J. F. Kennedy sq. 6
10000 Zagreb, Croatia
Tel +385(0)1 238 3333
www.efzg.hr/wps
wps@efzg.hr

EFZG WORKING PAPER SERIES
EFZG SERIJA ČLANAKA U NASTAJANJU
ISSN 1849-6857
UDC 33:65

No. 24-01

Zrinka Lukač, Krunoslav Puljić, Vedran Kojić

Maximizing tax revenue for profit maximizing monopolist with the Cobb-Douglas production function and linear demand as a bilevel programming problem

Maximizing tax revenue for profit maximizing monopolist with the Cobb-Douglas production function and linear demand as a bilevel programming problem

Zrinka Lukač
zlukac@efzg.hr
Faculty of Economics and Business
University of Zagreb
Trg J. F. Kennedy 6
10 000 Zagreb, Croatia

Krunoslav Puljić
kpuljic@efzg.hr
Faculty of Economics and Business
University of Zagreb
Trg J. F. Kennedy 6
10 000 Zagreb, Croatia

Vedran Kojić
vkojic@efzg.hr
Faculty of Economics and Business
University of Zagreb
Trg J. F. Kennedy 6
10 000 Zagreb, Croatia

Abstract

Optimal taxation and profit maximization are two very important problems, naturally related to one another since companies striving operates given tax system. However, in the literature these two problems are usually considered separately, either by studying optimal taxation or by studying profit maximization. This paper links the two problems together by formulating a bilevel model in which government acts as a leader and profit maximizing follower act as a follower. The exact form of the tax revenue function as well as optimal tax amount and optimal input levels are derived in cases when returns of scale take on values 0.5, 1 and 2. Several numerical examples and accompanying illustrations are given.

Key words

optimal taxation, government, profit maximization, monopolist, Cobb-Douglas production function, linear demand, bilevel programming

JEL classification

C61, C72, D42, H21

Maximizing tax revenue for profit maximizing monopolist with the Cobb-Douglas production function and linear demand as a bilevel programming problem

Zrinka Lukač[1,*], **Krunoslav Puljić**[1], **Vedran Kojić**[1]

[1] *Faculty of Economics and Business, University of Zagreb*

Trg J. F. Kennedyja 6, 10000 Zagreb, Croatia

E-mail: <{zlukac, kpuljic, vkojic}@efzg.hr>

* *Corresponding author.*

Abstract. Optimal taxation and profit maximization are two very important problems, naturally related to one another since companies striving operates given tax system. However, in the literature these two problems are usually considered separately, either by studying optimal taxation or by studying profit maximization. This paper tries links the two problems together by formulating a bilevel model in which government acts as a leader and profit maximizing follower act as a follower. The exact form of the tax revenue function as well as optimal tax amount and optimal input levels are derived in cases when returns of scale take on values 0.5, 1 and 2. Several numerical examples and accompanying illustrations are given.

Key words: optimal taxation, government, profit maximization, monopolist, Cobb-Douglas production function, linear demand, bilevel programming

JEL Classification: C61, C72, D42, H21

1. Introduction

Taxations is one of the most important instruments of any government. Its basic functions is to fund government's expenditure. However, it also has profound impact on competitiveness of companies, thus affecting economic growth and investments as well as supply and demand. Therefore, devising optimal tax policy is of key importance to any government. The problem of devising optimal taxation obviously contains hierarchical structure since government makes tax decision first and then companies independently operate under a given tax system trying to achieve what is personally best for them. Hence, it is natural to model the problem as a bilevel programing problem. Bilevel programming problems were first described by von Stackleberg (1934), with the first mathematical model given by Bracken and McGill (1972; 1973). They model problems with hierarchical structure, having two players located at different levels of hierarchy. The decision maker on the upper level is called the leader, while the one on the lower level called the follower. Decisions are made sequentially, assuming perfect information. The leader acts first, with the goal of optimizing his objective. However, in making his decision, he has to anticipate the response of the follower who, once having the leader's decision, acts independently, trying to optimize the objective of his own. Therefore, one of the constraints of the leader's optimization problem is the optimization problem of the follower.

