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Event-triggered adaptive robust fault-tolerant control for a class of uncertain switched nonlinear systems

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ABSTRACT

In this paper, the adaptive robust H_∞ fault-tolerant control problem for a class of switched nonlinear systems with parameter uncertainty, disturbances and actuator failures is concerned based on event-triggered control strategy. The adaptive laws based on state-dependent switched strategy are designed to eliminate the effects of actuator faults and parameter uncertainties by using the estimations of the unknown upper bounds of uncertain parameters. Then, the robust H_∞ fault-tolerant technique and multiple Lyapunov functions method are used, the designed controller can guarantee that all signals of the switched closed-loop systems are uniformly bounded. Meanwhile, the desired H_∞ performance of the systems is promised. Finally, the simulation results are given to illustrate the effectiveness of the proposed design method.

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Event-triggered control; adaptive robust control; fault-tolerant control; state-dependent switched strategy

1. Introduction

As the complexity of industrial systems grows, faults are inevitable to occur. Actuator failure is the most common failure in the dynamic system, which will directly reduce the performance of the system and even destroy the stability of the system (1). In addition, the uncertainties and external disturbances also have an important impact on system performance (2–5). Due to the existence of these factors, there is an urgent need for high-reliability methods to ensure acceptable performance in case of failure or external disturbances. Therefore, it is necessary to continue exploring adaptive fault-tolerant control problems.

Adaptive fault-tolerant control is a highly reliable method to ensure system performance, which has made a great contribution to the development of industrial automation system (6–8). In recent years, literature on this issue has been widely concerned, especially in nonlinear control dynamic systems (9–12). Many successful adaptive schemes have been developed to achieve the expected control objective of the closed-loop system in the case of parameter changes and actuator failure. For instance, in Liu et al. (13) linear matrix inequality (LMI) technology was introduced to deal with the uncertainty of mismatch, and adaptive technology was used to compensate actuator fault. A nonlinear fault compensation function was constructed to solve the adaptive fault-tolerant control problem of uncertain switched nonaffine nonlinear systems with actuator faults and time delays in Wu and Park (14). With the in-depth study, fault-tolerant control methods are used in more complex systems, such as multi-agent systems (15), fractional order control systems (16) and linear quantum stochastic systems (17; 18), Takagi–Sugeno systems (19).

Because of its wide applications in practical systems, network control has recently received a large amount of attention (20; 21). Continuous signal input wastes resources and increases the burden on the system in the actual control system. Event-triggered control is used to reduce the need for feedback while ensuring the stability and expected performance level of the closed-loop system (22–24). Mishra et al. (25) presented an optimized algorithm for event-triggered control that keeps the system states on a stable trajectory. Kaneba et al. (26) used a hybrid event-triggered control scheme for the fault-tolerant control system with actuator failure and random parameter uncertainty.

As is known to all, switched system is a hybrid system with coordinated switched rules, which is composed of a group of continuous or discrete subsystems. Even if some subsystems are unstable, choosing appropriate switched rules can stabilize the whole system, which is a remarkable feature of its stability. It is this characteristic that makes the switched system extensive attention in the field of control. Chadli and Darouach (27) gave sufficient conditions for robust admissibility of uncertain switched singular systems in strict linear matrix inequality formulas. The switched rules of switched systems have attracted more attention, such as average dwell time strategy (28), minimum dwell time method (29), arbitrary switched (30) and so

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on. With the in-depth study of switched systems, based on these basic research methods, the event-triggered strategy was used to study discrete-time switched systems based on piecewise Lyapunov theory and average dwell time control, and the trajectory of the closed-loop system enters the bounded switched region (31). However, there are few research results using fault-tolerant control based on the event-triggered method in switched systems.

Inspired by the above considerations, this paper deals with the problem of adaptive robust fault-tolerant control for a class of uncertain switched nonlinear systems based on event-triggered control. Compared with the existing works, the main contributions of this paper can be summarized as follows:

(1) Different from the existing results (23; 32), an adaptive robust fault-tolerant controller for a class of uncertain switched nonlinear systems is considered in this paper.
(2) An adaptive H_∞ fault-tolerant control scheme based on event-triggered control is constructed to reduce the effects of disturbances and parameter uncertainty. At the same time, the adaptive law constructed in this paper can effectively deal with the actuator faults.
(3) The control strategy proposed in this paper can ensure the boundedness of all signals of the switched closed-loop nonlinear systems and reduce the transmission count between the controller and the actuator.

The remainder of this paper is structured as follows. Some preliminaries and problem statement are introduced in Section 2. A robust adaptive switched fault-tolerant controller based on event-triggered control is designed in Section 3. The simulation results are shown in Section 4 to prove the effectiveness of the proposed method. Finally, the conclusions are drawn in Section 5.

2. Preliminaries and problem statement

2.1. System description

The following switched nonlinear system with unknown parameters and actuator failures is considered:

$$\begin{aligned}\dot{x}(t) &= (A_\sigma + \Delta A_\sigma)x(t) + B_\sigma u(t) + B_\sigma u_{l\sigma}(t)\\ &\quad + B_{1\sigma}\omega(t) + f_\sigma(x,t) \qquad (1)\\ Z &= C_\sigma x(t),\end{aligned}$$

where $x(t) \in R^n$ is the state vector, $u_{l\sigma}(t) \in R^m$ denotes the time-varying float fault, $u(t) \in R^m$ is the control input with the switching signal $\sigma(t) : [0, +\infty) \to K = \{1, 2, \ldots, s\}$, $\omega(t) \in L_2[0, +\infty)$ is the system external disturbance, $f_\sigma \in R^n$ is the continuous nonlinear function satisfying $f_\sigma(0, t) = 0$ and $Z \in R^q$ represents the output of the system. For $i \in K$, A_i, B_i, B_{1i} and C_i are known real constant matrices with appropriate dimensions. The switching signal $\sigma(t)$ can be expressed as the following sequence:

$$\sum = \{(i_0, t_0), (i_1, t_1) \cdots (i_k, t_k), \ldots | i_k \in K,\ k = 0, 1, 2, \ldots\},$$

when $t \in [t_k, t_{k+1})$, $\sigma(t) = i_k$ means that the i_kth subsystem is activated.

