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Dynamic Pricing in a Blockchain-Enabled Dual-Channel Supply Chain

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Abstract: With the rapid development of Internet technology and digital economy, many manufacturers sell their products through various channels. This paper develops Stackelberg game models by backward induction to analyze dynamic pricing decisions in a dual-channel supply chain enabled with blockchain technology. These models account for strategic consumer purchasing behaviors across sales periods. Analytical and numerical results provide insights into the pricing impacts of blockchain adoption under centralized and decentralized channel structures. Key findings indicate that blockchain adoption incentivizes channels to raise prices in early sales periods but lower prices later on. The results also reveal how consumer strategic behavior influences optimal pricing. This research contributes new knowledge on incorporating emerging blockchain technologies into retail channel design and operations.

Keywords: blockchain; dual-channel system; pricing strategy; strategic consumption

1 INTRODUCTION

In recent years, Internet technology and digital economy have changed the consumers' shopping environment, prompting them to significantly improve their rationality in making consumption decisions. More and more consumers can quickly and conveniently obtain product quality information and market consumption feedback with the help of the Internet [1-3]. According to analysts' prediction, the market penetration rate of global online shopping will increase by 25% in 2026 [4]. Statistics from Chinese government departments show that the online shopping market in China reached 13.1 trillion RMB in 2021. The number of online shopping users in China has exceeded 840 million by the end of December 2021 [5].When they purchase products, they will not only consider the utility obtained from the current purchase, but also estimate the utility obtained from the purchase when the price is reduced in the next period [6]. This kind of consumers are generally called strategic consumers in academic research [7]. However, the product quality information obtained by consumers through the Internet is limited, and the authenticity of various product quality information is not entirely trustworthy. Therefore, strategic consumers, with such information disadvantage, may decide to give up their purchase or postpone purchasing based on prudent consideration [8].

Meanwhile, Internet technology and digital economy have also changed the marketing activities of enterprises, forcing many enterprises to adopt blockchain, big data and other advanced technologies to expand sales channels and build green and sustainable supply chain systems [9-15]. Many manufacturing enterprises have introduced online channels in addition to the traditional wholesale sales channels in the fiercely competitive market. At the same time, there are also many traditional manufacturing enterprises, such as Haier and Midea which have opened up many traditional physical stores all over China, as well as many online sales channels on Internet platforms [16].

In addition, in order to ease online consumers' doubts about product quality information, more and more manufacturing enterprises have introduced blockchain to stimulate consumers' desire to purchase. They use blockchain to improve consumers' consumption experience and the utility level, thus expanding the scale of market [17]. For example, in order to enhance the transparency of commodity information, Walmart has joined hands with IBM to integrate blockchain into the food supply chain [18]; Amazon and Nestle are working together to display details of the origin and quality of all kinds of coffee with the help of blockchain [19]. Therefore, more and more manufacturers use blockchain alone or jointly with retailers to increase the transparency of product information. By doing so, they can not only bring more benefits to consumers, but also expand the market scale of enterprises. However, there are still many manufacturing enterprises that refuse to use blockchain for different reasons. Therefore, in the marketing environment where the volume of strategic consumers is increasing and the tendency of online shopping is becoming more and more popular, we attempt to address the following important research questions:

(1) If a dual-channel system invests in blockchain, what impacts will it have on the strategic consumers who are increasingly fond of online shopping?

(2) What impacts will blockchain have on the product pricing and optimal profit of enterprises in a dual-channel supply chain?

(3) Why might some dual-channel supply chain systems refuse to invest in blockchain?

In order to answer these research questions, in this paper we construct a dual-channel supply chain system to study the Stackelberg games between a supplier and a retailer. Aiming at analysing the impact of blockchain on the dual-channel supply chain, we conduct our research from the perspective of centralized and decentralized model, respectively. At the same time, we assume that the supplier may or may not adopt blockchain in the second period. Therefore, based on different combinations of blockchain and decision-making conditions, we have constructed four different game models as follows: a centralized model without blockchain; a centralized model with blockchain; a decentralized model without blockchain; a decentralized model with blockchain. By using backward induction, we have solved the equilibrium solutions of the above four models. In addition, we have also studied the impacts of the change of consumers' strategic degree on the product pricing and optimal profit of the dual-channel system during two periods. For further analysis, we have carried out numerical simulation on the above four models.

The remainder of this paper is organised as follows. Section 2 presents the related literature. Section 3 presents the problem descriptions and model setting. Section 4 provides the analysis of four models. Section 5 provides numerical simulation of these models. Section 6 presents our conclusions and proposals. All the proof details are presented in the appendices of this paper.