This paper formulates the problem of devising optimal tax policy as a bilevel programming problem in which the leader is the government deciding about tax amount with the objective of maximizing its tax revenue. The tax is modelled as an amount per unit product. The follower is the monopolist who, once having the tax decision of the leader, chooses the level of production which maximize his profit. The model assumes that production is described by Cobb-Douglas production function and that the market price follows linear demand.

By itself, the problem of profit maximizing monopolist is so important that it is covered in any intermediate microeconomics textbook (see for example Mas-Collel et al., 1995). As for production functions, Avvakumov et al. (2010) offer extensive coverage of the properties of the Cobb-Douglas and CES production function. Maximization of tax revenue function is also an important and well-studied problem (Lott and Miller, 1974; Gahvari, 1989). However, to the best of our knowledge, these two problems were not considered together as a single problem as a bilevel programming problem.

The paper is organized as follows. After notation in the second section, the third and the fourth sections formulate and solve the problem for case of two inputs. Conclusions are given in the last section.

2. Notation

The notation used in this paper is as follows:

$x_1, x_2 > 0$	inputs (labor and capital)
$w_1, w_2 > 0$	input prices
$\alpha_1, \alpha_2 > 0$	output elasticities of inputs
$q > 0$	output level
$t > 0$	the tax amount per unit product
$p > 0$	price of the product received by producer
$p + t > 0$	market price of the product
$a, b > 0$	coefficients of demand function

3. Model

The assumptions of the model are as follows. The monopolist's output quantity is described by the Cobb-Douglas production function of two inputs, labor and capital, denoted as x_1 and x_2 respectively:

$$q = q\left(x_1, x_2\right) = x_1^{\alpha_1} x_2^{\alpha_2} . \tag{1}$$

According to Zevelev (2014), it is known that the firm's cost function in case of the Cobb-Douglas technology is given by

$$c(q) = \omega q^{1/\varepsilon}, \tag{2}$$

where

$$\omega = \varepsilon \rho^{-1/\varepsilon}, \ \varepsilon = \alpha_1 + \alpha_2, \ \rho = \left(\frac{\alpha_1}{w_1}\right)^{\alpha_1} \left(\frac{\alpha_2}{w_2}\right)^{\alpha_2}. \tag{3}$$

Additionally, firm's conditional input demand functions in case of the Cobb-Douglas technology are given by

$$x_i = \frac{\alpha_i}{w_i} \rho^{-1/\varepsilon} q^{1/\varepsilon}, \ i = 1, 2. \tag{4}$$

Furthermore, in our model the monopolist receives price p for the product. However, the government charges tax amount t per unit product, and therefore the price buyers pay at the market for the product is equal to $p+t$. The demand on the market is linear, described by the inverse demand function

$$p + t = a - bq \ \Rightarrow \ p = p(q) = a - t - bq, \tag{5}$$

where

$$a - t > 0 \ \text{ and } \ a - t - bq > 0. \tag{6}$$

The leader makes the tax decision per unit product *t*. However, in doing this, he has to take into account the response of the follower who, given the leader's tax decision *t*, does what is personally best for him. In other words, the government' tax decision affects the supply at the market. Therefore, the leader's tax revenue function is a function of *t* as well as of output level *q*, and it is equal to

$$T(t,q) = t \cdot q. \tag{7}$$

Variable *t* is controlled by the leader (i.e. government), while variable *q* is controlled by the follower (i.e. monopolist).

For the given tax decision *t*, the follower maximizes his profit function. Since the price the monopolist receives is equal to *p*, from (2) and (5) it follows that his profit function is equal to

$$\Pi(t,q) = p(q)q - c(q) = (a-t)q - bq^2 - \omega q^{1/\varepsilon}. \tag{8}$$

The problem of determining optimal tax policy for profit maximizing monopolist with the Cobb-Douglas production function and linear demand function can now be stated as the following bilevel programming problem:

$$\max_{0<t<a} T(t,q) = tq \tag{9}$$

$$\text{s.t.} \ \max_{q>0} \Pi(t,q) = (a-t)q - bq^2 - \omega q^{1/\varepsilon}. \tag{10}$$

The tax decision *t* made by the leader affects the choices available to the follower. Given the leader's choice *t*, the follower acts independently and does what is personally best for him and not for the leader, i.e. the monopolist chooses level of output which maximizes his profit.

