping to process a class of nonlinear time-delay systems with a state feedback control problem. In [29], aiming at the input-to-state stability (ISS) of nonlinear time-delay systems, a class of LRFs with a more relaxed requirement on derivative was constructed for the first time. Based on this, the stability of a class of event-triggered stabilization of switched nonlinear time-varying systems was studied. An iterative robust controller for a class of SISO nonlinear time-delay systems had been designed with the novel use of Lyapunov–Krasovskii functionals see [30]. In [31,32], unknown functions containing time-delayed states could be resolved into a series of continuous functions with the utilization of the separation technique, which canceled out the constraints of assumption about the functions containing delayed states. The

CONTACT Youguo He hyg197715@163.com Automotive Engineering Research Institute, Jiangsu University, Zhenjiang 212013, People's Republic of China

desired control performance of nonlinear time-delay systems was achieved in [31,33] due to the combination of LKFs and adaptive NNs backstepping. A class of strict-feedback nonlinear systems with time delays were presented in [32,34,35] and adaptive fuzzy control based on the backstepping technique as well as LKFs had been developed to handle the problem of output tracking. In practice, time delay and state constraints usually exist in practical engineering systems simultaneously, how to address the control problem of nonlinear systems with both the state constraints and time delay remains urgent.

It is noteworthy that the problem of symmetric constant state constraints in time delay systems can be solved by using the tangent barrier Lyapunov function (TBLF) in [9], or NNs based on BLF in [31,33,36]. However, symmetric and non-time-varying state constraints considered in [30,31,33,36] were just some special cases of our scheme and may not meet the requirements when applied to the actual engineering systems. And the method proposed in [30,31,32,33], which used a traditional backstepping algorithm, suffered from a large amount of calculation caused by repetitive differentiations. To overcome these problems, the dynamic surface control (DSC) technique, which obtained the information of the virtual control by introducing a first-order low-pass filter at each step of the design procedure was presented in [37,38] and adopted in [39–43]. Therefore, through the study of previous papers, it can be concluded that there are few works combining DSC technology to solve the problem of time delay and time-varying full-state constraint at the same time, which motivates our research.

In this paper, the time-varying asymmetric Barrier Lyapunov Functionals (ABLF)-based control is employed to stabilize a class of nonlinear strict-feedback systems with the time-delays and the time-varying full state constraints. The main contributions of this paper are summarized as follows: (i) Based on the separation technique, a novel unified framework is proposed, in the meantime, time delay and the full state time-varying constraint problems are taken into account more comprehensively in this current work, which means that the limitation on initial conditions can be relaxed and is able to satisfy the constraint requirements of the state variables better in practical engineering systems. (ii) Compared with approaches using traditional iterative backstepping, repetitive differentiation of stabilizing functions, which will lead to tedious and complicated calculation, especially in higher order systems, can be averted by choosing the DSC strategy. (iii) With the utilization of the time-varying asymmetric BLFs, all the states are always within the prescribed time-varying scopes.

The rest of this paper is organized as follows: Section 2 formulates the problem, and some necessary preliminary knowledge is provided in this section as well. The design process of the controller and stability analysis are given in Section 3. The effectiveness of the proposed approach is illustrated by the example provided in Section 4. The conclusions are drawn in Section 5.

2. Problem formulations and preliminaries

2.1. Problem formulation

Consider the following strict-feedback nonlinear time-delay systems with output constraints

$$\begin{cases} \dot{x} = f_1(x_1) + g_1(x_1)x_2(t) \\ \qquad +h_1(x_1(t-\tau_1)) \\ \dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1}(t) \\ \qquad +h_i(\bar{x}_i(t-\tau_i)), \qquad i = 2, \cdots n-1 \\ \dot{x}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n)u(t) \\ \qquad +h_n(\bar{x}_n(t-\tau_n)) \\ y = x_1 \end{cases} \quad (1)$$

where $\bar{x}_i = [x_1, x_2, \cdots x_i]^T \in R^n, i = 1, 2, \ldots n$ represents system states, $y \in R, u \in R$ are system output and control input respectively; $g_i(\bar{x}_i^T)$ and $f_i(\bar{x}_i)$ are smooth functions; $h_i(\cdot)$ are unknown smooth functions affected by time delays; $\tau_i, i = 1, 2, \cdots n$ denote time-delay constants. The control objective of this paper is to design an adaptive controller u for system (1) to ensure that the system output y tracks the desired trajectory y_d, while all signals in the close-loop system remain bounded and all the corresponding constraints $\underline{k}_{ci} < x_i < \bar{k}_{ci}, \forall t \geq 0$ are not violated.

The necessary Assumptions for establishing constraint satisfaction and performance bounds are as follows.

Assumption 2.1 ([31]): For the uncertain non-linear time-delay functions, the following inequality holds

$$|h_i(\bar{x}_i(t-\tau_i))| \leq \sum_{j=1}^{i} |s_j(t-\tau_i)| q_{ij}(\bar{s}_j(t-\tau_i)) \quad (2)$$

where $q_{ij}(\cdot)$ are known continuous functions, $\bar{s}_i(t) = [s_1(t), s_2(t), \cdots s_i(t)]^T$.

Assumption 2.2 ([39]): The functions $g_i(\bar{x}_i), i = 1, \ldots, n$ are positive and there exists a class of positive constants $g_{i\min}$ and $g_{i\max}$ such that $0 < g_{i\min} \leq g_i(\bar{x}_i) \leq g_{i\max}$.

Assumption 2.3 ([11]): There exist constants $\underline{K}_{ci}$, $\bar{K}_{ci}$, $\underline{d}_{c_{ij}}, \bar{d}_{c_{ij}} i, j = 1, \ldots n$ satisfying $\underline{k}_{ci}(t) > \underline{K}_{ci}$ and $\bar{k}_{ci}(t) \leq \bar{K}_{ci}$ and their time derivatives satisfy $|\underline{k}_{ci}^{(j)}(t)| \geq \underline{d}_{c_{ij}}$, $|\bar{k}_{ci}^{(i)}(t)| \leq \bar{d}_{c_{ij}}$.

Assumption 2.4 ([11]): There exist function $\underline{Y}_0(t) : R_+ \to R_+, \bar{Y}_0(t) : R_+ \to R_+$ satisfy $\underline{Y}_0(t) > \underline{k}_{c1}(t)$ and

$\bar{Y}_0(t) < \bar{k}_{c1}(t)$, $\forall t > 0$. Furthermore, there exist positive constants Y_i, $i = 1, \ldots, n$ such that the reference signal $y_d(t)$ and its time derivatives satisfy $\underline{Y}_0(t) \leq y_d(t) \leq \bar{Y}_0(t)$ and $|y_d^i(t)| \leq Y_i, \forall t > 0$.

2.2. Preliminaries

Lemma 2.1 ([44]): *For all $|\xi| < 1$ and any positive integer p, the inequality $\log \frac{1}{1-\xi^p} < \frac{1}{1-\xi^p}$ holds in the set $\eta \in N$, and σ, c are positive constants, then $s_i(t)$ remain in the open set $S_i, \forall t \in [0, \infty)$.*

Lemma 2.2 ([31]): *For $1 \leq j \leq n$, define the set Ω_{c1} as $\Omega_{c1} = \{s_j||s_j| < 0.5549v_j\}$ with v_j is a design constant.*

Lemma 2.3: *Even function $k(\cdot) : R \to R$*

$$k(x) = \frac{x^2 \cosh(x)}{1 + x^2}, \forall x \in R \tag{3}$$

is continuous, and monotonic, i.e. for any $|x| > c$, where c is a positive constant, $k(x) \geq k(c)$.

Remark 1: Observing the system studied in this paper, each state variable has a separate derivative expression, so we design a BLF function for each state variable based on the backstepping technique, and the function will involve the constraint boundaries of that state variable, the LKFs, and the design terms of the dynamic surfaces. The controller is then designed based on the derivatives of each BLF so that the final overall BLF derivative satisfies our requirements.

3. Controller design and stability analysis

In this section, a combination of backstepping DSC design and ABLF will be proposed to develop a controller for strict-feedback nonlinear time-delay systems. The specific design process is shown below.