In addition, we assume the parameter uncertainty satisfies the condition:

$$\Delta A_i = B_i N_i(t), \qquad (2)$$

where $\|N_i(t)\| \le N_i^*$ with $N_i^* > 0$ being an unknown constant.

2.2. Basic assumptions and lemmas

Definition 2.1 (Yang and Ye (33)): Consider the following system:

$$\begin{aligned}\dot{x} &= A_\sigma x + B_\sigma \omega,\\ z &= C_\sigma x, \qquad (3)\end{aligned}$$

where A_σ, B_σ and C_σ are the parameter matrices with approximate dimensions, and $x(0) = 0$. Let $\gamma > 0$ be a given constant, then the system is said to be with an adaptive H_∞ performance index no larger than γ_0, if for any $\varepsilon > 0$, the following inequality holds:

$$\int_0^\infty z^T(t)z(t)\mathrm{d}t \le \gamma_0{}^2 \int_0^\infty \omega^T(t)\omega(t)\mathrm{d}t + \varepsilon. \qquad (4)$$

Assumption 2.1: The time-varying fault function is piecewise continuous bounded, that is $u_{l\sigma}(t)$ satisfies $\|u_{l\sigma}\| \le S_i^*$ with $S_i^* > 0$ being an unknown constant.

Assumption 2.2 (Li et al. (34)): For the nonlinear term $f_i(x, t)$ of the ith subsystem, there exists a function $\sum_i(x, t)$ such that $f_i(x, t) = B_i \sum_i(x, t)$ and

$$\left\| \sum\nolimits_i (x, t) \right\| \le M_i \|x\| + G_i^*, \qquad (5)$$

$\|M_i(t)\| \le M_i^*$ with M_i^* and G_i^* are unknown positive constants.

Remark 2.1: Assumption 2.1 is a standard assumption that is common in fault-tolerant control references. Assumption 2.2 can be understood that the adaptive laws are designed to compensate nonlinear disturbance effectively, under the condition of matching.

Lemma 2.1 (Conte (35)): *For any matrices A and B with appropriate dimensions, the following inequality holds:*

$$A^T B + B^T A \le cA^T A + \frac{1}{c} B^T B, \quad \forall c > 0. \qquad (6)$$

Constructing a robust adaptive H_∞ control scheme and corresponding rules to ensure the boundedness of all signals of closed-loop nonlinear switched systems is the main control objective of this paper, and the H_∞ performance index is not greater than γ_0 when there are parameter uncertainty, external interference and nonlinear function.

Remark 2.2: It should be noted that the method proposed by Zhang et al. (36) does not address the nonlinear fault-tolerant control problem of actuator failure. The switched system in this paper is more flexible and robust than the switched uncertain nonlinear systems in Cui and Xiang (37). Compared with Jin et al. (2) and Aouaouda and Chadli (27), the control method proposed in this paper considers a broader range of applications and effectively reduces system redundancy. The control scheme developed in this paper can ensure the boundedness of all signals and the desired H_∞ performance switched closed-loop system while saving resources.

3. Event-triggered adaptive switched fault-tolerant control

In this section, to achieve the desired control objectives given in Section 2, the adaptive controller is designed as

$$\alpha(t) = \left(K_i - \frac{B_i{}^T P_i \hat{\theta}_{i_1}^2}{\|x^T P_i B_i\| \hat{\theta}_{i1} + \delta_i} - \frac{1}{2}\eta_i B_i^T P_i \hat{\theta}_{i2} \right) x, \tag{7}$$

where K_i is a set gain matrix. In addition, $\hat{\theta}_{i1}$ and $\hat{\theta}_{i2}$ are the estimates of θ_{i1} and θ_{i2}, orderly, satisfying $\hat{\theta}_{i1} = \tilde{\theta}_{i1} + \theta_{i1}$ and $\hat{\theta}_{i2} = \tilde{\theta}_{i2} + \theta_{i2}$, θ_{i1} and θ_{i2} are unknown constants, which are designed in (12) and (13). $\delta_i(t)$ is a positive uniform continuous and bounded function satisfying

$$\lim_{t\to\infty} \int_{t_0}^{t} \delta_i(\xi)\mathrm{d}\xi \le \bar{\xi}_i < \infty. \tag{8}$$

The triggering event is defined as

$$\begin{aligned} u(t) &= \alpha(t_k), \quad \forall t \in [t_k, t_{k+1}), \\ t_{k+1} &= \inf\{t \in R |\, |e_i(t)| \ge d_i\}, \end{aligned} \tag{9}$$

where d_i is a positive design parameter, $e_i(t) = \alpha_i(t) - u_i(t)$, $i = 1, 2, \ldots, n$, $t_1 = 0$, $t_k, k \in Z^+$ is the controller update time, the time t_k will be updated as t_{k+1} when (9) is triggered, at the same time the controller $u(t)$ will be used in the systems (1). For $t \in [t_k, t_{k+1})$, if $|e_i(t)| \le d_i$, there exists a continuous time-varying parameter $\lambda_i(t)$, satisfying $\lambda_i(t_k) = 0$, $\lambda_i(t_{k+1}) = 1$ and $|\lambda_i(t)| \le 1$, such that

$$\alpha(t) = u(t) + \lambda(t), \tag{10}$$

where $\lambda(t) = [\lambda_1(t)d_1, \lambda_2(t)d_2, \ldots, \lambda_n(t)d_n]^T$, $\|\lambda(t)\| \le d^*$, d^* is an unknown constant. By Assumption 2.1, there are positive constants θ_1 and θ_2 satisfying

$$\begin{aligned} \left\| G^* + u_{li}(t) - \lambda(t) \right\| &\le G^* + S^* + d^* \le \theta_1, \\ N_i^{*2} + M_i^{*2} &\le \theta_2. \end{aligned} \tag{11}$$

