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Final remedy of the excitation in the analysis of open planar circuits

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SUMMARY

The Spectral Domain Method (SDM) is widely applied to analyse planar microwave circuits, but open planar circuit analysis with SDM requires an efficient excitation modeling technique. Recent excitation modeling technique is very efficient but needs an improvement at low frequencies. This paper introduces an interpolation technique to improve the deficiency of the recent excitation modelling at low frequencies.

***Key words**: Spectral Domain Method (SDM), planar circuits, excitation, antennas, interpolation.*

1. INTRODUCTION

Since 80's planar microwave circuits and planar antennas are becoming essential components of electronic communication systems utilising the microwave region of electromagnetic spectrum because of their potential applications in the design of new devices and components. Meanwhile the rapid increase in the use of higher frequencies for communication and computation over recent years has created a demand for accurate design tools, because the simplified circuit theory which is valid at low frequencies cannot simply be employed for the analysis of the microwave circuits of interest. Existing CAD tools allow rapid design, but accuracy is not sufficient in todays applications. This situation forced the researchers to full-wave techniques.

The high-speed computer has also influenced the computation of electromagnetic problems to the point that most practical computations of fields are now done numerically by computers. This is because most of the practical problems in the electromagnetics can be solved numerically but can not be done analytically. Therefore computers are necessary for numerical solutions. As a result the science of numerically computation of electromagnetics is a mixture of electromagnetic theory, mathematics and numerical analysis.

A number of numerical full-wave techniques are reported in the literature for the analysis of microstrip antennas [1], resonators [2] and circuits [3-5]. The details available in the literature for the extraction of the S -parameters of planar circuits using SDM or other related techniques are limited in scope as the topic is avoided or ignored in most papers. For shielded planar microwave circuits, the S-parameters at a spot frequency are derived in Ref. [6, Chapter 5] from the solution of a matrix equation. In Ref. [6], the tangential electric field is assumed to be identical to zero because of the side-walls. Since they do not exist in open structures, a different method must be sought. In addition, the effects of the port/feed arrangement is assumed to be negligible. The extraction of the S-parameters from the knowledge of the surface current distribution is achieved by applying transmission line theory to the feedlines (de-embedding algorithm) [7], but this method can only be used when the line length is bigger than a half-wavelength. A method was introduced by Jackson [8] to calculate S-parameters of gap-discontinuities in 1985 for open planar circuits, but his technique, in contrast to the method described in Ref. [3], is not complete and efficient at relatively low frequencies. This is because the cosine portion of the travelling wave is truncated one-quarter of a guide wavelength from a zero of the sine. The length of the truncated portion is a function of the operating frequency. At low frequencies, its length becomes larger than the entire circuit's dimension and a large number of extra rooftop functions are required in order to avoid spurious numerical reflections.

In Ref. [3], semi-infinite feedlines, which extend to infinity from the ports, was connected to the ports of the circuit to simplify the excitation. In order to complete the algorithm, however, an additional set of basis functions was required to define the unknown current distribution on the feedlines. Following Ref. [9], travelling current waves were used as basis functions. With this choice, the S-parameters of the circuit could be directly derived from the coefficients of the used travelling waves. But, the use of the travelling wave as a current basis function on the feedline caused a current discontinuity in the interface between the port and feedline. This difficulty was overcome in Refs. [8, 9] by truncating the cosine (real part) portion one-quarter of wavelength from a zero of the sine (imaginary part). But this truncation requires an extra number of rooftop functions which is a large number at low frequencies. In Ref. [3], in order to overcome this difficulty compensation functions were introduced at the interface between the port of the circuit and the adjacent feedline.

Balik in Ref. [3] remedied some existing drawbacks. SDM is a frequency domain technique and requires repeated impedance matrix calculation at each spot frequency, therefore to speed up the impedance matrix calculation is crucial to the efficiency of the technique. The impedance matrix calculation requires two-dimensional numerical integration over an infinite surface. The infinite integration is limited to the finite computer resources and speeded up by Adaptive integration technique in Ref. [5]. The current wave of a finite length which is a number of half wavelength is used to excite the open microwave circuit. The length of the current wave is a function of the operating frequency and at low frequencies this length is bigger and its Fourier transform is finer. Therefore fine integration steps are unnecessary to use in order to accurately model the excitation. The requirement to use fine integration steps are eliminated and course steps which are enough to model the rest of the parameters are used. The value of the integration corresponding to the fine step location is interpolated. With this choice up to *96 %* improvement is gained. The accuracy and efficiency of the new method is demonstrated by means of numerical examples.