Step 1:

Suppose the tracking error as $s_1(t) = x_1 - y_d$, its time derivative according to the first equation of (1) is given by

$$\dot{s}_1(t) = f_1(x_1) + g_1(x_1)x_2(t) + h_1(x_1(t - \tau_1)) - \dot{y}_d \tag{4}$$

then the virtual error of the next step is defined as $s_2(t) = x_2 - z_2$, simultaneously, letting a designed stabilizing function α_1 passing through a first-order filter that contains a time constant ϑ_2 in the following way

$$\alpha_1(s)\frac{1}{\vartheta s + 1} = z_2(s) \tag{5}$$

and the corresponding time domain expression

$$\vartheta_2 \dot{z}_2 + z_2 = \alpha_1, z_2(0) = \alpha_1(0) \tag{6}$$

wherefore we could get the first-order filter error $\chi_2 = z_2 - \alpha_1$, the derivative of z_2 can be defined as

$$\dot{z}_2 = \frac{\alpha_1 - z_2}{\vartheta_2} = -\frac{\chi_2}{\vartheta_2} \tag{7}$$

then, referring to [6] we choose the time-varying asymmetric barrier function candidate as

$$V_1 = \frac{1}{2}(1 - q(s_1)) \log \frac{k_{a1}^2(t)}{k_{a1}^2(t) - s_1^2} + \frac{1}{2}q(s_1) \times \log \frac{k_{b1}^2(t)}{k_{b1}^2(t) - s_1^2} + \int_{t-\tau_1}^{t} s_1^2(\tau) q_{11}^2(\bar{s}_1(\tau))d\tau + \frac{1}{2}\chi_2^2 \tag{8}$$

where

$$q(\bullet) = \begin{cases} 0, & \bullet \leq 0 \\ 1, & \bullet > 0 \end{cases} \tag{9}$$

for ease of notation, we abbreviate $q(\bullet)$ by q_i throughout this paper, unless otherwise stated. And the time-varying barriers in the above formula are given by

$$\begin{cases} k_{a1}(t) = y_d(t) - \underline{k}_{c1}(t) \\ k_{b1}(t) = \bar{k}_{c1}(t) - y_d(t) \end{cases} \tag{10}$$

and due to assumptions 2.3 and 2.4, we know that positive constants $\underline{k}_{b1}, \bar{k}_{b1}, \underline{k}_{a1}, \bar{k}_{a1}$ exist, which leads to

$$\underline{k}_{a1} \leq k_{a1}(t) \leq \bar{k}_{a1},\ \underline{k}_{b1} \leq k_{b1}(t) \leq \bar{k}_{b1},\ \forall t > 0 \tag{11}$$

Remark 2: $\log(\cdot)$ denotes the natural logarithm of $\cdot$, define a set $\Omega_s := \{s = (s_1, \ldots, s_n)^T \subset R^n, -k_{ai}(t) < s_i(t) < k_{bi}(t), i = 1, \ldots, n, \forall t > 0\}$, where $k_{ai}(t)$, $s_i(t)$, $k_{bi}(t), i = 2, \ldots n$.

It is clear that V_1 is positive definite and continuously differentiable, then the time derivative of V_1 is given by

$$\dot{V}_1 = \left(\frac{1 - q_1}{k_{a1}^2(t) - s_1^2} + \frac{q_1}{k_{b1}^2(t) - s_1^2}\right) s_1 \times \left(\dot{s}_1 - \frac{\dot{k}_{a1}(t)}{k_{a1}(t)} s_1(1 - q_1) - \frac{\dot{k}_{b1}(t)}{k_{b1}(t)} s_1 q_1\right) + s_1^2(t) q_{11}^2(\bar{s}_1(t)) - s_1^2(t - \tau_1) q_{11}^2(\bar{s}_1(t - \tau_1)) + \chi_2 \dot{\chi}_2 \tag{12}$$

similarly, for ease of notation, the following notation definitions will be used

$$\begin{cases} \mu_1 = \left(\frac{1-q_1}{k_{a1}^2(t)-s_1^2} + \frac{q_1}{k_{b1}^2(t)-s_1^2}\right) \\ \lambda_1 = (1 - q_1) \cdot (k_{a1}^2(t) - s_1^2) + q_1 \bullet (k_{b1}^2(t) - s_1^2) \end{cases} \tag{13}$$

where the time-varying gain is given by

$$\bar{k}_1(t) = \sqrt{(1 - q_1)\left(\frac{\dot{k}_{a1}(t)}{k_{a1}(t)}\right)^2 + q_1\left(\frac{\dot{k}_{b1}(t)}{k_{b1}(t)}\right)^2 + \beta_1} \tag{14}$$

and denote $\bar{K}_1(t)$ as follows

$$\bar{K}_1(t) = \bar{k}_1(t) + \frac{\dot{k}_{a1}(t)}{k_{a1}(t)}(1 - q_1(s_1)) + \frac{\dot{k}_{b1}(t)}{k_{b1}(t)}q_1(s_1) \tag{15}$$

in (14), β_1 is a positive constant, which ensures that $\bar{K}_1(t) > 0$ even when $\dot{k}_{a1}(t)$ and $\dot{k}_{b1}(t)$ are both zero. We choose G_1

$$G_1 = \int_{t-\tau}^{t} s_1^2(\tau)q_{11}^2(\bar{s}_1(\tau))d\tau \tag{16}$$

Design stabilizing function α_1 as

$$\alpha_1 = \frac{1}{g_1(x_1)}\left(-f_1(x_1) + \dot{y}_d - (\bar{k}_1(t) + k_1)s_1 - \frac{1}{4}\mu_1 s_1 - s_1\lambda_1 {q_{11}}^2(\bar{s}_1(t)) - \cosh(s_1)\varepsilon_1\lambda_1 G_1\frac{s_1}{1+s_1^2}\right) \tag{17}$$

where k_1 and ε_1 are positive design parameters, substituting (12), (13), (15), and (17) into (11), we could obtain that

$$\begin{aligned}\dot{V}_1 = &-(k_1 + \bar{K}_1(t))\mu_1 s_1^2 - \frac{1}{4}\mu_1^2 s_1^2 - \mu_1 s_1^2\lambda_1 {q_{11}}^2 \\ &\times (\bar{s}_1(t)) - \mu_1\cosh(s_1)\varepsilon_1\lambda_1 G_1\frac{s_1^2}{1+s_1^2} \\ &+ \mu_1 s_1 h_1(x_1(t-\tau_1)) + \mu_1 g_1 s_1 s_2 + \mu_1 s_1 g_1\chi_2 \\ &+ s_1^2(t)q_{11}^2(\bar{s}_1(t)) - s_1^2(t-\tau_1)q_{11}^2(\bar{s}_1(t-\tau_1)) \\ &+ \chi_2\dot{\chi}_2\end{aligned} \tag{18}$$

where $\mu_1\lambda_1 = 1$ and the function D_1 is designed as

$$D_1 = h_1^2(x_1(t-\tau_1)) - s_1^2(t-\tau_1)q_{11}^2(\bar{s}_1(t-\tau_1)) \tag{19}$$

with respect to $\chi_2\dot{\chi}_2$, according to first-order filter error $\chi_2 = z_2 - \alpha_1$ and (3), we have the time derivative of $\dot{\chi}_2$

$$\dot{\chi}_2 = \dot{z}_2 - \dot{\alpha}_1 = -\frac{\chi_2}{\vartheta_2} + B_2 \tag{20}$$

where $B_2(x_1, y_d \cdots) = -\dot{\alpha}_1$, due to assumptions 2.3 and 2.4, we note that $|B_2| \le M_2$, which means that B_2 is bounded and continuous with a maximum absolute value. Thus, we can rewrite $\chi_2\dot{\chi}_2$ with following Young's inequality

$$|B_2\chi_2| \le \frac{1}{2o_2}\chi_2^2 B_2^2 + \frac{1}{2}o_2 \le \frac{1}{2o_2}\chi_2^2 M_2^2 + \frac{1}{2}o_2, o_2 > 0 \tag{21}$$

we have

$$\begin{aligned}\chi_2\dot{\chi}_2 &= \chi_2\left(-\frac{\chi_2}{\vartheta_2} + B_2\right) = \chi_2 B_2 - \frac{\chi_2^2}{\vartheta_2} \\ &\le \frac{1}{2o_2}\chi_2^2 M_2^2 + \frac{1}{2}o_2 - \frac{\chi_2^2}{\vartheta_2}\end{aligned} \tag{22}$$

combining (22) and the following Young's inequality

$$\begin{cases}\mu_1 s_1 h_1(x_1(t-\tau_1)) \le \frac{1}{4}\mu_1^2 s_1^2 + h_1^2(x_1(t-\tau_1)) \\ \mu_1 s_1 g_1(x_1)\chi_2 \le \mu_1 g_{1\max}(x_1)(s_1^2 + \frac{1}{4}\chi_2^2)\end{cases} \tag{23}$$

we obtain

$$\begin{aligned}\dot{V}_1 \le\ &\mu_1 g_1 s_1 s_2 - \mu_1 s_1^2(k_1 + \bar{K}_1(t) - g_{1\max}(x_1)) \\ &- \chi_2^2\left(-\frac{1}{2o_2}M_2^2 + \frac{1}{\vartheta_2} - \frac{1}{4}\mu_1 g_{1\max}(x_1)\right) \\ &- \cosh(s_1)\varepsilon_1 G_1\frac{s_1^2}{1+s_1^2} + \frac{1}{2}o_2 + D_1\end{aligned} \tag{24}$$