4. Main results

In order to solve the bilevel programming problem (9)-(10), one first needs to solve the follower's problem for an arbitrary but fixed tax amount *t* and thus obtain optimal output level as a function of *t*. Knowing the optimal follower's decision for any choice of *t*, the leader's problem can be solved. This gives the solution of the overall bilevel programming problem.

Theorem 1. The stationary point of the follower's problem (10) for an arbitrary but fixed tax amount *t* is defined implicitly as the solution of the equation

$$(a-t)-2bq=\frac{\omega}{\varepsilon}q^{\frac{1}{\varepsilon}-1}\ , \tag{11}$$

where condition

$$(a-t)-2bq>0 \tag{12}$$

must be satisfied.

Proof. Let tax amount *t* per unit product be arbitrary but fixed. For a given *t*, from the first order condition on the follower's optimization problem given in the form (10), the following equation is obtained

$$\Pi_q\left(t,q\right)=(a-t)-2bq-\frac{\omega}{\varepsilon}q^{\frac{1}{\varepsilon}-1}=0\,, \tag{13}$$

from where the equation (11) follows directly. Since the right hand side of (11) is positive because of the economic reasons, the left hand side of (11) must be positive too, i.e. condition (12) must hold. ■

The stationary point of the follower's problem (10) cannot be obtained explicitly for any choice of the sum of the output elasticities $\varepsilon=\alpha_1+\alpha_2$. Instead, it is obtained by solving equation (11) for given elasticities α_1 and α_2.

Once the solution of the follower's problem for an arbitrary but fixed tax amount *t* is known, the leader's optimization problem can be solved. Let us illustrate the solution of the problem (9)-(10) through the three following numerical cases.

Corollary 1. (Constant returns of scale) For the constant returns of scale $\varepsilon=\alpha_1+\alpha_2=1$ bilevel programming problem (9)-(10) becomes

$$\max_{0<t<a} T\left(t,q\right)=tq \tag{14}$$

$$\text{s.t. } \max_{q>0}\Pi\left(t,q\right)=\left(a-\omega-t\right)q-bq^2\,. \tag{15}$$

The optimal solution of the problem (14)-(15) is given by

$$t^* = \frac{a-\omega}{2}, \tag{16}$$

$$q^* = \frac{a-\omega}{4b}, \tag{17}$$

where t^* and q^* are the optimal leader's and follower's decisions respectively. The optimal leader's tax is equal to

$$T^* = \frac{(a-\omega)^2}{8b}, \tag{18}$$

while the optimal follower's profit and input levels are, respectively, equal to

$$\Pi^* = \frac{(a-\omega)^2}{16b}, \tag{19}$$

$$x_i^* = \frac{\alpha_i}{w_i \rho} \cdot \frac{a-\omega}{4b},\ \ i = 1,2\,. \tag{20}$$

Proof. For $\varepsilon = 1$ equation (13) becomes

$$a - t - \omega - 2bq = 0\,, \tag{21}$$

which implies

$$q = \frac{a-t-\omega}{2b}. \tag{22}$$

Note that (22) implies the following condition:

$$a - t - \omega > 0 \ \ \Leftrightarrow \ \ t \in \langle 0,\, a-\omega \rangle\,. \tag{23}$$

Furthermore, since

$$\Pi_{qq}(t,q) = \frac{\partial}{\partial q}(a - t - \omega - 2bq) = -2b < 0\,, \tag{24}$$

the follower's profit function is concave overall its domain in q and the maximum profit is achieved. By substituting (22) into (14) we have

$$T\left(t, \frac{a-\omega-t}{2b}\right) = -\frac{t^2}{2b} + \frac{a-\omega}{2b}t\ \ . \tag{25}$$