2. FINAL EXCITATION MODEL

This section starts with a brief information on the recent excitation model of the open planar circuits introduced by Balik in Ref. [3]. The interpolation technique is developed to improve the efficiency at relatively low frequencies although it works fine compared with other available excitation mechanisms for open structures such as Jackson's technique [9]. The fundamental microstrip mode is assumed to propagate on the feedlines and thus the travelling current waves are chosen as current basis functions. These functions are given in both space and spectral domain by the equations (1), where L is the length of the feedlines which theoretically extends to infinity, but in practice it is chosen to be an integer number of half wavelengths [9], k_n is the pre-calculated wavenumber of the feedline, z_s ($s = i, o$) is the offset of the port from the origin. The letters i, t, r indicate the incident, transmitted and reflected current waves respectively and the unknown coefficients a_r and a_t are coefficients of the reflected and transmitted current waves which are to be calculated. The ports of the circuit are assumed to be placed at the zero origin and shifted to z_{off} by multiplying the Fourier transform by $e^{jZ_{off}k_z}$.

$$
\begin{aligned}
J_i(z)&=\begin{cases} e^{-jk_n(z-z_i)} & -L+z_i \le z \le z_i \\ 0 & \text{otherwise}\end{cases} \Rightarrow \boldsymbol{J}_i(k_n,k_z)=\frac{2}{k_z-k_n} sin\left((k_z-k_n)\frac{L}{2}\right)e^{-j(k_z-k_n)\frac{L}{2}}e^{jk_z z_i} \\
J_r(z)&=\begin{cases} -a_r e^{-jk_n(z-z_i)} & -L+z_i \le z \le z_i \\ 0 & \text{otherwise}\end{cases} \Rightarrow \boldsymbol{J}_r(k_n,k_z)=\frac{2}{k_z+k_n} sin\left((k_z+k_n)\frac{L}{2}\right)e^{-j(k_z+k_n)\frac{L}{2}}e^{jk_z z_i} \\
J_t(z)&=\begin{cases} a_t e^{-jk_n(z-z_0)} & z_0 \le z \le L+z_0 \\ 0 & \text{otherwise}\end{cases} \Rightarrow \boldsymbol{J}_t(k_n,k_z)=\frac{2}{k_z-k_n} sin\left((k_z-k_n)\frac{L}{2}\right)e^{j(k_z-k_n)\frac{L}{2}}e^{jk_z z_0}
\end{aligned}
\tag{1}
$$

For the excited port, the current is:

$$J_{input} = J_i + J_r = \\ =(1-a_r)\cos(k_n(z-z_i)) - j(1+a_r)\sin(k_n(z-z_i)) \quad (2)$$

and for the ports which are not excited, the current is:

$$J_{output} = J_t = \\ =a_t\,[\cos(k_n(z-z_0)) - j\sin(k_n(z-z_0))] \quad (3)$$

The Fourier transforms of the current basis functions for the feedlines as defined in Eq. (1) are given by:

$$\boldsymbol{J}_i(k_n,k_z) = \frac{2}{k_z-k_n}\sin\left((k_z-k_n)\frac{L}{2}\right)e^{-j(k_z-k_n)\frac{L}{2}}e^{jk_z z_i} \quad (4)$$

$$\boldsymbol{J}_r(k_n,k_z) = \frac{2}{k_z+k_n}\sin\left((k_z+k_n)\frac{L}{2}\right)e^{-j(k_z+k_n)\frac{L}{2}}e^{jk_z z_i} \quad (5)$$

$$\boldsymbol{J}_t(k_n,k_z) = \frac{2}{k_z-k_n}\sin\left((k_z-k_n)\frac{L}{2}\right)e^{j(k_z-k_n)\frac{L}{2}}e^{jk_z z_0} \quad (6)$$