And then, the adaptive update control laws are designed as

$$\begin{aligned} \dot{\hat{\theta}}_{i1} &= -l_{i1}\delta_i\hat{\theta}_{i1} + 2l_{i1}\|x^T P_i B_i\|, \\ \dot{\hat{\theta}}_{i2} &= -l_{i2}\delta_i\hat{\theta}_{i2} + \eta_i l_{i2}\|x^T P_i B_i\|^2, \end{aligned} \tag{12}$$

l_{i1} and l_{i2} are positive designed parameters. Setting $\tilde{\theta}_{ij} = \hat{\theta}_{ij} - \theta_{ij}$, $i = 1, 2$, the following equation is established

$$\begin{aligned} \dot{\tilde{\theta}}_{i1} &= -l_{i1}\delta_i\hat{\theta}_{i1} + 2l_{i1} \left\| x^T P_i B_i \right\| - l_{i1}\delta_i\theta_{i1}, \\ \dot{\tilde{\theta}}_{i2} &= -l_{i2}\delta_i\hat{\theta}_{i2} + \eta_i l_{i2} \left\| x^T P_i B_i \right\|^2 - l_{i2}\delta_i\theta_{i2}. \end{aligned} \tag{13}$$

4. Stability analysis

Theorem 4.1: *On the basis of Assumptions 2.1 and 2.2, for the given positive design parameters γ_0, η_i if there exist matrices $P_i > 0$, $Q_i > 0$, K_i and positive constants $\pi_{ik} > 0$, $i, k = 1, 2, \ldots, s$ such that the following matrix inequality holds:*

$$\begin{bmatrix} \Phi_i + \sum_{k=1}^{s} \pi_{ik}(P_k - P_i) & P_i B_{1i} & C_i^T \\ * & -\gamma_0^2 I & 0 \\ * & * & -I \end{bmatrix} < 0, \tag{14}$$

where $\Phi_i = (A_i P_i^{-1} + B_i K_i P_i^{-1})^T + (A_i P_i^{-1} + B_i K_i P_i^{-1}) + \frac{2}{\eta_i} I$. *Then the adaptive H_∞ fault-tolerant controller (7) with adaptive laws (11) and switching law*

$$\sigma(x(t)) = \arg\min_{i \in K} x^T(t) P_i x(t) \tag{15}$$

can guarantee that signals of the switched close-loop nonlinear system (1) are bounded and the H_∞ performance index is no larger than γ_0.

Proof: Substituting (10) into (1), we can obtain the following system:

$$\begin{aligned} \dot{x}(t) &= (A_\sigma + \Delta A_\sigma)x(t) + B_\sigma(\alpha(t) - \lambda(t)) \\ &\quad + B_\sigma u_{l\sigma}(t) + B_{1\sigma}\omega(t) + f_\sigma(x, t), \\ Z &= C_\sigma x(t). \end{aligned} \tag{16}$$

Then choose a Lyapunov function candidate as follows:

$$V_i(x) = x^T P_i x + \frac{1}{2} l_{i1}^{-1}\tilde{\theta}_{i1}^2 + \frac{1}{2} l_{i2}^{-1}\tilde{\theta}_{i2}^2. \tag{17}$$

If the ith subsystem is active, we have

$$\dot{V}_i(x) + Z^T Z - \gamma_0^2 \omega^T \omega$$

$$\leq x^T(P_iA_i + A_i^TP_i)x + 2x^TP_iB_iN_i(t)x + 2x^TP_iB_i(\alpha(t) - \lambda(t)) + 2x^TP_iB_iu_{li}(t) + 2x^TP_iB_{1i}\omega(t) + 2x^TP_iB_i\sum_i(x,t) + l_{i1}^{-1}\tilde{\theta}_{i1}\dot{\hat{\theta}}_{i1} + l_{i2}^{-1}\tilde{\theta}_{i2}\dot{\hat{\theta}}_{i2} + Z^TZ - \gamma_0^2\omega^T\omega. \tag{18}$$

With (7) we can get

$$\dot{V}_i(x) + Z^TZ - \gamma_0^2\omega^T\omega \leq x^T(P_i(A_i + B_iK_i) + (A_i + B_iK_i)^TP_i)x + 2x^TP_iB_i(u_{li}(t) - \lambda(t)) - \frac{2x^TP_iB_iB_i^TP_ix\hat{\theta}_{i1}^2}{\|x^TP_iB_i\|\hat{\theta}_{i1} + \delta_i} - x^TP_iB_i\eta_iB_i^TP_ix\hat{\theta}_{i2} + 2\left\|x^TP_iB_i\right\|\|N_i(t)\|\|x\| + 2x^TP_iB_{1i}\omega(t) + 2\left\|x^TP_iB_i\right\|\|M_i\|\|x\| + 2x^TP_iB_iG_i^* + l_{i1}^{-1}\tilde{\theta}_{i1}\dot{\hat{\theta}}_{i1} + l_{i2}^{-1}\tilde{\theta}_{i2}\dot{\hat{\theta}}_{i2} + Z^TZ - \gamma_0^2\omega^T\omega. \tag{19}$$

From (6), we can obtain the following conclusion

$$2\left\|x^TP_iB_i\right\|\|N_i(t)\|\|x\| \leq \eta_i\left\|x^TP_iB_i\right\|^2\|N_i(t)\|^2 + \frac{1}{\eta_i}\|x\|^2,$$
$$2\left\|x^TP_iB_i\right\|\|M_i\|\|x\| \leq \eta_i\left\|x^TP_iB_i\right\|^2\|M_i\|^2 + \frac{1}{\eta_i}\|x\|^2,$$
$$2x^TP_iB_i\omega \leq \gamma_0^{-2}x^TP_iB_iB_i{}^TP_ix + \gamma_0^2\omega^T\omega. \tag{20}$$