It is easy to see that the maximum of the function (25) equals (18) and that it is obtained for (16). That is, the optimal leader's tax is equal to (18) and it is achieved at level (16). Now, substituting (16) into (22) we get (17). Finally, by substituting (16) and (17) into (15), we get (19). Furthermore, for constant returns to scale $\varepsilon = 1$, from (17) and (4) follows (20). This proves the theorem. ■

Example 1. Let the input prices be $w_1 = 4$ and $w_2 = 9$ respectively, let the output elasticities of inputs be $\alpha_1 = 0.3$ and $\alpha_2 = 0.7$ respectively. Furthermore, let the coefficients of demand function be $a = 50$ and $b = 30$ respectively.

Note that the production function (1)

$$q = q(x_1, x_2) = x_1^{\alpha_1} x_2^{\alpha_2} = x_1^{0.3} x_2^{0.7} \tag{26}$$

has constant returns of scale homogeneity $\varepsilon = \alpha_1 + \alpha_2 = 0.3 + 0.7 = 1$

Note also that according to (3)

$$\rho = \left(\frac{\alpha_1}{w_1}\right)^{\alpha_1} \left(\frac{\alpha_2}{w_2}\right)^{\alpha_2} = \left(\frac{0.3}{4}\right)^{0.3} \left(\frac{0.7}{9}\right)^{0.7} \approx 0.077 \tag{27}$$

and

$$\omega = \varepsilon \rho^{-1/\varepsilon} = \rho^{-1} \approx 12.998 \tag{28}$$

Since for $\varepsilon = 1$

$$\Pi(t, q) = (a - \omega - t) q - bq^2 = (37.002 - t) q - 30q^2, \tag{29}$$

problem (14)-(15) or (9)-(10) can now be stated as the following bilevel programming problem:

$$\max_{0<t<a} T(t, q) = tq \tag{30}$$

$$\text{s.t.} \max_{q>0} \Pi(t, q) = (37.002 - t) q - 30q^2. \tag{31}$$

First, we solve the follower's problem (31) for a given but fixed tax amount t. From the first order conditions on (31)

$$\Pi_q(t, q) = (37.002 - t) - 60q = 0 \tag{32}$$

we obtain the optimal output level

$$q^*(t) = \frac{37.002 - t}{60}. \tag{33}$$

Furthermore, since $\Pi_{qq}(t, q) = -60 < 0$, the follower's profit function is concave overall its domain in q and the maximum profit is achieved.

By substituting $q^*(t) = \frac{37.002 - t}{60}$ into the leader's optimization problem (30) $\max_{0<t<a} T(t, q) = tq$ we get the following tax revenue function of only tax amount t:

$$T\left(t, \frac{37.002 - t}{60}\right) = \frac{-t^2 + 37.002t}{60}. \tag{34}$$

The maximum of the leader's tax revenue function $T(t)$ is obtained for $T'(t) = -2t + 37.002 = 0$ because $T''(t) = -2 < 0$. It is easy to see that $t^* \approx 18.501$ and $T^* \approx 5.705$.