2 LITERATURE REVIEW

2.1 Strategic Consumption Behaviours

Besanko and Winston [20] pointed out that if monopoly manufacturers take into account consumers' strategic behaviours when they determine the prices of products, this will bring a profit increase of about 20% to the enterprises. Levin et al. [21] also believed that the evaluation of products by strategic consumers will have different impacts on the product pricing decisions of monopoly manufacturers with the passage of time. Many scholars have also studied strategic consumers on the basis of a dual-channel system. Li et al. [22] pointed out that with the increase in the proportion of short-sighted consumers, the price competition between a physical channel and a network channel under decentralized decision-making will become more intense, with the manufacturer dominating the competition at this time; in addition, the collusion between supply-chain members can make the prices of two channels reach high prices, with the retailer dominating the competition at this time. Cao and Huang [23] believed that if the number of strategic consumers in the market is small, and their influence on the supply chain is very small, manufacturers mainly consider the behaviours of retailers when making pricing decisions. However, if the number of strategic consumers in the market is large, and their influence on the supply-chain is great, manufacturers mainly consider the behaviours of strategic consumers when pricing their products. The conclusion of Li and Wei [24] is similar to that of Cao. They believed that if the overall influence of strategic consumers on the dual-channel system is very small, the main problem faced by the manufacturer is channel competition. However, if the influence of strategic consumers on the dual-channel system is very large, the main problem faced by manufacturers is that of dealing with the loss of profits caused by consumers' strategic consumption behaviours. But Xu and Li [25] believed that the efficiency of a dual-channel system mainly depends on two factors. The first is the purchase risk in using a network channel, and the second is the search cost in using a retail channel.

2.2 Blockchain in the Supply Chain

Eljazzar et al. [26] believed that blockchain, with its unique technological advantages, can effectively solve the difficult problems of complicated information and data and the high processing cost of implementing enterprise strategies and plans, can greatly increase the transparency of various data in the process of market transactions, and can expand the extent and scope of resource sharing. Niu et al. [27] studied the issue of multinational companies introducing blockchain into the global sales network to provide product quality verification, and they believe that blockchain will have a double impact on multinational companies. On the one hand, blockchain will increase the wholesale profits of multinational companies, but on the other hand, blockchain will reduce their retail profits and international tax benefits. Li [28] studied the problem of reducing the channel cost of supply-chain through blockchain. In addition, he studied the boundary conditions of investment blockchain for the supply-chain system as well as the coordination and efficiency of the supply-chain system. Pietro's [29] research compared the roles of blockchain technology and intelligent contracts in the supply chain. Specifically, he pointed out that blockchain can help reduce the business risks and transaction costs of the supply-chain, although blockchain is sometimes not economically feasible. Christoph and Wagner [30] studied the roles of blockchain in supply chain from the perspective of transaction cost, and they believed that blockchain can limit the impacts of opportunistic behaviour, environment and behavioural uncertainty, in addition to reducing transaction costs through a transparent and effective transaction process.

In particular, some scholars have studied the impacts of blockchain based on the perspective of the dual-channel supply chain. Under the scenario of a dual-channel system, Zhang et al. [31] studied the impacts of blockchain on the risk-averse members of market transactions. They further concluded that the system strategy involving investment in blockchain mainly depended on three factors. The first is the unit blockchain operation cost of the system, the second is the direct selling cost, and the third is demand fluctuation. Under the condition of a dual-channel supply chain, Jiang et al. [32] studied the impacts of blockchain on market transactions for the fresh agricultural products in depth. They pointed out that the effectiveness of the investment in blockchain depended on the multiple effects of variable investment cost thresholds, fixed investment cost threshold and cost sensitivity threshold; the difference in the proportion of channels and the increase in the price elasticity coefficient both produce positive improvements in profits and dynamically affect the blockchain investment conditions. Liang and Xiao [33] believed that if a dual-channel system adopted blockchain in the market, the manufacturer's profit in direct selling mode would be higher than that in distribution mode.

Most of the existing literature concentrate on analysing the impacts of a single factor, either that of strategic consumers or of blockchain, on the pricing strategy. We also found that existing research considering both factors mentioned above is rare. Close to the research in this paper is Li et al. [22] research, which considers both short-sighted and strategic consumption behaviors in the dual-channel supply chain system. They divide the whole market demand into five parts according to the balance between the utility and valuation of products by strategic consumers, and derive different demand functions. This paper further introduces the blockchain on the basis of their dual-channel supply chain game model, and focuses on the influence of blockchain on the pricing and profit of dual-channel supply chain system members. In addition, in the aspect of describing the cost of blockchain, this paper adopts the results in Sun et al. [34] and Li and Wan [35]. They pointed out that the total investment cost of blockchain is in the form of a quadratic function, which is also in line with commercial practice. Thus, in contrast to the existing research, in this paper, we explore the impacts of blockchain on pricing decisions under the scenario of a dual-channel system during two periods. In order to better promote the wide application of blockchain for the dual-channel system, we constructed a Stackelberg game model under the strategic consumer market environment.