According to (20) and Assumption 2.2, it can be gained that

$$\dot{V}_i(x) + Z^TZ - \gamma_0^2\omega^T\omega \leq x^T(P_i(A_i + B_iK_i) + (A_i + B_iK_i)^TP_i)x - \frac{2x^TP_iB_iB_i^TP_ix\hat{\theta}_{i1}^2}{\|x^TP_iB_i\|\hat{\theta}_{i1} + \delta_i} + 2x^TP_iB_i\left(G_i^* + u_{li}(t) - \lambda(t)\right) - x^TP_iB_i\eta_iB_i^TP_ix\hat{\theta}_{i2} + \frac{2}{\eta_i}\|x\|^2 + \eta_i\left\|x^TP_iB_i\right\|^2\left(N_i^{*2} + M_i^{*2}\right) + \gamma_0^{-2}x^TP_iB_{1i}B_{1i}^Tx + l_{i1}^{-1}\tilde{\theta}_{i1}\dot{\hat{\theta}}_{i1} + l_{i2}^{-1}\tilde{\theta}_{i2}\dot{\hat{\theta}}_{i2} + Z^TZ + \gamma_0^2\omega^T\omega - \gamma_0^2\omega^T\omega. \tag{21}$$

Combining (1) and (14), we have

$$\dot{V}_i(x) + Z^TZ - \gamma_0^2\omega^T\omega \leq x^T(P_i(A_i + B_iK_i) + (A_i + B_iK_i)^TP_i + C_i^TC_i)x - \frac{2x^TP_iB_iB_i^TP_ix\hat{\theta}_{i1}^2}{\|x^TP_iB_i\|\hat{\theta}_{i1} + \delta_i} + 2x^TP_iB_i\left(G_i^* + u_{li}(t) - \lambda(t)\right) - x^TP_iB_i\eta_i \times B_i^TP_ix\hat{\theta}_{i2} + \frac{2}{\eta_i}\|x\|^2 + \eta_i\left\|x^TP_iB_i\right\|^2\left(N_i^{*2} + M_i^{*2}\right) + \gamma_0^{-2}x^TP_iB_{1i}B_{1i}^Tx + l_{i1}^{-1}\tilde{\theta}_{i1}\dot{\hat{\theta}}_{i1} + l_{i2}^{-1}\tilde{\theta}_{i2}\dot{\hat{\theta}}_{i2}$$
$$\leq x^T(P_i(A_i + B_iK_i) + (A_i + B_iK_i)^TP_i + C_i^TC_i + \frac{1}{\gamma_0^2}P_iB_{1i}^TB_{1i}P_i + \frac{2}{\eta_i})x + \frac{2\delta_i\left\|x^TP_iB_i\right\|\hat{\theta}_{i1}}{\|x^TP_iB_i\|\hat{\theta}_{i1} + \delta_i} - \sum_{k=1}^{2}\delta_i\tilde{\theta}_{ik}\hat{\theta}_{ik}. \tag{22}$$

By invoking the Schur complement, we can draw the following conclusion

$$\dot{V}_i(x) + Z^TZ - \gamma_0^2\omega^T\omega \leq -x^T\sum_{k=1}^{s}\pi_{ik}(P_k - P_i)x - \frac{2\delta_i\left\|x^TP_iB_i\right\|\hat{\theta}_{i1}}{\|x^TP_iB_i\|\hat{\theta}_{i1} + \delta_i} - \sum_{k=1}^{2}\delta_i\tilde{\theta}_{ik} - \sum_{k=1}^{2}\delta_i\tilde{\theta}_{ik}\theta_{ik}. \tag{23}$$

Applying the switching law (15) and the inequality $0 \leq \frac{ab}{a+b} \leq a,\ \forall a, b > 0$, (23) becomes

$$\dot{V}_i(x) + Z^TZ - \gamma_0^2\omega^T\omega \leq \delta_i\left(\frac{\theta_{i1}^2}{4} + \frac{\theta_{i2}^2}{4} + 2\right) \leq \delta_i\mu_{i0}. \tag{24}$$

where $\mu_{i0} = \frac{\theta_{i1}^2}{4} + \frac{\theta_{i2}^2}{4} + 2$. If the external $d(t) = 0$, it can be inferred that the states of the switched closed-loop system are bounded. Arrange and integrate (24)

$$\int_0^\infty z^T(t)z(t)\mathrm{d}t \leq \gamma_0^2\int_0^\infty \omega^T(t)\omega(t)\mathrm{d}t + \kappa_i, \tag{25}$$

where $\kappa_i = V_i(0) + \bar{\delta}_i\mu_i$. Setting $V(x) = \sum_{i=1}^{s}V_i(\tilde{x})$ where $\tilde{x} = [x^T, \tilde{\theta}_{i1}, \tilde{\theta}_{i2}]^T$ and $\kappa = \frac{V(0)+\sum_{i=1}^{s}\bar{\delta}_i\mu_i}{s}$. We can get that the H_∞ performance index of the switched closed-loop system (1) is less than or equal to γ_0 as in (4). ■

Remark 4.1: By multiplying both sides of (14) by the inverse matrix

$$\begin{bmatrix} P_i^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix},$$

it is easy to get the following inequation

$$\begin{bmatrix} \Psi_i + X_iQ_iX_i & B_{1i} & P_i^{-1}C_i^T & \sqrt{\pi_{ik}}P_i^{-1} & \sqrt{\frac{2}{\eta_i}}P_i^{-1} \\ * & -\gamma_0^2 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -P_i^{-1} & 0 \\ * & * & * & * & -I \end{bmatrix} < 0,$$

where $\Psi_i = (A_iP_i^{-1} + B_iK_iP_i^{-1})^T + (A_iP_i^{-1} + B_iK_i P_i{}^{-1}) + \frac{2}{\eta_i}(P_i^{-1})^2$, with $X_i = P_i{}^{-1}$, and $Q_i = \sum_{i=1}^{s} \pi_{ik} (P_i - P_k)$. The inverse matrix X_i, matrix K_i and matrix P_i can be obtained by solving the corresponding inequalities by using Schur complement theorem.