In order to guarantee the closed-loop stability, the design of constant gain k_1 and time-delay constant τ_i should make sure $k_1 + \bar{K}_1(t) - g_{1\max}(x_1) > 0$ and $-\frac{1}{2o_2}M_2^2 + \frac{1}{\vartheta_2} - \frac{1}{4}\mu_1 g_{1\max}(x_1) > 0$, and the term $\mu_1 g_1 s_1 s_2$ will be canceled in the subsequent step.

Step i, $2 \le i \le n-1$

Define the tracking error as $s_{i+1}(t) = x_{i+1} - z_{i+1}$, we introduce a filtering virtual control z_{i+1} and let designed stabilizing function α_i passing through a first-order filter that contains a time constant τ_{i+1}, similar to step 1, Where α_i is a stabilizing function to be designed. The first-order filter error in this step is $\chi_{i+1} = z_{i+1} - \alpha_i$. Thus we could obtain

$$\dot{z}_{i+1} = \frac{\alpha_i - z_{i+1}}{\vartheta_{i+1}} = -\frac{\chi_{i+1}}{\vartheta_{i+1}} \tag{25}$$

choose the time-varying asymmetric barrier function candidate as

$$\begin{aligned}V_i = V_{i-1} &+ \frac{1}{2}(1-q_i)\log\frac{k_{ai}^2(t)}{k_{ai}^2(t) - s_i^2} \\ &+ \frac{1}{2}q_i\log\frac{k_{bi}^2(t)}{k_{bi}^2(t) - s_i^2} + \int_{t-\tau_i}^{t} s_i^2(\tau)q_{i1}^2(\bar{s}_i(\tau))d\tau \\ &+ \frac{1}{2}\chi_{i+1}^2\end{aligned} \tag{26}$$

noting that V_i is positive definite and continuously differentiable, then the differentiating of V_i yields

$$\begin{aligned}\dot{V}_i = \dot{V}_{i-1} &+ \left(\frac{1-q_i}{k_{ai}^2(t) - s_i^2} + \frac{q_i}{k_{bi}^2(t) - s_i^2}\right)s_i \\ &\times\left\{\dot{s}_i - \frac{\dot{k}_{ai}(t)}{k_{ai}(t)}s_i(1-q_i) - \frac{\dot{k}_{bi}(t)}{k_{bi}(t)}s_i q_i\right\} \\ &+ s_i^2(t)q_{i1}^2(\bar{s}_i(t)) - s_i^2(t-\tau_i)q_{i1}^2(\bar{s}_i(t-\tau_i)) \\ &+ \chi_{i+1}\dot{\chi}_{i+1}\end{aligned} \tag{27}$$

for ease of notation, we have following notation definitions

$$\mu_i = \left(\frac{1-q_i}{k_{ai}^2(t) - s_i^2} + \frac{q_i}{k_{bi}^2(t) - s_i^2}\right) \tag{28}$$

$$\lambda_i = (1-q_i)(k_{ai}^2(t) - s_i^2) + q_i(k_{bi}^2(t) - s_i^2) \quad (29)$$

where the time-varying gain $\bar{k}_i(t)$ and $\bar{K}_i(t)$ are given by

$$\begin{cases} \bar{k}_i(t) = \sqrt{(1-q_i(s_i))\left(\frac{\dot{k}_{ai}(t)}{k_{ai}(t)}\right)^2 + q_i(s_i)\left(\frac{\dot{k}_{bi}(t)}{k_{bi}(t)}\right)^2 + \beta_i} \\ \bar{K}_i(t) = \bar{k}_i(t) + \frac{\dot{k}_{ai}(t)}{k_{ai}(t)}(1-q_i(s_i)) + \frac{\dot{k}_{bi}(t)}{k_{bi}(t)}q_i(s_i) \end{cases} \quad (30)$$

β_i is a positive constant, which ensures that $\bar{K}_i(t) > 0$. The time-varying barriers $k_{ai}(t),\ k_{bi}(t)$ are given as

$$k_{ai}(t) = z_i - \underline{k}_{ci}(t),\ k_{bi}(t) = \bar{k}_{ci}(t) - z_i \quad (31)$$

As mentioned above, it is easy to know that $s_i(t) = x_i - z_i$, $\dot{s}_i(t) = \dot{x}_i - \dot{z}_i$ and $\dot{z}_i = -\frac{\chi_i}{\vartheta_i}$, so combining with the second equation in (1) we have

$$\dot{s}_i(t) = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1}(t) + h_i(\bar{x}_i(t-\tau_i)) + \frac{\chi_i}{\vartheta_i} \quad (32)$$

due to tracking error and first-order filter error, $x_{i+1}(t)$ can be represented as follows

$$x_{i+1}(t) = s_{i+1} + \chi_{i+1} + \alpha_i \quad (33)$$

stabilizing function is designed as

$$\alpha_i = \frac{1}{g_i(\bar{x}_i)}\begin{pmatrix} -f_i(\bar{x}_i) - (\bar{k}_i(t) + k_i)s_i - \frac{\chi_i}{\vartheta_i} - \frac{1}{4}\mu_i s_i \\ -s_i\lambda_i {q_{i1}}^2(\bar{s}_i(t)) - \cosh(s_i)\varepsilon_i\lambda_i G_i\frac{s_i}{1+s_i^2} \\ -\frac{g_{i-1}(\bar{x}_{i-1})\mu_{i-1}s_{i-1}}{\mu_i} \end{pmatrix} \quad (34)$$

where k_i, ε_i are designed positive constant. We choose G_i

$$G_i = \int_{t-\tau}^{t} s_i^2(\tau)q_{i1}^2(\bar{s}_i(\tau))d\tau \quad (35)$$

Substituting (31), (32), (34) and (37) into (30), it leads to

$$\begin{aligned} \dot{V}_i = {} & \dot{V}_{i-1} + \mu_i s_i g_i s_{i+1} - \mu_i(\bar{K}_i(t) + k_i)s_i^2 - \frac{1}{4}\mu_i^2 s_i^2 \\ & - \mu_i s_i^2\lambda_i {q_{i1}}^2(\bar{s}_i(t)) - \mu_i\lambda_i\cosh(s_i)\varepsilon_i G_i\frac{s_i^2}{1+s_i^2} \\ & - g_{i-1}(\bar{x}_{i-1})\mu_{i-1}s_{i-1}s_i + \mu_i s_i g_i\chi_{i+1} \\ & + \mu_i s_i h_i(\bar{x}_i(t-\tau_i)) + s_i^2(t)\cdot q_{i1}^2(\bar{s}_i(t)) \\ & - s_i^2(t-\tau_i)\cdot q_{i1}^2(\bar{s}_i(t-\tau_i)) + \chi_{i+1}\dot{\chi}_{i+1} \end{aligned} \quad (36)$$

where $\mu_i\lambda_i = 1$, and the function D_i is designed as

$$D_i = h_i^2(\bar{x}_i(t-\tau_i)) - s_i^2(t-\tau_i)q_{i1}^2(\bar{s}_i(t-\tau_i)) \quad (37)$$

with respect to the term $\chi_{i+1}\dot{\chi}_{i+1}$, according to first-order filter error $\chi_{i+1} = z_{i+1} - \alpha_i$ and (19), we have the time derivative of χ_{i+1}