3 PROBLEM DESCRIPTION AND MODEL SETTING

We consider a supply chain system in which a supplier and a retailer sell products during two periods. The supplier sells products to consumers through a traditional sales channel, i.e., the retailer's physical store. Meanwhile, the supplier also sells products to consumers through a network channel, i.e., self-operated online stores. During the full-price sales period, the supplier is required to dynamically set online channel prices for the consumers and wholesale prices for the retailer. Furthermore, the retailer determines its retail prices for the physical channel based on the wholesale prices set by the supplier. Both the supplier and the retailer follow a basic principle, that is, to maximize their profits during two periods. The model assumes that the product has the characteristics of perishable goods, so the manufacturer and the retailer are expected to implement price discounts for all the remaining products through their respective sales channels in the second period. They each decide their own prices and make discounted sales to the consumers in the discounted sales period.

In addition, we assume that the supplier is in the leading position; and is regarded as the leader of the dual-channel system. In contrast, we assume that the retailer is in the following position, observing the actions of the supplier and then making their own best decisions. Therefore, the retailer is regarded as the follower of the system. Fig. 1 below is a schematic diagram of the game model.

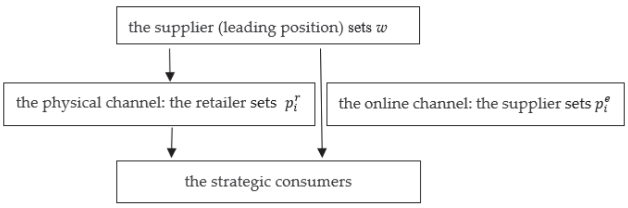


Figure 1 The schematic diagram of the dual-channel system model

Referring to the practices of Li et al. [22], we assume that the supplier sells products to the retailer at wholesale prices, and the price is denoted by w. The retailer sells these products to the end consumer through a physical channel at a certain markup rate. More specifically, in the first period, the supplier sells these products to the retailer at the wholesale price, w, while determining the retail price p_1^e for its network channels. The retailer decides to determine the physical channel retail price p_1^r at the beginning of the sales period. The consumers decide on whether to purchase the products during the full-price sales period and through which channel according to the comparison between the product price and the utility. After entering the discounted sales period, the supplier and the retailer respectively make discounted sales, in which the supplier decides its own network channel price p_2^e and the retailer decides its own physical channel price p_2^r. We set the unit product cost of the supplier to a constant c ($0 < c < 1$) [6, 16, 22, 33].

We consider that all the consumers are strategic consumers with intertemporal consumption behaviour characteristics. The retailer's physical channel price for each sales period is p_i^r (i = 1, 2); and the supplier's network channel price for each sales period is p_i^e (i = 1, 2). We assume that the utility discount of a product in the second period is δ, and the condition of $0 < \delta < 1$ needs to be satisfied [6, 16, 22]. For convenience of description, we use the subscript "M" to represent the supplier and the subscript "R" to represent the retailer. The superscript "C" indicates a centralized decision, and the superscript "D" indicates a decentralized decision. The superscript "r" denotes a physical channel, and the superscript "e" denotes a network channel; the superscript "B" indicates the condition under the use of blockchain. The subscripts “1” and "2" denote the full-price sales period and the discounted sales period, respectively.

In addition, we assume that all the consumers have special preferences with respect to the network channel, and we describe the channel preference behaviour through the channel preference coefficient ε ($0 < \varepsilon < 1$) [22]. Because the model assumes that the supplier and the retailer sell products at a discount after entering the discounted sales period, the prices of the discounted sales period are lower than that of the full-price sales period. In addition, because the sales cost of online channels is lower than that of physical channels, the prices of online channels are also lower than that of physical channels [22]. Therefore, there is a quantitative relationship between the prices involved in the model, which is expressed as follows: $p_1^r > p_1^e > p_2^r > p_2^e > w > c$. The assumption is also consistent with the actual situation. For example, the price of a Dell laptop with the same configuration in JD.COM is about 5% lower than that in physical channels. The game sequence of events is summarized in Fig. 2.