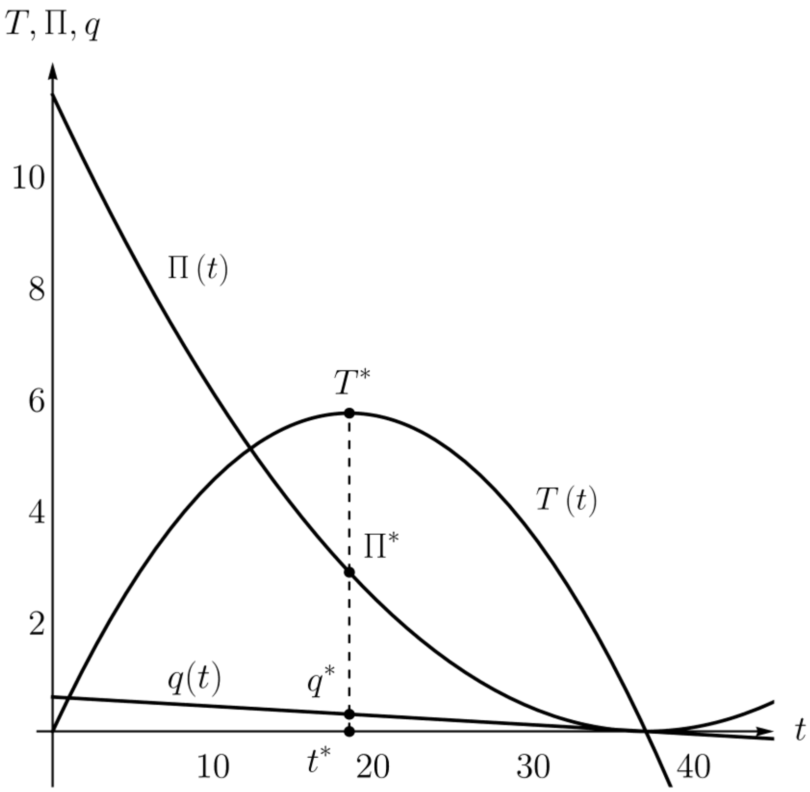

Figure 1. 2D graphical representation of the bilevel problem (30)-(31) solution

Furthermore, the optimal output level

$$q^*(t^*) = \frac{37.002 - t^*}{60} = \frac{37.002 - 18.501}{60} \approx 0.308. \tag{35}$$

Finally, by substituting $t^* = 18.501$ and $q^* = 0.308$ back into the follower's profit function, we get the optimal follower's profit

$$\Pi^*\left(t^*, q^*\right) = \left(37.002 - t^*\right)q^* - 30(q^*)^2 \approx 2.852. \tag{36}$$

Furthermore, for constant returns to scale $\varepsilon = 1$, from the optimal output level $q^* = 0.308$ and (4) follows the optimal inputs (labor and capital) levels

$$x_1^* = \frac{\alpha_1}{w_1}\rho^{-1/\varepsilon}q^{1/\varepsilon} = \frac{0.3}{4}\cdot 0.077^{-1}\cdot 0.308^1 \approx 556.795, \tag{37}$$

and

$$x_2^* = \frac{\alpha_2}{w_2}\rho^{-1/\varepsilon}q^{1/\varepsilon} = \frac{0.7}{9}\cdot 0.077^{-1}\cdot 0.308^1 \approx 164.976. \tag{38}$$

Corollary 2. (Decreasing returns of scale) For the decreasing returns of scale $\varepsilon = \alpha_1 + \alpha_2 = 0.5$ bilevel programming problem (9)-(10) becomes

$$\max_{0<t<a} T(t,q) = tq \tag{39}$$

$$\text{s.t. } \max_{q>0} \Pi(t,q) = -(b+\omega)q^2 + (a-t)q\,. \tag{40}$$

The optimal solution of the problem (14)-(15) is given by

$$t^* = \frac{a}{2}, \tag{41}$$

$$q^* = \frac{a}{4(b+\omega)}, \tag{42}$$

where t^* and q^* are the optimal leader's and follower's decisions respectively. The optimal leader's tax is equal to

$$T^* = \frac{a^2}{8(b+\omega)}, \tag{43}$$

while the optimal follower's profit and input levels are, respectively, equal to

$$\Pi^* = \frac{a^2}{16(b+\omega)}, \tag{44}$$

$$x_i^* = \frac{\alpha_i}{w_i} \cdot \frac{a^2}{4\rho^2 (b+\omega)^2},\ \ i = 1,2\,. \tag{45}$$