$$\dot{\chi}_{i+1} = \dot{z}_{i+1} - \dot{\alpha}_i = -\frac{\chi_{i+1}}{\vartheta_{i+1}} + B_{i+1} \quad (38)$$

where

$$B_{i+1}(\bar{x}_i, y_d, \dot{y}_d, \ddot{y}_d) = -\frac{\partial\alpha_i}{\partial\bar{x}}\dot{\bar{x}} - \frac{\partial\alpha_i}{\partial y_d}\dot{y}_d - \frac{\partial\alpha_i}{\partial\dot{y}_d}\ddot{y}_d \quad (39)$$

we note that B_{i+1} is bounded and continuous with a maximum absolute value under assumptions 2.3 and 2.4, which can be expressed as $|B_{i+1}| \le M_{i+1}$. Thus we can rewrite $\chi_{i+1}\dot{\chi}_{i+1}$ with following Young's inequality

$$\begin{aligned} |B_{i+1}\chi_{i+1}| \le {} & \frac{1}{2o_{i+1}}\chi_{i+1}^2 B_{i+1}^2 + \frac{1}{2}o_{i+1} \le \frac{1}{2o_{i+1}} \\ & \times \chi_{i+1}^2 M_{i+1}^2 + \frac{1}{2}o_{i+1}, o_{i+1} > 0 \end{aligned} \quad (40)$$

we have

$$\begin{aligned} \chi_{i+1}\dot{\chi}_{i+1} = {} & \chi_{i+1}\left(-\frac{\chi_{i+1}}{\vartheta_{i+1}} + B_{i+1}\right) = \chi_{i+1}B_{i+1} - \frac{\chi_{i+1}^2}{\vartheta_{i+1}} \\ \le {} & \frac{1}{2o_{i+1}}\chi_{i+1}^2 M_{i+1}^2 + \frac{1}{2}o_{i+1} - \frac{\chi_{i+1}^2}{\vartheta_{i+1}} \end{aligned} \quad (41)$$

combining (41) and the following Young's inequality

$$\begin{cases} \mu_i s_i h_i(\bar{x}_i(t-\tau_i)) \le \frac{1}{4}\mu_i^2 s_i^2 + h_i^2(\bar{x}_i(t-\tau_i)) \\ \mu_i s_i g_i(\bar{x}_i)\chi_{i+1} \le \mu_i g_{i\max}(\bar{x}_i)(s_i^2 + \frac{1}{4}\chi_{i+1}^2) \end{cases} \quad (42)$$

we can simplify (36) as

$$\begin{aligned} \dot{V}_i \le {} & \dot{V}_{i-1} + \mu_i g_i s_i s_{i+1} - \mu_i s_i^2(k_i + \bar{K}_i(t) - g_{i\max}(\bar{x}_i)) \\ & - \chi_{i+1}^2\left(-\frac{1}{2o_{i+1}}M_{i+1}^2 + \frac{1}{\vartheta_{i+1}} - \frac{1}{4}\mu_i g_{i\max}(\bar{x}_i)\right) \\ & - \cosh(s_i)\varepsilon_i G_i\frac{s_i^2}{1+s_i^2} + \frac{1}{2}o_{i+1} + D_i \end{aligned} \quad (43)$$

In order to guarantee the closed-loop stability, the design of constant gain k_i and time-delay constant τ_i should make sure $k_i + \bar{K}_i(t) - g_{i\max}(\bar{x}_i) > 0$ and $-\frac{1}{2o_{i+1}}M_{i+1}^2 + \frac{1}{\vartheta_{i+1}} - \frac{1}{4}\mu_i g_{i\max}(\bar{x}_i) > 0$, the term $\mu_i g_i s_i s_{i+1}$ will be canceled in the subsequent step.

Step n

Define the tracking error as $s_n(t) = x_n - z_n$, we can get a time derivative of $s_n(t)$

$$\dot{s}_n(t) = \dot{x}_n - \dot{z}_n \quad (44)$$

In step i, when $i = n-1$, we can obtain

$$\vartheta_n\dot{z}_n + z_n = \alpha_{n-1}, \dot{z}_n = -\frac{\chi_n}{\vartheta_n} \quad (45)$$

choose the time-varying asymmetric barrier function candidate as

$$\begin{aligned} V_n = {} & V_{n-1} + \frac{1}{2}(1-q(s_n))\log\frac{k_{an}^2(t)}{k_{an}^2(t) - s_n^2} + \frac{1}{2}q(s_n) \\ & \times \log\frac{k_{bn}^2(t)}{k_{bn}^2(t) - s_n^2} + \int_{t-\tau_n}^{t} s_n^2(\tau)q_{n1}^2(\bar{s}_n(\tau))d\tau \end{aligned} \quad (46)$$

then differentiating V_n yields

$$\dot{V}_n = \dot{V}_{n-1} + \left(\frac{1-q_n(s_n)}{k_{an}^2(t)-s_n^2} + \frac{q_i(s_i)}{k_{bn}^2(t)-s_n^2}\right)s_n \times \left(\dot{s}_n - \frac{\dot{k}_{an}(t)}{k_{an}(t)}s_n(1-q_n(s_n)) - \frac{\dot{k}_{bn}(t)}{k_{bn}(t)}s_n q_n(s_n)\right) + s_n^2(t)q_{n1}^2(\bar{s}_n(t)) - s_n^2(t-\tau_n)q_{n1}^2(\bar{s}_n(t-\tau_n)) \tag{47}$$

for ease of notation, we have the following notation definitions

$$\mu_n = \left(\frac{1-q_n}{k_{an}^2(t)-s_n^2} + \frac{q_n}{k_{bn}^2(t)-s_n^2}\right) \tag{48}$$

$$\lambda_n = (1-q_n)(k_{an}^2(t)-s_n^2) + q_n(k_{bn}^2(t)-s_n^2) \tag{49}$$

where the time-varying gain $\bar{k}_n(t)$ and $\bar{K}_n(t)$ are given by

$$\begin{cases} \bar{k}_n(t) = \sqrt{(1-q_n)\left(\frac{\dot{k}_{an}(t)}{k_{an}(t)}\right)^2 + q_n\left(\frac{\dot{k}_{bn}(t)}{k_{bn}(t)}\right)^2 + \beta_n} \\ \bar{K}_n(t) = \bar{k}_n(t) + \frac{\dot{k}_{an}(t)}{k_{an}(t)}(1-q_n) + \frac{\dot{k}_{bn}(t)}{k_{bn}(t)}q_n \end{cases} \tag{50}$$

β_n is a positive constant, which ensures that $\bar{K}_n(t) > 0$, even when $\dot{k}_{an}(t)$, $\dot{k}_{bn}(t)$ are both zero. The time-varying barriers $k_{an}(t)$, $k_{bn}(t)$ of virtual error s_n are defined as

$$k_{an}(t) = z_n - \underline{k}_{cn}(t), \quad k_{bn}(t) = \bar{k}_{cn}(t) - z_n \tag{51}$$

Then, according to the third equation in (1) and (44), (45), we could obtain

$$\dot{s}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n)u(t) + h_n(\bar{x}_n(t-\tau_n)) + \frac{\chi_n}{\vartheta_n} \tag{52}$$

introduce the actual controller $u(t)$ as

$$u(t) = \frac{1}{g_n(\bar{x}_n)}\begin{pmatrix} -f_n(\bar{x}_n) - (\bar{k}_n(t)+k_n)s_n - \frac{\chi_n}{\vartheta_n} \\ -\frac{1}{4}\mu_n s_n - s_n\lambda_n q_{n1}{}^2(\bar{s}_n(t)) \\ -\cosh(s_n)\varepsilon_n\lambda_n G_n\frac{s_n}{1+s_n^2} \\ -\frac{g_{n-1}(\bar{x}_{n-1})\mu_{n-1}s_{n-1}}{\mu_n} \end{pmatrix} \tag{53}$$