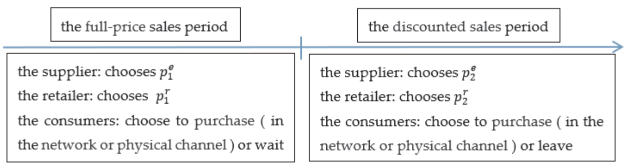


Figure 2 The event sequence diagram

We believe that the dual-channel system has a wait-and-see and acceptance process with the use of blockchain. For this reason, we assume that the supplier may adopt blockchain in the discounted sales period [36]. According to Jiang et al. [32], the intelligent contracts based on blockchain can shorten transaction time, speed up circulation and reduce double losses, thus reducing the unit cost of products. Christoph and Wagner [30] believed that blockchain can limit the impacts of opportunistic, environmental and behavioural uncertainties prevailing in the supply chain, thus reducing transaction costs. Therefore, the unit production cost of a product will be reduced to ($c - b$) after the supplier's blockchain input during the discounted sales period, where we use b to represent the intensity of the blockchain input (i.e., the cost reduction value per unit product), and it needs to meet $0 < b < c$, with the total cost of the blockchain input being $B_r\left(b\right)=\frac{1}{2}k_r b^2$ (where) $0 < k_r < 1$ [34, 35]. According to the principle of increasing marginal cost and referring to the researches [34, 35], we set the manufacturer's blockchain investment cost as a quadratic function. Among them, k_r represents the manufacturer's blockchain innovation ability. The smaller the value of k_r, the stronger the cost reduction ability that manufacturers can get from investing in blockchain, that is, the less innovation cost that manufacturers bear in order to reduce a certain unit variable cost. This hypothesis shows that if the manufacturer bears blockchain investment cost $\frac{1}{2}k_r b^2$, it can reduce its unit variable cost by b.

In addition, we assume that the utility valuation per unit product by strategic consumers is V, and V obeys the uniform distribution on [0, 1] [22, 37]. Hofmann et al. [38] pointed out that with the help of blockchain, consumers are more likely to obtain effective and reliable product quality information, thus having more confidence in the authenticity of product quality, and ultimately promoting the utility level of unit products. Kshetr [39] pointed out that blockchain can help the supply chain improve the transparency and sense of responsibility of product manufacturers, increase the consumers' trust in product characteristics, thus improving the utility level of each unit product. Therefore, we assume that after adopting blockchain, the consumer's utility valuation per unit product in the discounted sales period will increase to $V + e$ $(0 < e < 1)$. Next, we can divide the market into five different components, as presented in Fig. 3.

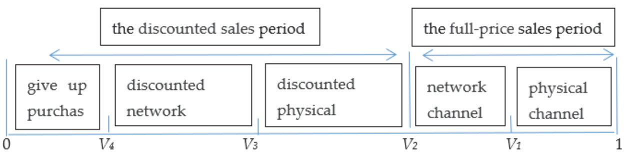


Figure 3 Analysis chart of strategic consumers' purchasing decisions

The supplier's demands through the network channel during two periods are:

$$D_{M1}=V_1-V_2 \tag{1}$$

$$D_{M2}=V_3-V_4 \tag{2}$$

Among them, V_1 represents the undifferentiated purchase point of strategic consumers through physical channels and online channels during the full-price sales period, V_2 represents the undifferentiated purchase point of strategic consumers during the full-price sales period and the discount sales period, V_3 represents the undifferentiated purchase point of strategic consumers through physical channels and online channels during the discount sales period, and V_4 represents the undifferentiated purchase point of strategic consumers during the discount sales period. See Appendix A and C for the specific calculation process of V_1, V_2, V_3 and V_4.

The retailer's demands through the physical channel during two periods are:

$$D_{R1}=1-V_1 \tag{3}$$

$$D_{R2}=V_2-V_3 \tag{4}$$

In the above Eq. (3) Eq. (4), the meaning of V_1, V_2 and V_3 is the same as that in the previous Eq. (1) and Eq. (2).

4 ANALYSIS OF THE MODELS

4.1 Centralized Decision-Making Model

When the dual-channel system makes centralized decisions, the supplier and the retailer play a cooperative game. They jointly decide on the prices of the product during two periods. Their common goal is to gain the maximum profit for the whole system. The consumers make decisions about whether to purchase, in which period to purchase and through which channel to purchase according to their own valuation of the product.