In addition, there exists a time $t^* > 0$ satisfies $\{t_{k+1} - t_k\} \geq t^*$. According to (7) we can get

$$\dot{\alpha} = \frac{\partial \alpha}{\partial x}\dot{x} + \sum_{j=1}^{2} \frac{\partial \alpha}{\partial \hat{\theta}_{ij}}\dot{\hat{\theta}}_{ij} + \frac{\partial \alpha}{\partial \delta_i}\dot{\delta}_i \tag{26}$$

Owing to $x, \hat{\theta}_{i1}, \hat{\theta}_{i2}$ are continuous and bounded, which implies δ_i and $\dot{\alpha}$ are also continuous and bounded functions. Hence, there exists a constant $\vartheta > 0$ satisfying $\|\dot{\alpha}\| \leq \vartheta$. Considering $e_i(t_k) = 0$, $e_i(t_{k+1}) = d_i$, and $e_i(t) = \alpha_i(t) - u_i(t)$, the following inequality holds

$$\frac{\mathrm{d}}{\mathrm{d}t}|e_i| \leq \frac{\mathrm{d}}{\mathrm{d}t}|d_i| = |\dot{\alpha}| < \vartheta \tag{27}$$

we can obtain that there exists the inter-execution intervals t^* satisfies $t^* \geq \frac{d_i}{\vartheta}$, this means the Zeno behaviour is excluded. The proof is completed.

5. Simulation studies

In this section, two examples are used to prove the effectiveness of this method.

Example 5.1: Consider the following uncertain switched nonlinear system with unknown parameters and actuator failures:

$$\begin{aligned} \dot{x}(t) &= (A_\sigma + \Delta A_\sigma)x(t) + B_\sigma u(t) + B_\sigma u_{l\sigma}(t) \\ &\quad + B_{1\sigma}\omega(t) + f_\sigma(x,t) \\ Z &= C_\sigma x(t), \end{aligned}$$

where $\sigma = 1, 2$ and

$$A_1 = \begin{bmatrix} 0 & 1 & 0.0802 & 1.0415 \\ -0.1980 & -0.115 & -0.0318 & 0.3 \\ -3.0500 & 1.1880 & -0.4650 & 0.9 \\ 0 & 0.0805 & 1 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 & 1.55 & 0.75 \\ 0.975 & 0.8 & 0.85 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0.25 \\ 0.1 & -0.1 & 0.8 & -0.23 \\ -4.25 & 1.5 & -0.5 & 1 \\ 0 & 0 & 1 & 0.5 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -1 & 2 & 0.25 \\ 0.5 & 0.1 & -0.5 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix},$$

$$B_{11} = \begin{bmatrix} -1 & 0 & 2 & 0 \end{bmatrix}^T,$$
$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix},$$
$$B_{12} = \begin{bmatrix} 1 & 0 & 0 & -3 \end{bmatrix}^T,$$
$$C_2 = \begin{bmatrix} 1 & 0 & -1 & 0 \end{bmatrix}.$$

In addition, it is assumed that

$$u_{l1} = \begin{bmatrix} 0.1\cos(t) & 0 & 0 \end{bmatrix}^T,$$
$$u_{l2} = \begin{bmatrix} 0 & 0.15\sin(t) & 0 \end{bmatrix}^T,$$
$$\omega = \begin{cases} 0.01\cos(t), & 0 < t < 15 \\ 0, & 15 < t < 45 \end{cases},$$
$$N_1 = \begin{bmatrix} 0.6\cos 2t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$
$$N_2 = \begin{bmatrix} 0.7\sin 2t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$
$$f_1(x,t) = \begin{bmatrix} 0.6\sin(x1) & -0.6\sin(x2) & 0 & 0 \end{bmatrix}^T,$$
$$f_2(x,t) = \begin{bmatrix} 0.5\sin(x1) & 0 & -0.5\sin(x3) & 0 \end{bmatrix}^T.$$

By selecting appropriate parameters and solving matrix inequality (14), we can get that

$$P_1 = \begin{bmatrix} 0.0451 & -0.0237 & 0.0045 & 0.0076 \\ -0.0237 & 0.0558 & -0.0186 & -0.0053 \\ 0.0045 & -0.0186 & 0.0687 & 0.0150 \\ 0.0076 & -0.0053 & 0.0150 & 0.0535 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 0.0434 & -0.0193 & 0.0082 & 0.0001 \\ -0.0193 & 0.0548 & -0.0173 & -0.0078 \\ 0.0082 & -0.0173 & 0.0572 & 0.0134 \\ 0.0001 & -0.0078 & 0.0134 & 0.0487 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} -3.7152 & -4.6720 & 5.8003 & 14.0077 \\ -23.2626 & 10.2456 & -2.4274 & 3.5442 \\ 6.8320 & -17.0706 & -14.2392 & -3.5365 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 14.2638 & 7.2817 & -32.5840 & -17.8032 \\ 4.4694 & -9.0061 & 4.8546 & -15.8124 \\ -27.1916 & 36.9011 & -38.2348 & -5.7158 \end{bmatrix}.$$

In the simulation, the parameters of the system are selected as $x(0) = [-1, 0, 1, 0, 0, 0]^T$, $\hat{\theta}_{ij}(0) = 0$, i, $j = 1, 2$, $\eta_1 = 20$, $\eta_2 = 50$, $l_{i1} = 15$, $l_{i2} = 30$, where $i = 1, 2, \pi_{12} = 200, \pi_{21} = 100, d_1 = 5, d_2 = 5, d_3 = 2$, and the H_∞ performance index is chosen as $\gamma_0 = 0.15$.