where k_n, ε_n are designed positive constants and define

$$G_n = \int_{t-\tau}^{t} s_n^2(\tau)q_{n1}^2(\bar{s}_n(\tau))d\tau \tag{54}$$

substituting (48), (49), (51), (52), and (53) into (47) leads to

$$\dot{V}_n = \dot{V}_{n-1} + \mu_i s_i g_i s_{i+1} - \mu_n(\bar{K}_n(t)+k_n)s_n^2 - \frac{1}{4}\mu_n^2 s_n^2 - \mu_n s_n^2\lambda_n q_{n1}{}^2(\bar{s}_n(t)) - \cosh(s_n)\varepsilon_n\lambda_n G_n\frac{\mu_n s_n^2}{1+s_n^2} - g_{n-1}(\bar{x}_{n-1})\mu_{n-1}s_{n-1}s_n + \mu_n s_n h_n(\bar{x}_n(t-\tau_n)) + s_n^2(t)\cdot q_{n1}^2(\bar{s}_n(t)) - s_n^2(t-\tau_n)\bullet q_{n1}^2(\bar{s}_n(t-\tau_n)) \tag{55}$$

where $\mu_n\lambda_n = 1$ and define

$$D_n = h_n^2(\bar{x}_n(t-\tau_n)) - s_n^2(t-\tau_n)\bullet q_{n1}^2(\bar{s}_n(t-\tau_n)) \tag{56}$$

using following Young's inequality

$$\mu_n s_n h_n(\bar{x}_n(t-\tau_n)) \le \frac{1}{4}\mu_n^2 s_n^2 + h_n^2(\bar{x}_n(t-\tau_n)) \tag{57}$$

we could obtain

$$\dot{V}_n = \dot{V}_{n-1} - \mu_n(\bar{K}_n(t)+k_n)s_n^2 - \cosh(s_n)\varepsilon_n G_n\frac{s_n^2}{1+s_n^2} - g_{n-1}(\bar{x}_{n-1})\mu_{n-1}s_{n-1}s_n + D_n \tag{58}$$

where $\bar{K}_n(t) + k_n > 0$. Combining (7), (26), (46) and (24), (43), (58) we could obtain V_n and $\dot{V}_n$ respectively

$$V_n = \sum_{i=1}^{n}\left(\frac{1}{2}(1-q(s_i))\log\frac{k_{ai}^2(t)}{k_{ai}^2(t)-s_i^2} + \frac{1}{2}q(s_i)\log\frac{k_{bi}^2(t)}{k_{bi}^2(t)-s_i^2}\right) + \sum_{i=2}^{n}\frac{1}{2}\chi_{i+1}^2 + \sum_{i=1}^{n}\int_{t-\tau_i}^{t}s_i^2(\tau)q_{i1}^2(\bar{s}_i(\tau))d\tau \tag{59}$$

$$\dot{V}_n \le -\sum_{i=1}^{n-1}\mu_i s_i^2(k_i + \bar{K}_i(t) - g_{i\max}(\bar{x}_i)) - \sum_{i=1}^{n-1}\chi_{i+1}^2 \times \left(-\frac{1}{2o_{i+1}}M_{i+1}^2 + \frac{1}{\vartheta_{i+1}} - \frac{1}{4}\mu_i g_{i\max}(\bar{x}_i)\right) - \sum_{i=1}^{n}\cosh(s_i)\varepsilon_i G_i\frac{s_i^2}{1+s_i^2} + \sum_{i=1}^{n-1}\frac{1}{2}o_{i+1} + \sum_{i=1}^{n}D_i - \mu_n(\bar{K}_n(t)+k_n)s_n^2 \tag{60}$$

where

$$\sum_{i=1}^{n}D_i = \sum_{i=1}^{n}h_i^2(\bar{x}_i(t-\tau_i)) - \sum_{i=1}^{n}s_i^2(t-\tau_i)q_{i1}^2(\bar{s}_i(t-\tau_i)) \tag{61}$$

according to assumption 2.1, we obtain $\sum_{i=1}^{n}D_i = 0$.

So far, the controller design process has been completed, and the schematic diagram of the proposed control scheme is illustrated in Figure 1.

Theorem 3.1: *Consider the nth nonlinear system consisting of the plant* (1) *with time delays and the full*

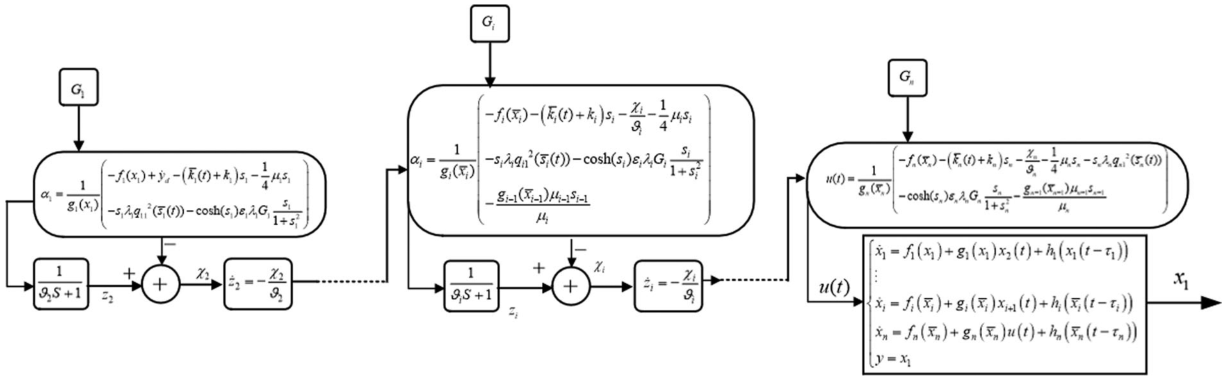

Figure 1. Control schematic diagram.

state constraints with Assumptions 2.1, 2.3 *and* 2.44. *By constructing the stabilizing functions* (17), (34) *and choosing the actual controller* (53), *it can be guaranteed that: the error signals can converge to a compact set; the asymmetric time-varying state constraint:* $\underline{k}_{ci}(t) < x_i(t) < \bar{k}_{ci}(t), \forall t > 0$ *are never violated and all the signals in the resulting closed-loop system are bounded for any* $t > 0$.

Proof: In order to express more conveniently, we present gain parameters in the following way:

$$\begin{cases} E_i = k_i + \bar{K}_i(t) - g_{i\max}(\bar{x}_i) > 0, i = 1\cdots,n-1 \\ E_n = \bar{K}_n(t) + k_n > 0 \\ e_i = -\frac{1}{2o_{i+1}}M_{i+1}^2 + \frac{1}{\vartheta_{i+1}} - \frac{1}{4}\mu_i g_{i\max}(\bar{x}_i) > 0 \\ c = \sum_{i=1}^{n-1} \frac{1}{2}o_{i+1} \end{cases} \tag{62}$$

due to the parameter selection requirements in the previous article, we can see $E_i > 0, i = 1\cdots,n$, $e_i > 0$, $i = 1\cdots,n-1$, $\dot{V}_n$ can be rewritten as

$$\dot{V}_n \le -\sum_{i=1}^{n} E_i\mu_i s_i^2 - \sum_{i=1}^{n-1} e_i\chi_{i+1}^2 - \sum_{i=1}^{n}\cosh(s_i)\varepsilon_i G_i \frac{s_i^2}{1+s_i^2} + c \tag{63}$$

from lemma 2.1, we can obtain

$$\begin{aligned} \sum_{i=1}^{n} E_i\mu_i s_i^2 &= \sum_{i=1}^{n} E_i\left((1-q_i(s_i))\frac{s_i^2}{k_{ai}^2(t)-s_i^2} + q_i(s_i)\frac{s_i^2}{k_{bi}^2(t)-s_i^2}\right) \\ &> \sum_{i=1}^{n}\left(\frac{1}{2}(1-q(s_i))\log\frac{k_{ai}^2(t)}{k_{ai}^2(t)-s_i^2} + \frac{1}{2}q(s_i)\log\frac{k_{bi}^2(t)}{k_{bi}^2(t)-s_i^2}\right) \end{aligned} \tag{64}$$

regarding G_i, for any $\tau_j \le \tau$, $[t-\tau_i, t] \subset [t-\tau, t]$ hold, it is easy to obtain

$$\sum_{i=1}^{n}\int_{t-\tau_i}^{t} s_i^2(\tau)q_{i1}^2(\bar{s}_i(\tau))d\tau \le \sum_{i=1}^{n}\int_{t-\tau}^{t} s_i^2(\tau)q_{i1}^2(\bar{s}_i(\tau))d\tau \tag{65}$$

therefore, (63) further becomes

$$\begin{aligned} \dot{V}_n \le &-\sum_{i=1}^{n} E_i\left(\frac{1}{2}(1-q(s_i))\log\frac{k_{ai}^2(t)}{k_{ai}^2(t)-s_i^2} + \frac{1}{2}q(s_i)\log\frac{k_{bi}^2(t)}{k_{bi}^2(t)-s_i^2}\right) \\ &-\sum_{i=1}^{n}\varepsilon_i\cosh(s_i)\frac{s_i^2}{1+s_i^2}\int_{t-\tau_i}^{t} s_i^2(\tau)q_{i1}^2(\bar{s}_i(\tau))d\tau \\ &-\sum_{i=1}^{n-1} e_i\chi_{i+1}^2 - c \end{aligned} \tag{66}$$

depending on the size of s_j, concerning the analysis of stability, there are three cases that need to be considered. ■

Remark 3: Based on Lemma 2.2, there are three cases of error variables in the control process, all in the predetermined set, all not in the predetermined set, and partially in the predetermined set. The proof is developed for these three cases.