4.1.1 Centralized Decision-Making without Blockchain (C)

The profits of the dual-channel system during two periods are expressed as follows:

$$\pi_1^C=\left(1-V_1\right)\left(p_1^r-c\right)+\left(V_1-V_2\right)\left(p_1^e-c\right) \tag{5}$$

$$\pi_2^C=\left(V_2-V_3\right)\left(p_2^r-c\right)+\left(V_3-V_4\right)\left(p_2^e-c\right) \tag{6}$$

The total profit of the system during two periods is expressed as follows:

$$\begin{aligned}\pi^C&=\pi_1^C+\pi_2^C=\left(1-V_1\right)\left(p_1^r-c\right)+\left(V_1-V_2\right)\left(p_1^e-c\right)+\\&+\left(V_2-V_3\right)\left(p_2^r-c\right)+\left(V_3-V_4\right)\left(p_2^e-c\right)\end{aligned} \tag{7}$$

Among them, we have: $V_1=\frac{p_1^r-p_1^e}{1-\varepsilon}$; $V_2=\frac{p_1^e-p_2^r}{\varepsilon-\delta}$; $V_3=\frac{p_2^r-p_2^e}{\delta\left(1-\varepsilon\right)}$; $V_4=\frac{p_2^e}{\delta\varepsilon}$. In particular, V_1, V_2, V_3 and V_4 represent the undifferentiated purchase points under different channels without blockchain, respectively. Their solution details are presented in Appendix A.

In the discounted sales period, the decision model of the system is expressed as follows:

$$\begin{aligned}&\max\pi_2^C=\left(V_2-V_3\right)\left(p_2^r-c\right)+\left(V_3-V_4\right)\left(p_2^e-c\right)\\&\text{s.t. } 0\le\frac{p_2^e}{\delta\varepsilon}\le\frac{p_2^r-p_2^e}{\delta\left(1-\varepsilon\right)}\le\frac{p_1^e-p_2^r}{\varepsilon-\delta}\le 1\end{aligned} \tag{8}$$

In the full-price sales period, the decision model of the system is expressed as follows:

$$\begin{aligned}&\max\pi^C=\left(1-V_1\right)\left(p_1^r-c\right)+\left(V_1-V_2\right)\left(p_1^e-c\right)+\\&+\left(V_2-V_3\right)\left(p_2^r-c\right)+\left(V_3-V_4\right)\left(p_2^e-c\right)\\&\text{s.t. } 0\le\frac{p_2^e}{\delta\varepsilon}\le\frac{p_2^r-p_2^e}{\delta\left(1-\varepsilon\right)}\le\frac{p_1^e-p_2^r}{\varepsilon-\delta}\le\frac{p_1^r-p_1^e}{1-\varepsilon}\le 1\end{aligned} \tag{9}$$

By constructing Lagrange function, we use KKT condition to solve the decision model of the dual-channel system, and obtain the conclusion of Lemma 1. The proof details of Lemma 1 are presented in Appendix B.

Lemma 1. *When the dual-channel system makes centralized decisions without blockchain, using the backward induction calculation, the equilibrium results of the game can be obtained as follows:*

$$p_1^{eC*} = \frac{2\varepsilon^2 - 2\varepsilon\delta - c\delta + 2c\varepsilon}{4\varepsilon - 3\delta} \tag{10}$$

$$p_1^{rC*} = \frac{4\varepsilon + 4c\varepsilon - \varepsilon\delta - 2c\delta - 3\delta}{8\varepsilon - 6\delta} \tag{11}$$

$$p_2^{eC*} = \frac{1}{2}\left(c + \frac{\delta c(2\varepsilon - \delta) + 2\delta\varepsilon(\varepsilon - \delta)}{4\varepsilon - 3\delta}\right) \tag{12}$$

$$p_2^{rC*} = \frac{2(2c + \delta)\varepsilon^2 - c\delta^2 - \delta\varepsilon(c + 2\delta)}{2\varepsilon(4\varepsilon - 3\delta)} \tag{13}$$

$$\pi_{M2}^{C*} = \frac{1}{24}c\left(\frac{12w}{\delta - \varepsilon} + \frac{2c(3 - \delta)}{\delta\varepsilon} + \frac{5(4c - 3\delta)}{4\varepsilon - 3\delta} - 9\right) \tag{14}$$

$$\pi_{R2}^{C*} = \frac{(\varepsilon - c)\left(c\delta^2 + \delta(c - 6w + 2\delta)\varepsilon - 2(2c - 4w + \delta)\varepsilon^2\right)}{2\varepsilon(3\delta - 4\varepsilon)^2} \tag{15}$$

$$\pi^{C*} = \frac{c^2\left(\delta(3 + \delta) - 4\varepsilon\right) + 8c\delta\varepsilon(\varepsilon - \delta) + \delta\varepsilon(3\delta + \delta\varepsilon - 4\varepsilon)}{4\delta\varepsilon(3\delta - 4\varepsilon)} \tag{16}$$