The simulation results are displayed in Figures 1–5. The switched signal is shown in Figure 1. From Figure 2, we can see that the adaptive switched fault-tolerant controller can ensure the state x_i of the system is bounded $(d(t) = 0)$. The boundedness of parameter estimations $\hat{\theta}_{ij}$ and the control signals u_{ik}, $i, j = 1, 2$, $k = 1, 2, 3$ are displayed in Figures 3–5.

Example 5.2: A simple mass-spring-damper switched system model from Long and Zhao (38) is used in this subsection, and the parameter matrices of the corresponding state space equation are $A_i = \left[\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right]$, $B_i = [0, 1]^T$, $C_i = [1, 0]$, $B_{ji} = [0, 0]^T$, $i = 1, 2$

$$P_1 = \begin{bmatrix} 2.3368 & 0.1116 \\ 0.1116 & 0.0656 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 1.7952 & 0.2852 \\ 0.2852 & 0.1413 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} -30.0830 & -12.7948 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -30.6224 & -20.3110 \end{bmatrix}.$$

In addition, set $f_i(x, t) = -\frac{1}{m}(f(x_1) - g(x_2) + \Delta f_i(x))$, $i = 1, 2$. $u_{l1} = 0.3 \sin(2t)$, $u_{l2} = 0.2 \cos(t)$, $m = 1/5$,