Proof. For $\varepsilon = 0.5$ equation (13) becomes

$$a - t - 2(b+\omega)q = 0\,, \tag{46}$$

which implies

$$q = \frac{a-t}{2(b+\omega)}. \tag{47}$$

Note that (34) implies the following condition:

$$a - t > 0 \ \Leftrightarrow\ t \in \langle 0,\, a \rangle\,. \tag{48}$$

Furthermore, since

$$\Pi_{qq}(t,q) = \frac{\partial}{\partial q}\big(a - t - 2(b+\omega)q\big) = -2(b+\omega) < 0\,, \tag{49}$$

the follower's profit function is concave overall its domain in q and the maximum profit is achieved. By substituting (34) into (26) we have

$$T\left(t, \frac{a-t}{2(b+\omega)}\right) = \frac{1}{2(b+\omega)}\left(-t^2 + at\right) . \tag{50}$$

It is easy to see that the maximum of the function (50) equals (43) and that it is obtained for (41). That is, the optimal leader's tax is equal to (50) and it is achieved at level (41). Now, substituting (41) into (47) we get (42). Finally, by substituting (41) and (42) into (40), we get (44). Furthermore, for decreasing returns to scale $\varepsilon = 0.5$, from (42) and (4) follows (45). This proves the theorem. ■

Example 2. Consider the similar problem as in the Example 1. There are the same input prices $w_1 = 4$ and $w_2 = 9$ respectively, and the same coefficients of demand function be $a = 50$ and $b = 30$ respectively. In this example the output elasticity α_1 is again set to $\alpha_1 = 0.3$ but this time the output elasticity α_2 is set to $\alpha_2 = 0.2$ because decreasing returns of scale $\varepsilon = \alpha_1 + \alpha_2 = 0.3 + 0.2 = 0.5$ is considered.

Note that the production function (1)

$$q = q(x_1, x_2) = x_1^{\alpha_1} x_2^{\alpha_2} = x_1^{0.3} x_2^{0.2} \tag{51}$$

now has decreasing returns of scale homogeneity $\varepsilon = \alpha_1 + \alpha_2 = 0.4 + 0.1 = 0.5$

Note also that according to (3)

$$\rho = \left(\frac{\alpha_1}{w_1}\right)^{\alpha_1} \left(\frac{\alpha_2}{w_2}\right)^{\alpha_2} = \left(\frac{0.3}{4}\right)^{0.3} \left(\frac{0.2}{9}\right)^{0.2} \approx 0.215 \tag{52}$$

and

$$\omega = \varepsilon \rho^{-1/\varepsilon} = 0.5\rho^{-2} \approx 10.845 \tag{53}$$

Since for $\varepsilon = 0.5$

$$\Pi(t,q) = -(b+\omega)q^2 + (a-t)q = -40.845q^2 + (50-t)q, \tag{29}$$

problem (39)-(40) or (9)-(10) can now be stated as the following bilevel programming problem:

$$\max_{0<t<a} T(t,q) = tq \tag{54}$$

$$\text{s.t. } \max_{q>0} \Pi(t,q) = (50-t)q - 40.845q^2 . \tag{55}$$

First, we solve the follower's problem (55) for a given but fixed tax amount t. From the first order conditions on (55)

$$\Pi_q(t,q) = -81.690q + 50 - t = 0 \tag{56}$$

we obtain the optimal output level

$$q^*(t) = \frac{50-t}{81.690}. \tag{57}$$

Furthermore, since $\Pi_{qq}(t,q) = -81.690 < 0$, the follower's profit function is concave overall its domain in q and the maximum profit is achieved.

By substituting $q^*(t) = \dfrac{50-t}{81.690}$. into the leader's optimization problem (54) $\max\limits_{0<t<a} T(t,q) = tq$ we get the following tax revenue function of only tax amount t:

$$T\left(t, \frac{50-t}{81.690}\right) = \frac{-t^2+50t}{81.690}. \tag{58}$$

The maximum of the leader's tax revenue function $T(t)$ is obtained for $T'(t) = -2t+50 = 0$ because $T''(t) = -2 < 0$. It is easy to see that $t^* = 25$ and $T^* = 7.651$.