As the above equations are seen, there is not truncation of the current wave existed and the compensation functions are used in Ref. [3] to transfer the effects of the excitation into the circuit. The compensation functions are defined for the input port in the space domain by:

$$J_z(z) = \begin{cases} 1-\dfrac{z-z_i}{l_z} & z_i \le z \le z_i + l_z \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

$$J_x(z) = \begin{cases} \dfrac{z-z_i}{l_z} & z_i \le z \le z_i + l_z \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

and for the output port:

$$J_z(z) = \begin{cases} 1+\dfrac{z-z_0}{l_z} & z_0 - l_z \le z \le z_0 \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

$$J_x(z) = \begin{cases} -\dfrac{z-z_0}{l_z} & z_0 - l_z \le z \le z_0 \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

where l_z is the size of the compensation functions in the direction of propagation. $J_z(z)$ in the Eq. (7) transfers the effect of the cosine portion of the incident current wave in the direction of current flow, whereas $J_z(z)$ in the Eq. (8) transfers the effect of the sine portion of the incident current wave in the direction perpendicular to the current flow. Similarly $J_z(z)$ in the Eq. (9) transfers the current wave to the output port in the direction of propagation whereas $J_x(z)$ in the Eq. (10) transfers the current wave in the transverse direction.

The compensation function is actually a semi-rooftop function with an offset from the origin in either $-z$ or $+z$ directions and the missing portion of the rooftop function has been completed by the current waves. Fourier transforms of the half rooftop functions are given by:

$$\boldsymbol{J}_{left}(k_z) = \frac{1}{k_z^2 l_z}\left[(1-\cos(k_z l_z)) + j(\sin(k_z l_z) - k_z l_z)\right] \quad (11)$$

$$\boldsymbol{J}_{right}(k_z) = \frac{1}{k_z^2 l_z}\left[(1-\cos(k_z l_z)) - j(\sin(k_z l_z) - k_z l_z)\right] \quad (12)$$

where $\boldsymbol{J}_{left}$ and $\boldsymbol{J}_{right}$ indicate the Fourier transform of the left and right hand side of a rooftop function. The derivation of the compensation functions from the left and right hand side of the rooftop function for the input port are given by:

$$\boldsymbol{J}_z(k_z) = \boldsymbol{J}_{right}(k_z)e^{jk_z z_{off}} \quad (13)$$

$$\boldsymbol{J}_x(k_z) = \boldsymbol{J}_{left}(k_z)e^{jk_z(z_{off}+l_z)} \quad (14)$$

and for the output port, they are given by:

$$\boldsymbol{J}_z(k_z) = \boldsymbol{J}_{left}(k_z)e^{jk_z z_{off}} \quad (15)$$

$$\boldsymbol{J}_x(k_z) = \boldsymbol{J}_{right}(k_z)e^{jk_z(z_{off}-l_z)} \quad (16)$$

Including all current basis functions of the entire system, the total current is expressed as:

$$\boldsymbol{J}_{total} = \boldsymbol{J}_{box}(k_x,k_z) + \\ + \sum_{n=1}^{N}\left(a_{pn}\boldsymbol{J}_{port_n}(k_x,k_n k_z) + a_{cn}\boldsymbol{J}_{comp_n}(k_x,k_z)\right) \quad (17)$$

where N is the number of ports, $\boldsymbol{J}_{box}$ refers to the basis functions of the microwave circuit, $\boldsymbol{J}_{port_n}$ is either the sum of the incident and reflected current wave for the excited port or the transmitted current wave for unexcited ports and $\boldsymbol{J}_{comp_n}$ is the Fourier transform of the compensation function. To calculate the S-parameters of the circuit a_{pn} must be known. The Method of Moments is employed to eliminate the electric field components and to find the unknown coefficients. A total of N weighting functions must be defined to complete the algorithm. These are chosen to be a triangle function which straddles the lines separating each port and the feedline in the direction of current flow and a pre-calculated basis function in the direction perpendicular to current flow as in Ref. [4]. After application of the Method of Moments, the matrix equation yields:

$$[\boldsymbol{Z}][\boldsymbol{I}] = [0] \quad (18)$$

As two travelling waves, which are the incident current wave with unit amplitude and the reflected current wave with unknown amplitude, are used for the feedline connected to the input port of the circuit, $\boldsymbol{Z}$ is not a square matrix and contains unknown coefficients, as well as the known unit amplitude. The column of the impedance matrix corresponding to the incident current wave products is moved to the right hand side of the equation and the Eq. (18) becomes:

$$[\boldsymbol{Z}]_n[\boldsymbol{I}]_n=[\boldsymbol{Z}]_i \tag{19}$$

where $\boldsymbol{Z}_n$, consists of the elements related to unknown current basis functions and $\boldsymbol{Z}_i$ is a column vector containing the elements related to incident current waves. Any root-finding procedure can be applied to solve the Eq. (19) and the unknown coefficients can be found. The S-parameters of the N-port circuit are actually the coefficients of the reflected and the transmitted current waves.