Corollary 1. *When the supply chain system makes centralized decisions without using blockchain, since $0 < \varepsilon < 1$ and $\varepsilon > \delta$ is satisfied, for any ε,*

(a) $\frac{\partial P_1^{eC}}{\partial \delta} > 0$;

(b) $\frac{\partial P_1^{rC}}{\partial \delta} > 0$ only if: $c > \varepsilon$; $\frac{\partial P_1^{rC}}{\partial \delta} < 0$ only if: $c < \varepsilon$;

(c) $\frac{\partial P_2^{eC}}{\partial \delta} > 0, \frac{\partial P_2^{rC}}{\partial \delta} > 0$ only if:

$$0 < \delta < \frac{2\left(2c\varepsilon + 4\varepsilon^2 - \sqrt{2}\sqrt{-c^2\varepsilon^2 - c\varepsilon^3 + 2\varepsilon^4}\right)}{3(c + 2\varepsilon)},$$

or

$$\frac{2\left(2c\varepsilon + 4\varepsilon^2 + \sqrt{2}\sqrt{-c^2\varepsilon^2 - c\varepsilon^3 + 2\varepsilon^4}\right)}{3(c + 2\varepsilon)} < \delta < 1;$$

$\frac{\partial P_2^{eC}}{\partial \delta} < 0, \frac{\partial P_2^{rC}}{\partial \delta} < 0$ only if:

$$\frac{2\left(2c\varepsilon + 4\varepsilon^2 - \sqrt{2}\sqrt{-c^2\varepsilon^2 - c\varepsilon^3 + 2\varepsilon^4}\right)}{3(c + 2\varepsilon)} < \delta < \frac{2\left(2c\varepsilon + 4\varepsilon^2 + \sqrt{2}\sqrt{-c^2\varepsilon^2 - c\varepsilon^3 + 2\varepsilon^4}\right)}{3(c + 2\varepsilon)};$$

(d) $\frac{\partial \pi^C}{\partial \delta} > 0$ only if:

$$0 < \delta < \frac{4\left(3c^2\varepsilon - 2\sqrt{2c^3\varepsilon^4 - c^2\varepsilon^5 - c^4\varepsilon^3}\right)}{9c^2 + 4c^2\varepsilon - 8c\varepsilon^2 + 4\varepsilon^3}$$

or $\frac{4\left(3c^2\varepsilon + 2\sqrt{2c^3\varepsilon^4 - c^2\varepsilon^5 - c^4\varepsilon^3}\right)}{9c^2 + 4c^2\varepsilon - 8c\varepsilon^2 + 4\varepsilon^3} < \delta < 1$;

$\frac{\partial \pi^C}{\partial \delta} < 0$ only if:

$$\frac{4\left(3c^2\varepsilon - 2\sqrt{2c^3\varepsilon^4 - c^2\varepsilon^5 - c^4\varepsilon^3}\right)}{9c^2 + 4c^2\varepsilon - 8c\varepsilon^2 + 4\varepsilon^3} < \delta < \frac{4\left(3c^2\varepsilon + 2\sqrt{2c^3\varepsilon^4 - c^2\varepsilon^5 - c^4\varepsilon^3}\right)}{9c^2 + 4c^2\varepsilon - 8c\varepsilon^2 + 4\varepsilon^3}.$$

Corollary 1 shows that when the system makes centralized decisions without blockchain, regardless of the value of the channel preference coefficient ε, and with the increase in the consumers' strategic degree δ, the supplier tends to raise its network channel price during the full-price sales period. If the cost per unit product c exceeds the consumers' channel preference coefficient ε, the retailer tends to raise the physical channel price of the full-price sales period with the increase in δ. On the contrary, when c is less than ε, the retailer tends to lower the physical channel price during the full-price sales period with the increase in δ. When δ is extremely low or extremely high, and if it keeps growing, both entities of the system tend to increase the prices of the two channels during the discounted sales period, with the total profit of the system also maintaining the growth trend. However, when δ is in the middle state, and if it keeps growing, both entities of the system tend to reduce the prices of the two channels during the discounted sales period. At this point, the total profit of the system also follows a downward trend.

Corollary 1 shows that when the system makes centralized decisions without blockchain, if a large number of strategic consumers choose to postpone their purchase, the system will increase the prices of the two channels during the discounted sales period, so as to ensure that the total profit of the system increases greatly.