Case 1: ($s_j \in \Omega_{c1}$, $\forall j = 1, 2\ldots, n$): it is easy to ensure the boundedness of s_j due to the positive constant v_j and $|s_j| < 0.5549v_j$, and we can also have $y_d^i(t)$ are bounded considering $|y_d^i(t)| \le Y_i$ in assumption 2.4. Because of $|y_d(t)| \le Y_0$ and $s_1 = x_1 - y_d$ we get $\underline{k}_{c1} = -|k_{a1}| + Y_0 < x_1 < |k_{b1}| + Y_0 = \bar{k}_{c1}$, we can also conclude that α_1 in (20) is bounded, therefore, $\underline{k}_{c2} = -|k_{a2}| + |\alpha_1| < x_2 < |k_{b2}| + |\alpha_1| = \bar{k}_{c2}$. Following the same way, we can get $\underline{k}_{ci}(t) \le x_i \le \bar{k}_{ci}(t)$ and the actual controller u is bounded, which means that all the closed-loop signals are bounded.

Case 2: ($s_j \notin \Omega_{c1}, \forall j = 1, 2 \ldots, n$): Considering lemma 2.3 we can know the monotonic increasing function and continuous $\frac{\cosh(s_i)s_i^2}{1+s_i^2} \geq \frac{(0.5549v_i)^2 \cosh(0.5549v_i)}{1+(0.5549v_i)^2}$ we define σ as follows

$$\sigma = \min_{1 \leq i \leq n} \left\{ 2e_i, E_i, \varepsilon_i \frac{(0.5549v_i)^2 \cosh(0.5549v_i)}{1 + (0.5549v_i)^2} \right\} \tag{67}$$

based on (59), we obtain

$$\dot{V}_n \leq -\sigma V_n + c \tag{68}$$

multiplying (61) by $e^{\sigma t}$ on its both sides yields to

$$\begin{cases} e^{\sigma t}\dot{V}_n \leq e^{\sigma t}(-\sigma V_n + c) \Rightarrow \frac{d(e^{\sigma t}V_n)}{dt} \leq ce^{\sigma t} \\ e^{\sigma t}V_n - V_n(0) \leq \frac{c}{\sigma}(e^{\sigma t} - 1) \\ 0 \leq V \leq V_n(0)e^{-\sigma t} + \frac{c}{\sigma}(1 - e^{-\sigma t}) \leq V_n(0) + \frac{c}{\sigma} \end{cases} \tag{69}$$

supposing $V_n(0) \leq \gamma$ we can get $V_n(t) \leq \gamma + \frac{c}{\sigma}, \forall t > 0$, V_n is bounded, meanwhile, it is easy to obtain

$$(1 - q(s_i)) \frac{k_{ai}^2(t)}{k_{ai}^2(t) - s_i^2} + q(s_i) \frac{k_{bi}^2(t)}{k_{bi}^2(t) - s_i^2} \leq e^{2[V_n(0)e^{-\sigma t} + \frac{c}{\sigma}(1 - e^{-\sigma t})]} \tag{70}$$

for $s_i > 0, q = 1$,(70) can be simplified to

$$\frac{k_{bi}^2(t)}{k_{bi}^2(t) - s_i^2} \leq e^{2[V_n(0)e^{-\sigma t} + \frac{c}{\sigma}(1 - e^{-\sigma t})]} \tag{71}$$

multiplying both sides of (71) by $k_{bi}^2(t) - s_i^2$, which yields to

$$s_i(t) \leq k_{bi}(t)\sqrt{1 - e^{-2[V_n(0)e^{-\sigma t} + \frac{c}{\sigma}(1 - e^{-\sigma t})]}} \tag{72}$$

Likewise, when $s_i \leq 0, q = 0$, we can obtain $s_i(t) \geq -k_{ai}(t)\sqrt{1 - e^{-2[V_n(0)e^{-\sigma t} + \frac{c}{\sigma}(1 - e^{-\sigma t})]}}$, thus the conclusion can be drawn that $-\underline{D}_{s_i} \leq s_i \leq \bar{D}_{s_i}$, where $\underline{D}_{s_i} = k_{ai}(t)\sqrt{1 - e^{-2[V_n(0)e^{-\sigma t} + \frac{c}{\sigma}(1 - e^{-\sigma t})]}}$, $\bar{D}_{s_i} = k_{bi}(t)\sqrt{1 - e^{-2[V_n(0)e^{-\sigma t} + \frac{c}{\sigma}(1 - e^{-\sigma t})]}}$. From the previous derivation, we have $\underline{k}_{c1}(t) = -k_{a1}(t) + y_d(t)$ and $\bar{k}_{c1}(t) = k_{b1}(t) + y_d(t)$, due to $s_1(t) = x_1 - y_d$ and $s_1(t) \in \Omega_s$, which has been proved, it can be learned that $\underline{k}_{c1}(t) \leq x_1 \leq \bar{k}_{c1}(t)$. Similarly, it can also be proved that $\underline{k}_{ci}(t) \leq x_i \leq \bar{k}_{ci}(t)$ on account of $x_i = s_i + z_{i-1}$, $s_i \in \Omega_s$ where $\bar{k}_{ci}(t) = k_{bi}(t) + z_{i-1}(t)$, $\underline{k}_{ci}(t) = z_{i-1}(t) - k_{ai}(t), i = 2, \ldots, n$. Hence, the conclusion can be drawn that the full state constraints are not breached. It has been proven that the states $x_i(t)$ and the error signals $s_i(t)$ are all bounded, according to assumption 2.4, y_d, $\dot{y}_d$ are bounded. From (13) we know that α_1 is a function of $x_1, \dot{y}_d$, so α_1 must be bounded, the upper bound $\bar{\alpha}_1$ of α_1 exists, so we can infer that the signals α_i, $i = 2, \ldots n - 1$ are bounded, furthermore $u(t)$ is bounded. Since z_1 and y_d are bounded, y is also bounded.

Case 3: ($s_j \notin \Omega_{c1}, s_i \in \Omega_{c1}$): Denote $s_j \notin \Omega_{c1}$ as $\sum J$, $s_i \in \Omega_{c1}$ as $\sum I$,define the Lyapunov function candidate as follow for $\sum J$:

$$V_{\sum J} = \sum_{j \in \sum J} \left(\frac{1}{2}(1 - q_i) \log \frac{k_{ai}^2(t)}{k_{ai}^2(t) - s_i^2} + \frac{1}{2} q_i \log \frac{k_{bi}^2(t)}{k_{bi}^2(t) - s_i^2} + \int_{t-\tau_i}^{t} s_i^2(\tau) q_{i1}^2(\bar{s}_i(\tau)) d\tau + \frac{1}{2}\chi_{i+1}^2 \right) \tag{73}$$

differentiating $V_{\sum J}$ yields

$$\dot{V}_{V_{\sum J}} \leq \sum_{j \in \sum J} (\mu_i g_i s_i s_{i+1} - \mu_{i-1} g_{i-1} s_i s_{i-1}) + \sum_{j \in \sum J} \begin{pmatrix} -\mu_i s_i^2 (k_i + \bar{K}_i(t) - g_{i\max}(\bar{x}_i)) \\ -\chi_{i+1}^2 (-\frac{1}{2o_{i+1}} M_{i+1}^2 + \frac{1}{\vartheta_{i+1}} \\ -\frac{1}{4}\mu_i g_{i\max}(\bar{x}_i) \end{pmatrix} \times \sum_{j \in \sum J} \left(-\cosh(s_i)\varepsilon_i G_i \frac{s_i^2}{1 + s_i^2} + c_j \right) \tag{74}$$

the term $\sum_{j \in \sum J} (\mu_i g_i s_i s_{i+1} - \mu_{i-1} g_{i-1} s_i s_{i-1})$ in (74) can be rewritten as

$$\begin{aligned} &\sum_{j \in \sum J} (\mu_i g_i s_i s_{i+1} - \mu_{i-1} g_{i-1} s_i s_{i-1}) \\ &\leq \sum_{\substack{j+1 \in \sum J \\ j \in \sum J}} \mu_i g_i s_i s_{i+1} + \sum_{\substack{j+1 \in \sum I \\ j \in \sum J}} \mu_i g_i s_i s_{i+1} \\ &\quad - \sum_{\substack{j-1 \in \sum J \\ j \in \sum J}} \mu_{i-1} g_{i-1} s_i s_{i-1} - \sum_{\substack{j-1 \in \sum I \\ j \in \sum J}} \mu_{i-1} g_{i-1} s_i s_{i-1} \end{aligned} \tag{75}$$