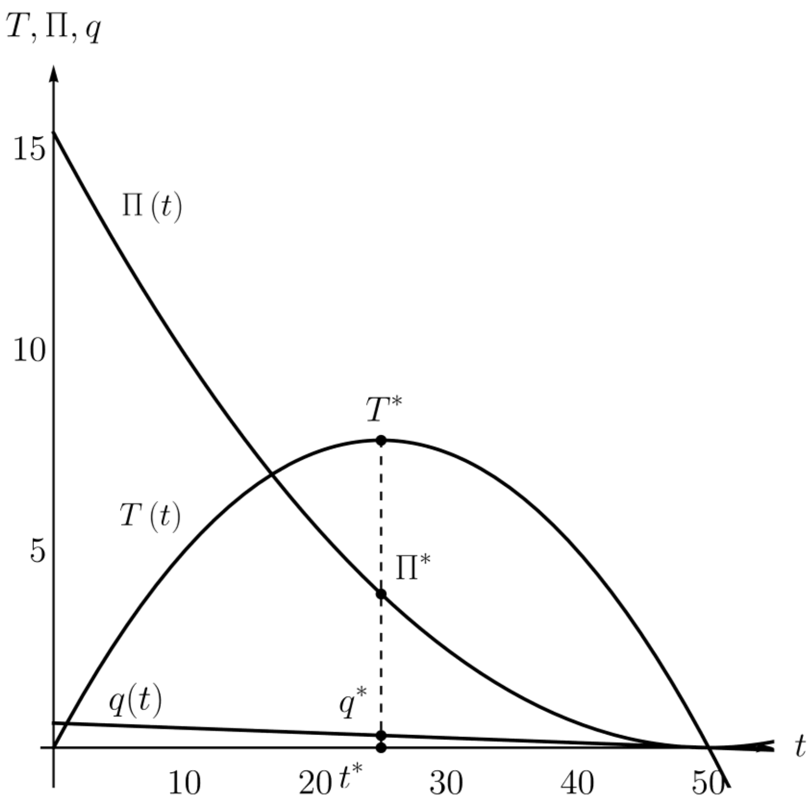

Figure 2. 2D graphical representation of the bilevel problem (54)-(55) solution

Furthermore, the optimal output level

$$q^*(t^*) = \frac{50-t^*}{81.690} = \frac{50-25}{144.022} \approx 0.306. \tag{59}$$

Finally, by substituting $t^* = 25$ and $q^* = 0.306$ back into the follower's profit function, we get the optimal follower's profit

$$\Pi^*\left(t^*,q^*\right)=\left(50-t^*\right)q^*-40.845(q^*)^2=3.825. \tag{60}$$

Furthermore, for decreasing returns to scale $\varepsilon=0.5$, from the optimal output level $q^*=0.306$ and (4) follows the optimal inputs (labor and capital) levels

$$x_1^*=\frac{\alpha_1}{w_1}\rho^{-1/\varepsilon}q^{1/\varepsilon}=\frac{0.3}{4}\cdot 0.215^{-2}\cdot 0.306^2\approx 1016.690, \tag{61}$$

and

$$x_2^*=\frac{\alpha_2}{w_2}\rho^{-1/\varepsilon}q^{1/\varepsilon}=\frac{0.1}{9}\cdot 0.215^{-2}\cdot 0.306^2\approx 301.242. \tag{62}$$

According to Lukač (2022), in case when a follower is a perfect competitor, the follower's maximum profit cannot be achieved for increasing returns of scale, i.e. for $\varepsilon>1$. However, in our model (9)-(10), since the follower is a monopolist with the linear demand (5), the solution of the bilevel programming problem (9)-(10) can be achieved for a certain numerical value of parameters in the model. Let us illustrate this situation with the following example.

Example 3. Consider the similar problem as in the Example 1 and Example 2. There are the same input prices $w_1=4$ and $w_2=9$ respectively, and the same coefficients of demand function be $a=50$ and $b=30$ respectively. In this example the output elasticity α_1 is again set to $\alpha_1=0.3$ but this time the output elasticity α_2 is set to $\alpha_2=1.7$ because decreasing returns of scale $\varepsilon=\alpha_1+\alpha_2=0.3+1.7=2$ is considered.