4.1.2 Centralized Decision-Making with Blockchain (CB)

When the supplier adopts blockchain in the discounted sales period, and the system makes centralized decisions, we assume that the supplier bears the cost of the blockchain input. Both entities of the system also play a cooperative game. They jointly decide on the prices of the products during the two periods to obtain the maximum profit for the whole system. The system can obtain profits with the following expressions:

$$\pi_1^{CB} = \left(1-V_1\right)\left(p_1^r - c\right) + \left(V_1 - V_2\right)\left(p_1^e - c\right) \tag{17}$$

$$\pi_2^{CB} = \left(V_2 - V_3\right)\left(p_2^r - \left(c-b\right)\right) + \\ +\left(V_3 - V_4\right)\left(p_2^e - \left(c-b\right)\right) - \frac{1}{2}k_r b^2 \tag{18}$$

The total profit of the system during the two periods is expressed as follows:

$$\pi^{CB} = \pi_1^{CB} + \pi_2^{CB} = \left(1-V_1\right)\left(p_1^r - c\right) + \left(V_1 - V_2\right)\left(p_1^e - c\right) + \\ +\left(V_2 - V_3\right)\left(p_2^r - \left(c-b\right)\right) + \left(V_3 - V_4\right)\left(p_2^e - \left(c-b\right)\right) - \\ -\frac{1}{2}k_r b^2 \tag{19}$$

Among them, we have: $V_1 = \frac{p_1^r - p_1^e}{1-\varepsilon}$; $V_2 = \frac{p_1^e - p_2^r + \delta e}{\varepsilon - \delta}$; $V_3 = \frac{p_2^r - p_2^e - \delta e\left(1-\varepsilon\right)}{\delta\left(1-\varepsilon\right)}$; $V_4 = \frac{p_2^e}{\delta\varepsilon} - e.$ Similarly, V_1, V_2, V_3 and V_4 represent the undifferentiated purchase points under different channels with blockchain, respectively. Their solution details are presented in Appendix C.

In the discounted sales period, the decision model of the system is expressed as follows:

$$\max \pi_2^{CB} = \left(V_2 - V_3\right)\left(p_2^r - \left(c-b\right)\right) + \\ +\left(V_3 - V_4\right)\left(p_2^e - \left(c-b\right)\right) - \frac{1}{2}k_r b^2 \\ \text{s.t. } 0 \le \frac{p_2^e}{\delta\varepsilon} - e \le \frac{p_2^r - p_2^e - \delta e\left(1-\varepsilon\right)}{\delta\left(1-\varepsilon\right)} \le \frac{p_1^e - p_2^r + \delta e}{\varepsilon - \delta} \le 1 \tag{20}$$

In the full-price sales period, the decision model of the system is expressed as follows:

$$\max \pi^{CB} = \left(1-V_1\right)\left(p_1^r - c\right) + \left(V_1 - V_2\right)\left(p_1^e - c\right) + \\ +\left(V_2 - V_3\right)\left(p_2^r - \left(c-b\right)\right) + \left(V_3 - V_4\right)\left(p_2^e - \left(c-b\right)\right) - \\ -\frac{1}{2}k_r b^2 \\ \text{s.t. } 0 \le \frac{p_2^e}{\delta\varepsilon} - e \le \frac{p_2^r - p_2^e - \delta e\left(1-\varepsilon\right)}{\delta\left(1-\varepsilon\right)} \le \\ \frac{p_1^e - p_2^r + \delta e}{\varepsilon - \delta} \le \frac{p_1^r - p_1^e}{1-\varepsilon} \le 1 \tag{21}$$

Similarly, by constructing Lagrange function, we also use KKT condition to solve the decision model of the dual-channel system, and obtain the conclusion of Lemma 2. The proof details of Lemma 2 are presented in Appendix D.

Lemma 2. *When the dual-channel system makes centralized decisions with blockchain, by using the backward induction method, the equilibrium results of the game can be obtained as follows:*

$$p_1^{rCB*} = \frac{E_1}{32\delta^3\varepsilon - 8\varepsilon\left(1+\varepsilon\right)^2 - 8\delta^2\left(1+\varepsilon\right)\left(1+5\varepsilon\right) + 2\delta\left(3+15\varepsilon+25\varepsilon^2+5\varepsilon^3\right)} \tag{22}$$

$$p_1^{eCB*} = \\ \frac{E_2}{3\delta - 4\delta^2 - 4\varepsilon + 14\delta\varepsilon - 12\delta^2\varepsilon - 8\varepsilon^2 + 15\delta\varepsilon^2 - 4\varepsilon^3} \tag{23}$$