Remark 4: $\sum_{j \in \sum J} \mu_i g_i s_i s_{i+1}$ and $\sum_{j \in \sum J} \mu_{i-1} g_{i-1} s_i s_{i-1}$ are eliminated in the backstepping design in both case 1 and case 2, but in the present case, these two terms have errors at different steps and are coupled terms, which cannot be canceled in the backstepping process when s_i and s_{i-1} or s_i and s_{i-1} do not happen to be in the same set, so a separate analysis is needed for this case, and a similar analysis can be found in the paper [31].

The terms $\sum_{\substack{j+1 \in \sum J \\ j \in \sum J}} \mu_i g_i s_i s_{i+1}$ and $\sum_{\substack{j-1 \in \sum J \\ j \in \sum J}} \mu_{i-1} g_{i-1} s_i s_{i-1}$ are eliminated during backstepping. Due to $0 < g_{i\min} \leq g_i(\bar{x}_i) \leq g_{i\max}$ in assumption 2.2, denote $g_{i\max}$ as $\bar{g}$ we can get

$$\sum_{j \in \sum J} (\mu_j g_j s_j s_{j+1} - \mu_{j-1} g_{j-1} s_j s_{j-1})$$

$$\leq \sum_{\substack{j+1\in\sum I\\ j\in\sum J}} \mu_j g_j s_j s_{j+1} - \sum_{\substack{j-1\in\sum I\\ j\in\sum J}} \mu_{j-1} g_{j-1} s_j s_{j-1}$$

$$\leq \sum_{\substack{j+1\in\sum I\\ j\in\sum I}} \left(\frac{1}{4\gamma}\mu_j s_j^2 + \mu_j\gamma\bar{g}^2 s_{j+1}^2\right)$$

$$+ \sum_{\substack{j-1\in\sum I\\ j\in\sum J}} \left(\frac{1}{4\gamma}\mu_j s_j^2 + \frac{\gamma\bar{g}^2 s_{j-1}^2\mu_{j-1}^2}{\mu_j}\right)$$

$$\leq \sum_{\substack{j-1\in\sum I\\ j+1\in\sum J}} \left(\mu_j\gamma\bar{g}^2(0.5549v_{j+1})^2\right.$$

$$\left. + \frac{\gamma\bar{g}^2(0.5549v_{j-1})^2\mu_{j-1}^2}{\mu_j}\right)$$

$$+ \sum_{j\in\sum J} \frac{1}{2\gamma}\mu_j s_j^2 \tag{76}$$

where μ_j is positive and bounded, according to (76), (74) can be rewritten as

$$\begin{cases} C_{\sum J} = \sum_{\substack{j-1\in\sum I\\ j+1\in\sum J}} \left(\mu_j\gamma\bar{g}^2(0.5549v_{j+1})^2 \right. \\ \quad \left. + \frac{\gamma\bar{g}^2(0.5549v_{j-1})^2\mu_{j-1}^2}{\mu_j}\right) + \sum_{j\in\sum J} c_j \\ \dot{V}_{V_{\sum J}} \leq \sum_{j\in\sum J} \begin{pmatrix} -\mu_i s_i^2(k_i + \bar{K}_i(t) - g_{i\max}(\bar{x}_i) \\ -\frac{1}{2\gamma}) - \chi_{i+1}^2(-\frac{1}{2o_{i+1}}M_{i+1}^2 \\ +\frac{1}{\vartheta_{i+1}} - \frac{1}{4}\mu_i g_{i\max}(\bar{x}_i) \end{pmatrix} \\ \quad + \sum_{j\in\sum J} -\cosh(s_i)\varepsilon_i G_i \frac{s_i^2}{1+s_i^2} + C_{\sum J} \end{cases} \tag{77}$$

With a similar reasoning in case 2, It is easy to conclude the boundedness of all the signals in the closed-loop system, the tracking errors are able to converge in a compact set and the time-varying full state constraints are never breached for $j \in \sum J$. In regard to $j \in \sum I$ The proof of the stability of the closed-loop system is the same as in case 1.

So far, we complete the proof of Theorem 3.1.

4. Simulation example

A simulation example containing terms affected by uncertain time delay is provided in this section in order to verify the effectiveness and feasibility of the proposed method, the proposed approach is used to control the following nonlinear systems with time-varying full state constraints:

$$\begin{cases} \dot{x}_1(t) = 0.1x_1^2(t) + x_2(t) + h_1(x_1(t-\tau_1)), \\ \dot{x}_2(t) = 0.1x_1(t)x_2(t) - 0.2x_1(t) + (1 + x_1^2(t))u(t) \\ \quad +h_2(x_1(t-\tau_1)) \\ y = x_1(t) \end{cases} \tag{78}$$

in (76), $x_1(t)$ and $x_2(t)$ are the state variables of systems, which need to be constrained, and denote initiate values $x_1(0) = 0.8$ and $x_2(0) = 0$, y and $u(t)$ are the output and the input of systems, the terms affected by uncertain time delay are described by

$$h_1(x_1(t-\tau_1)) = sin(x_1(t-\tau_1)), h_2(\bar{x}_2(t-\tau_2)) = sin(x_2(t-\tau_2)) \tag{79}$$

the relevant system parameters are chosen as $\tau_1 = 0.1$, $\tau_2 = 0.2$, $q_{11} = q_{12} = 1$. $y_d(t)$, the desired trajectory tracked by y is $0.5\cos(t)$. $\underline{k}_{c1}(t) < x_1(t) < \bar{k}_{c1}(t), \underline{k}_{c2}(t) < x_2(t) < \bar{k}_{c2}(t)$ are the asymmetric time-varying full state constraints, where

$$\begin{cases} \underline{k}_{c1}(t) = -0.5 + 0.4\cos(t) \\ \bar{k}_{c1}(t) = 0.7 + 0.2\cos(t) \\ \underline{k}_{c2}(t) = -2.7 + 0.5\cos(t) \\ \bar{k}_{c2}(t) = 1.5 + 0.1\cos(t) \end{cases} \tag{80}$$

the following time-varying asymmetric barrier function are chosen according to the design process above mentioned

$$V_2 = \sum_{i=1}^{2}\left(\frac{1}{2}(1-q(s_i))\log\frac{k_{ai}^2(t)}{k_{ai}^2(t)-s_i^2} + \frac{1}{2}q(s_i)\log\frac{k_{bi}^2(t)}{k_{bi}^2(t)-s_i^2}\right) \tag{1}$$

$$+ \sum_{i=1}^{2}\int_{t-\tau_i}^{t} s_i^2(\tau)q_{i1}^2(\bar{s}_i(\tau))d\tau + \frac{1}{2}\chi_2^2 \tag{81}$$

where

$$\begin{cases} k_{b1}(t) = \bar{k}_{c1}(t) - y_d(t), k_{a1}(t) = y_d(t) - \underline{k}_{c1}(t) \\ k_{b2}(t) = \bar{k}_{c2}(t) - z_1(t), k_{a2}(t) = z_1(t) - \underline{k}_{c2}(t) \end{cases} \tag{82}$$

DSC laws are designed as follows according to the DSC method demonstrated in the paper

$$\alpha_1 = \frac{1}{g_1(x_1)}\left(-f_1(x_1) + \dot{y}_d - (\bar{k}_1(t) + k_1)s_1 - \frac{1}{4}\mu_1 s_1 - s_1\lambda_1 {q_{11}}^2(\bar{s}_1(t)) - \varepsilon_1\lambda_1 G_1\right)$$

$$u(t) = \frac{1}{g_2(\bar{x}_2)}\left(-f_2(\bar{x}_2) - (\bar{k}_2(t) + k_2)s_2 - \frac{\chi_2}{\vartheta_2} - \frac{1}{4}\mu_2 s_2 - s_2\lambda_2 {q_{21}}^2(\bar{s}_2(t)) - \varepsilon_2\lambda_2 G_2 - \frac{g_1(x_1)\mu_1 s_1}{\mu_2}\right) \tag{83}$$

the design parameters are given as $k_1 = 12$, $k_2 = 20$, $\beta_1 = \beta_2 = 0.1$, $\varepsilon_1 = 0.01$, $\varepsilon_2 = 0.01$ and $\vartheta_2 = 0.5$.