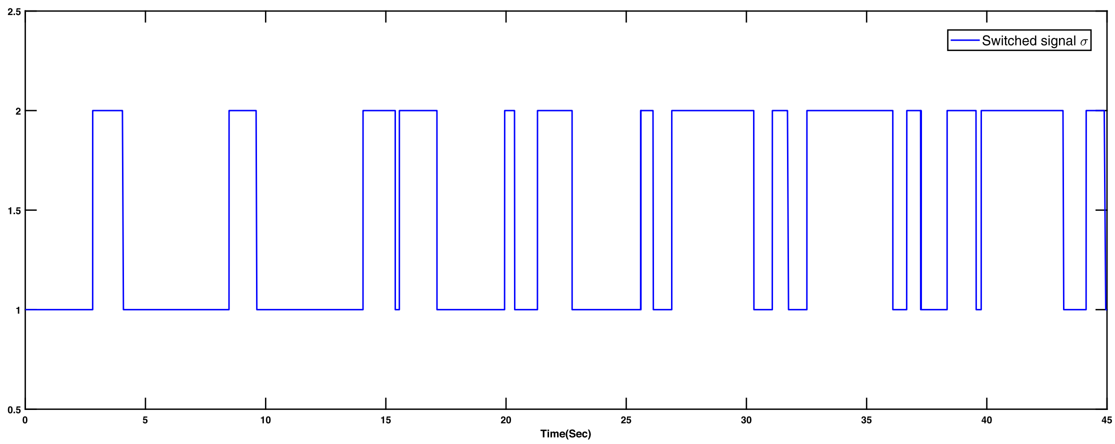

Figure 1. Switched rule σ of Example 5.1

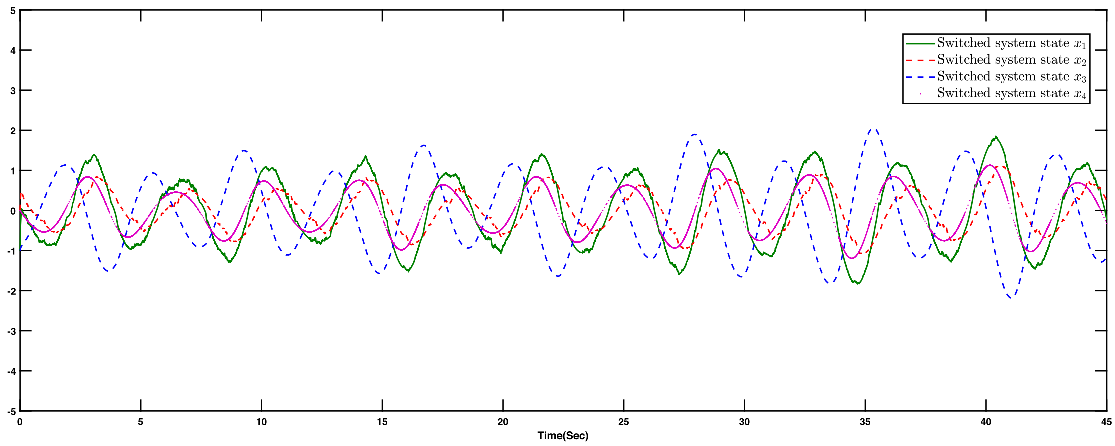

Figure 2. Switched system states $x_i(t), i = 1, 2, 3, 4$ in Example 5.1

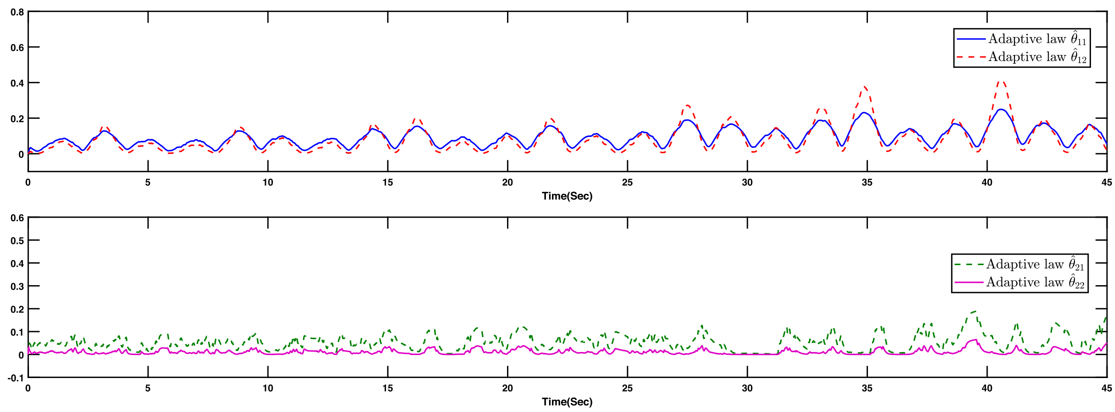

Figure 3. Adaptive laws $\hat{\theta}_{ij}, i, j = 1, 2$ in Example 5.1

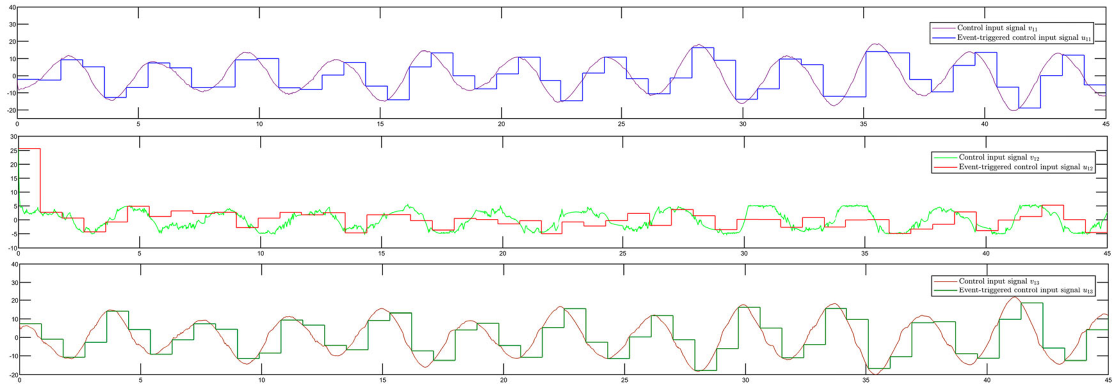

Figure 4. Control input signals of subsystem 1 in Example 5.1

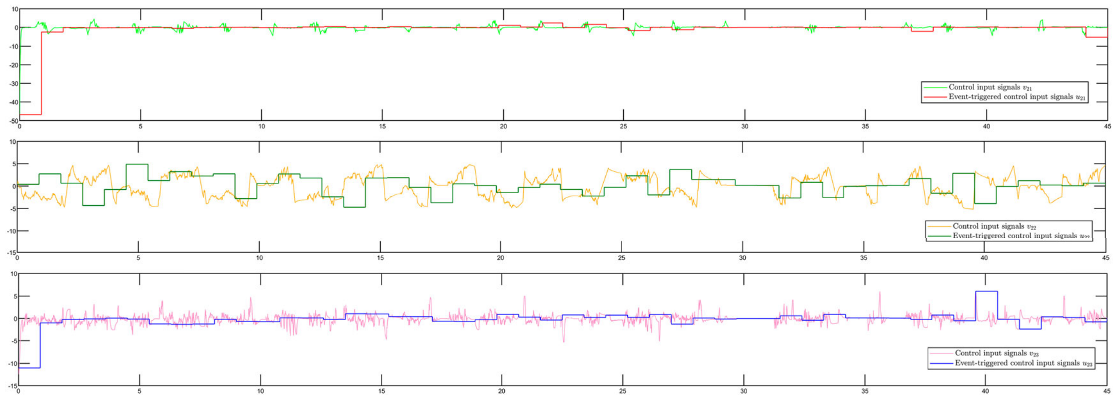

Figure 5. Control input signals of subsystem 2 in Example 5.1

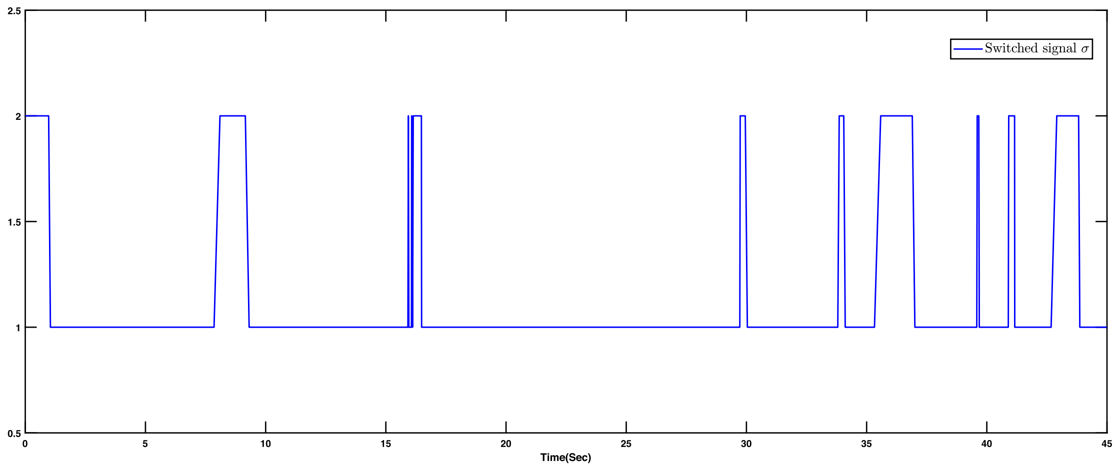

Figure 6. Switched rule σ of Example 5.2

$f(x_1) = 2{x_1}^2, g(x_2) = {x_2}^2 \cos(x_2), \Delta f_1 = {x_1}^2 \sin(x_1 x_2)$, $\Delta f_2 = x_1 \cos({x_1}^2)$ the simulation parameters are chosen as $x(0) = [1, -1, 2, 1]^T$, $\hat{\theta}_{11}(0) = 2$, $\hat{\theta}_{12}(0) = 1$, $\hat{\theta}_{21}(0) = 2$, $\hat{\theta}_{22}(0) = 1$, $\eta_1 = 10$, $\eta_2 = 15$, $l_{i1} = 10$, $l_{i2} = 15, \pi_{12} = 1, \pi_{21} = 0.25, d_1 = 5, d_2 = 3$.

The simulation results can be seen from Figures 6–9. Figure 6 demonstrates the switched signal. It can be easily found that the state x_i, the parameter estimations $\hat{\theta}_{ij}$ and the control signals of the switched system are all bounded in Figures 7–9.