Note that the production function (1)

$$q=q\left(x_1,x_2\right)=x_1^{\alpha_1}x_2^{\alpha_2}=x_1^{0.3}x_2^{1.7} \tag{63}$$

now has decreasing returns of scale homogeneity $\varepsilon=\alpha_1+\alpha_2=0.4+0.1=0.5$

Note also that according to (3)

$$\rho=\left(\frac{\alpha_1}{w_1}\right)^{\alpha_1}\left(\frac{\alpha_2}{w_2}\right)^{\alpha_2}=\left(\frac{0.3}{4}\right)^{0.3}\left(\frac{1.7}{9}\right)^{1.7}\approx 0.027 \tag{64}$$

and

$$\omega=\varepsilon\rho^{-1/\varepsilon}=2\rho^{-1/2}\approx 12.162 \tag{65}$$

Since for $\varepsilon=2$

$$\Pi\left(t,q\right)=-bq^2+\left(a-t\right)q-\omega q^{1/2}=-30q^2+(50-t)q-12.162q^{1/2}, \tag{66}$$

problem (39)-(40) or (9)-(10) can now be stated as the following bilevel programming problem:

$$\max_{0<t<a} T(t,q) = tq \tag{67}$$

$$\text{s.t. } \max_{q>0} \Pi(t,q) = -30q^2 + (50-t)q - 12.162q^{1/2}. \tag{68}$$

First, we solve the follower's problem (55) for a given but fixed tax amount t. From the first order conditions on (55)

$$\Pi_q(t,q) = -60q + 50 - t - 6.081q^{1/2} = 0 \tag{69}$$

we obtain the optimal output level …

…(To Be Done)…

Figure 3

Figure 3. 2D graphical representation of the bilevel problem (67)-(68) solution

...(To Be Done)…

5. Conclusion

This paper formulates the bilevel problem of determining the optimal tax amount, that is of maximizing a government's tax revenue function, in presence of profit maximizing monopolist whose output is described by Cobb-Douglas production function. The necessary conditions are first derived for the case of two inputs. Necessary conditions can be derived only in an implicit form, which needs to be solved for different values of output elasticities with respect to inputs. The exact form of the tax revenue function as well as optimal tax amount and optimal input levels are derived for the case when decreasing returns of scale equal 0.5 and for the case of the constant returns of scale. Additionally, since our model assumes monopolist with linear demand as a follower, we illustrated through numerical example that the optimal solution of our model can be achieved even in case when increasing returns of scale hold, which is not case if a follower is a perfect competitor.

References

[1] Avvakumov, S. N., Kiselev, Yu. N., Orlov, M. V., Taras'ev, A. M. (2010). Profit maximization problem for Cobb-Douglas and CES production function. Computational Mathematics and Modelling, 21 (3), 336 – 378.

[2] Bracken, J., McGill, J. T. (1972). Computer Program for Solving Mathematical Programs With Nonlinear Programs in the Constraints.

[3] Bracken, J., McGill, J. T. (1972). Mathematical Programs With Optimization Problems in the Constraints. Operations Research, 21 (1), 37-44.

[4] Gahvari, F. (1989). The nature of government expenditures and the shape of the Laffer curve. Journal of Public Eocnomics, 40 (2), 251-260.

[5] Lott, W., Miller, S. (1974). Excise Tax Revenue Maximization. Southern Economic Journal, 40 (4), 657-664. doi: 10.2307/1056384

[6] Lukač, Z. (2023). Optimal taxation of a perfectly competitive firm with Cobb-Douglas production function as a bilevel programming problem. Central European Journal of Operations Research, 31, 891-909. doi: 10.1007/s10100-022-00832-2

[7] Mas-Colell, A., Whinston, M. D., Green, J. R. (1995) Microeconomic Theory. New York: Oxford University Press.

[8] von Stackelberg, H. (1934). Marktform und Gleichgewicht (in German). Wien, Verlag von Julius Springer.

[9] Zevelev, A. A. (2014). Closed Form Solutions in Economics. Online: http://dx.doi.org/10.2139/ssrn.2354226