This Excitation Model is found by the author to be a significant improvement, but as seen in the Eq. (1), k_n is a function of the operating frequency and at low frequencies L tends to infinity whereas the length of the Fourier transform tends to zero. In the numerical integration unnecessary fine integration steps must be used to model this fine Fourier transform functions. The range of the numerical integration is determined by the current basis function which is a rooftop function as explained in Ref. [5].

In the numerical integration, an appropriate integration step is determined in this contribution for the integration by using the Fourier transform of the rooftop function and the requirement to use fine integration steps are eliminated. The value of the integration corresponding to the fine step location is interpolated.

3. NUMERICAL RESULTS

The present implementation is applied to the step-discontinuity which is modelled by Balik in Ref. [10]. The circuit shown in Figure 1 is completely open on a substrate thickness *1.272 mm* and relative permittivity *10*. The dimensions of the metallisation are given in Figure 1.

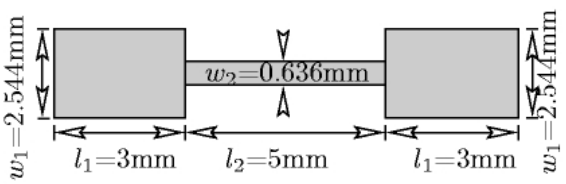

Fig. 1 Microstrip step discontinuity

For this analysis, the length of the feedlines is 4 half-wavelength of the current wave, the rooftop function dimensions are *0.318 mm* in *x* and *0.5* in *z* direction. The range of the integration is one cycle of the Fourier transform of the rooftop functions which is $2\pi/1_s$ ($s = x, z$). The number of integration steps are chosen to be *200* and the number of integration step for each cycle of the Fourier transform of the current wave is *20*. The minimum and maximum operation frequencies are *1 GHz* and *20 GHz* respectively.

At the minimum operating frequency, the length of the feedline is *224 mm* which is *448* times bigger than the rooftop function size in the propagation direction. Since *20* fine integration steps are set to model each cycle of the Fourier transformed current wave, the total number of fine integration steps are *4480* without interpolation. The saving with this enhancement is *96 %*. The operating frequency where no interpolation required is *20 GHz* which is maximum operating frequency. The number of interpolation decays exponentially as frequency increases. The *S*-parameter results are compared with the published date in Ref. [10] and plotted in Figure 2. As shown in Figure 2 the agreement is very good.

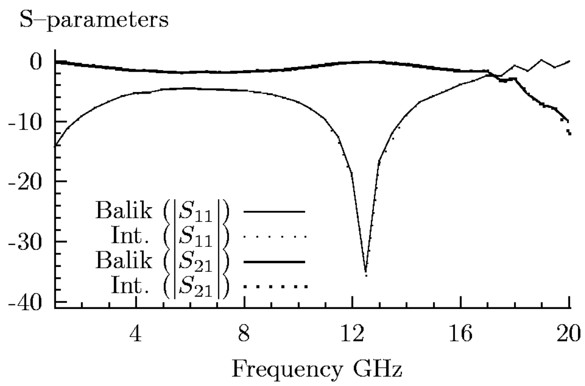

Fig. 2 Plot of S -parameter magnitude for step discontinuity

4. CONCLUSION

This paper presents an interpolation technique to improve the deficiency of the recent excitation modeling technique at low frequency. The improvement is up to *96 %* for the microstrip step discontinuity. In addition this improvement is gained without losing any accuracy.

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KONAČNO POBOLJŠANJE RJEŠENJA POBUDE PRI ANALIZI OTVORENIH RAVNINSKIH MIKROVALNIH KRUGOVA

SAŽETAK

Metoda spektralne domene uvelike se koristi za analizu ravninskih mikrovalnih krugova. Analiza otvorenih ravninskih krugova pomoću SDM zahtijeva djelotvorniju tehniku modeliranja pobude. Nova tehnika modeliranja pobude vrlo je djelotvorna ali zahtijeva poboljšanje u nižim frekvencijama. Ovaj rad opisuje jednu tehniku interploacije za odstranjivanje nedostataka modeliranja pobude u niskim frekvencijama.

Ključne riječi: *Metoda spektralne domene, planarni krugovi, pobuda, antene, interpolacija.*