The tracking performance is illustrated in Figure 2, it can be observed that the output trajectories are always within the scope of the asymmetric constraint $\underline{k}_{c1}(t) <$

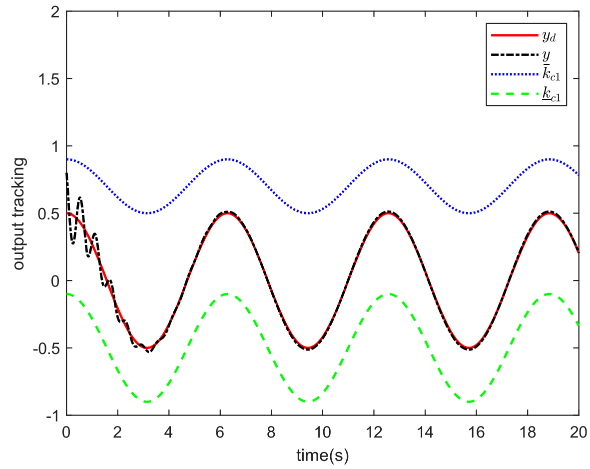

Figure 2. Trajectories of $y = x_1(t)$, $\bar{k}_{c1}(t)$, $\underline{k}_{c1}(t)$ and $y_d(t)$.

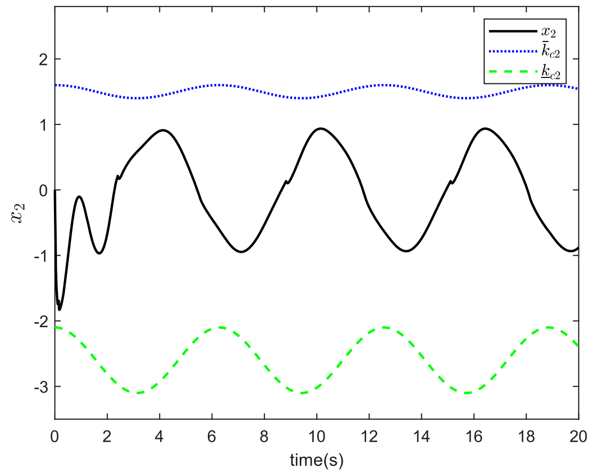

Figure 3. Trajectories of $x_2(t)$ and $\underline{k}_{c2}(t)$, $\bar{k}_{c2}(t)$.

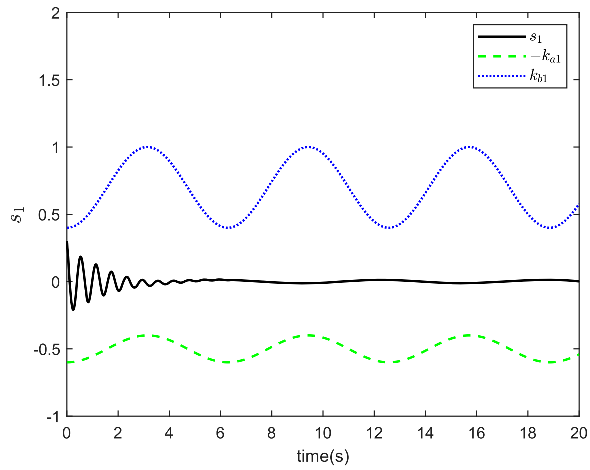

Figure 4. Trajectory of s_1.

$y(t) < \bar{k}_{c1}(t)$, in the meantime, good tracking performance is achieved. The trajectory of state variables shown in Figure 3, it is obvious that time-varying state constraints are not violated. From Figures 4 and 5, it can be seen that tracking errors are within their constraints. The boundedness trajectory of the actual controller is illustrated in Figure 6. Therefore, according to the

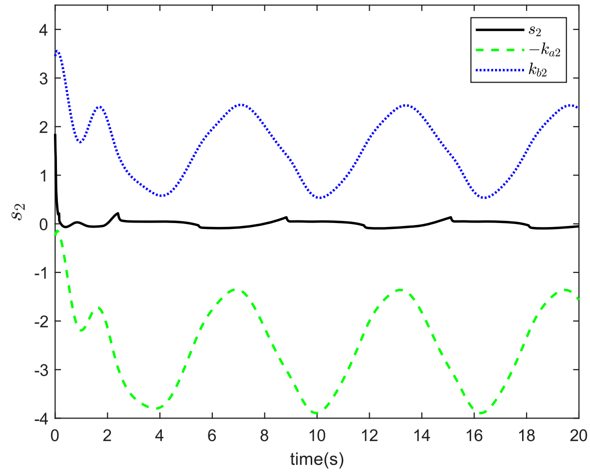

Figure 5. Trajectory of s_2.

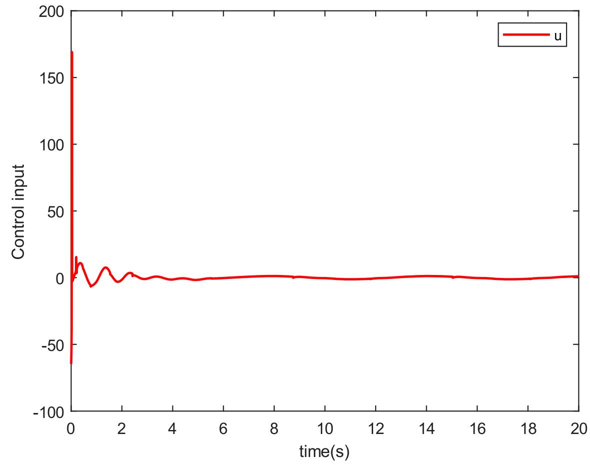

Figure 6. Trajectory of $u(t)$.

figures of the simulation results above, it can be learned the constraints are not violated and all the signals in the closed-loop system are bounded.

Remark 5: The common simulation example used in this paper is referenced from the following papers [30,31,33]. Compared with the papers [30,31,33,36] using non-time-varying BLF, this paper takes the time-varying boundary into account in the design process of the control algorithm, and the simulation results also show that the control algorithm can satisfy the tracking control of the state variables, and the state variables do not cross the time-varying boundary throughout the simulation process. The DSC technique used in this paper largely simplifies the design steps of the control algorithm as well as the computational effort, which can be compared with the papers [30,31,32,33] that also uses the backstepping control technique, but this cannot be presented in the final simulation results.

5. Conclusion

In this paper, an ABLF-based backstepping DSC strategy has been studied for a class of nonlinear systems with time delays and full state constraints. The influence of the time-delays terms was canceled by adopting appropriate assumptions. By employing the appropriate ABLFs, the full state constraints, which are asymmetric and time-varying, were ensured not to be breached. The result that the tracking errors and all the signals of the closed-loop system are asymptotic stable is guaranteed by the proposed approach and choosing appropriate parameters. In addition, compared with approaches using iterative backstepping, repetitive differentiation of stabilizing functions, which will lead to tedious and complicated calculations, especially in higher-order systems, can be averted by choosing the DSC strategy. Finally, the effectiveness of the DSC in conjunction with the ABLF in dealing with time delay problems can be verified by the simulation results and the desired performance can be achieved. Our future research work is to apply this control algorithm to the actual engineering system.

Disclosure statement

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