Remark 5.1: Based on event-triggered control strategy, a robust adaptive switched fault-tolerant controller with parameter updated laws is designed. Furthermore, compared with (23; 32) Figures 1–9 demonstrate that the proposed adaptive method can guarantee the system's expected H_∞ performance while effectively reducing system redundancy in the event of failure.

6. Conclusion

For a class of uncertain nonlinear switched systems with parameter uncertainties, actuator failures, and external disturbances, this paper investigates the problem of robust adaptive H_∞ event-triggered fault-tolerant control. The adaptive controller with

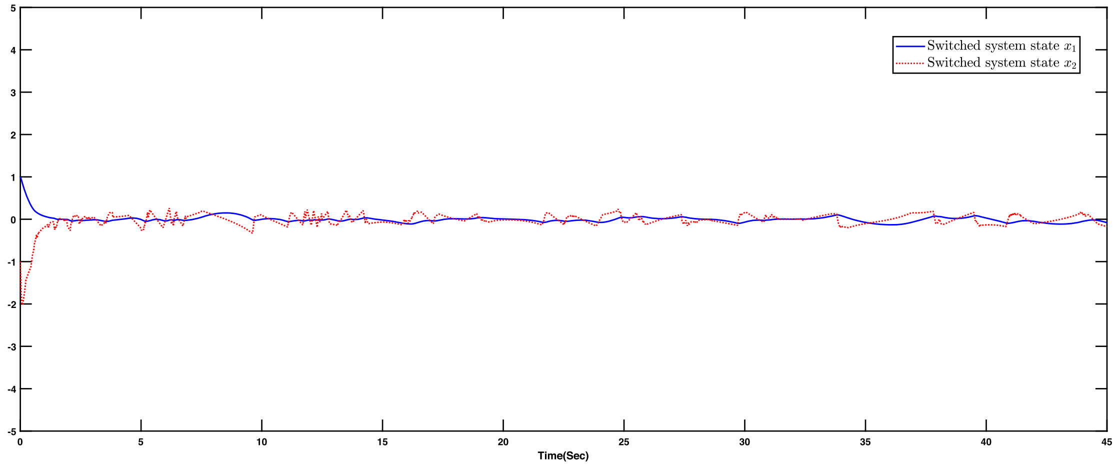

Figure 7. Switched system states $x_i(t), i = 1, 2$ in Example 5.2

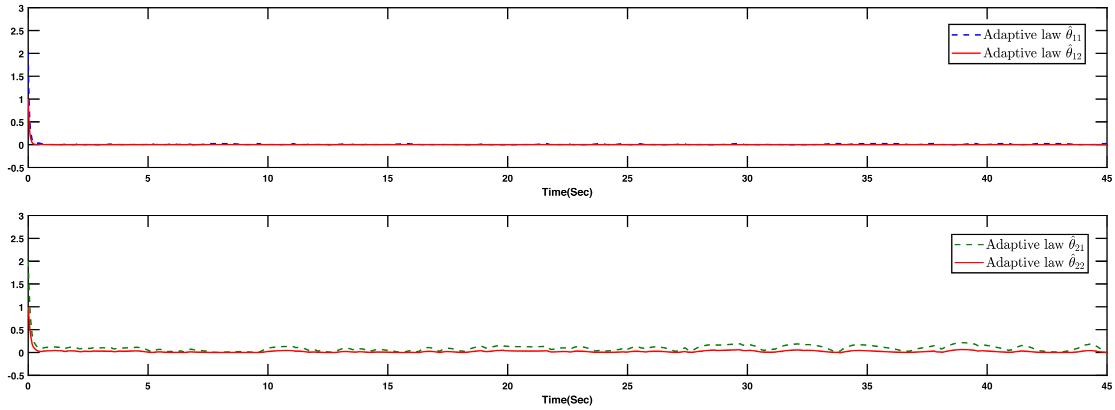

Figure 8. Adaptive laws $\hat{\theta}_{ij}, i, j = 1, 2$ in Example 5.2

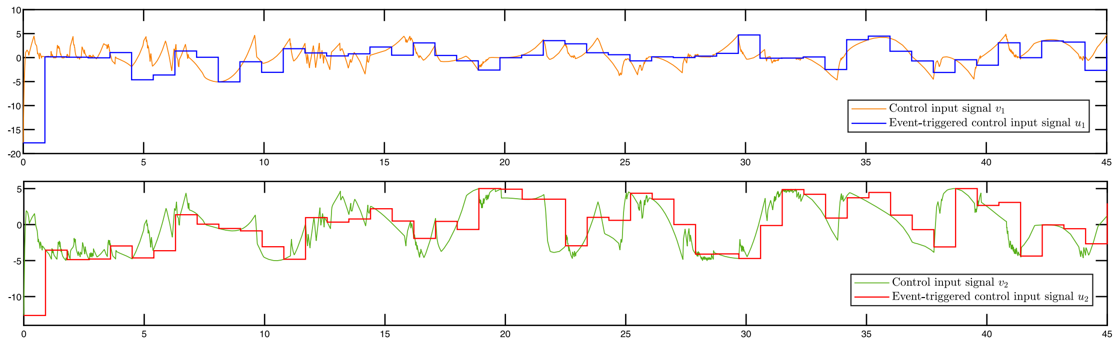

Figure 9. Control input signals in Example 5.2

the correlative parameter updated laws and state-dependent switching rule is designed to compensate for the actuator faults and eliminate the effects of nonlinear input disturbance by combining the linear matrix inequality and adaptive approach. It has been demonstrated that Zeno behaviour can be avoided and that all signals' boundedness and the expected H_∞ performances of switched closed-loop systems' can be guaranteed. Finally, two simulation examples are provided to show the viability of the proposed approach. In future work, for uncertain switched nonlinear systems with unknown control directions and full-state constraints, robust adaptive fault-tolerant tracking control design approaches will be taken into consideration.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Data availability statements

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

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