$$p_2^{rCB*} = \\ \frac{E_3}{2\varepsilon\left(\delta\left(1+3\varepsilon\right)\left(3+5\varepsilon\right) - 4\varepsilon\left(1+\varepsilon\right)^2 - 4\delta^2\left(1+3\varepsilon\right)\right)} \tag{24}$$

$$p_2^{eCB*} = \\ \frac{E_4}{2\delta\left(1+3\varepsilon\right)\left(3+5\varepsilon\right) - 8\varepsilon\left(1+\varepsilon\right)^2 - 8\delta^2\left(1+3\varepsilon\right)} \tag{25}$$

E_1, E_2, E_3 and E_4 in the above expressions are very complicated, so they are presented in Appendix E.

In addition, as the expressions for the respective profits of the system members are complicated, we show them full in Appendix F.

Corollary 2. *When the dual-channel system makes centralized decisions with blockchain, based on the above assumptions, since $0 < \varepsilon < 1$, $0 < \delta < 1$, $0 < b < 1$, $0 < e < 1$, and $\varepsilon > \delta$ is satisfied, the following can be determined:*

(e) $\frac{\partial P_1^{eCB*}}{\partial \delta} < 0; \frac{\partial P_1^{eCB*}}{\partial b} > 0; \frac{\partial P_1^{eCB*}}{\partial e} > 0;$

(f) $\frac{\partial P_1^{rCB*}}{\partial \delta} < 0; \frac{\partial P_1^{rCB*}}{\partial b} > 0; \frac{\partial P_1^{rCB*}}{\partial e} > 0;$

(g) $\frac{\partial P_2^{eCB*}}{\partial \delta} > 0; \frac{\partial P_2^{eCB*}}{\partial b} < 0; \frac{\partial P_2^{eCB*}}{\partial e} > 0;$

(h) $\frac{\partial P_2^{rCB*}}{\partial \delta} > 0; \frac{\partial P_2^{rCB*}}{\partial b} < 0; \frac{\partial P_2^{rCB*}}{\partial e} > 0.$

Corollary 2 shows that, when the system makes centralized decisions with blockchain, regardless of the value of the channel preference coefficient ε, with the increase in consumers' strategic degree δ, both entities of the system tend to reduce the prices of the two channels during the full-price sales period to stimulate the strategic consumers into making a purchase in advance. The prices of the two channels will increase during the full-price sales period as the cost reduction value per unit product b increases. When the system adopts blockchain, regardless of the value of ε, with the increase in δ, both entities of the system tend to raise the prices of the two channels. As b increases, the prices of the two channels will decrease during the discounted sales period.

Corollary 2 show that when the system makes centralized decisions with blockchain, blockchain can bring more utility to strategic consumers, thus giving the system a strong incentive to raise the prices of the two channels during two sales periods.

4.2 Decentralized Decision-Making Model

When the dual-channel system makes decentralized decisions, each of the entities of the system takes their own profit maximization as the decision-making objective, and each independently decides the prices of the products within two periods to obtain their own maximum profit. Similarly, the consumers make their own decisions about whether to purchase, in which period to purchase and through which channel to purchase.

4.2.1 Decentralized Decision-Making without Blockchain (D)

The profits that the supplier can obtain during two periods are expressed as follows:

$$\pi_{M1}^{D} = (1-V_1)(w-c) + (V_1-V_2)\left(p_1^e - c\right) \tag{26}$$

$$\pi_{M2}^{D} = (V_2-V_3)(w-c) + (V_3-V_4)\left(p_2^e - c\right) \tag{27}$$

The supplier's total profit during two periods is calculated as follows:

$$\begin{aligned}\pi_{M}^{D} = \pi_{M1}^{D} + \pi_{M2}^{D} = (1-V_1)(w-c) + (V_1-V_2)\left(p_1^e - c\right) + \\ +(V_2-V_3)(w-c) + (V_3-V_4)\left(p_2^e - c\right)\end{aligned} \tag{28}$$

The profits that the retailer can obtain during two periods are expressed as follows:

$$\pi_{R1}^{D} = (1-V_1)\left(p_1^r - w\right) \tag{29}$$

$$\pi_{R2}^{D} = (V_2-V_3)\left(p_2^r - w\right) \tag{30}$$

The retailer's total profit during two periods is calculated as follows:

$$\pi_{R}^{D} = \pi_{R1}^{D} + \pi_{R2}^{D} = (1-V_1)\left(p_1^r - w\right) + (V_2-V_3)\left(p_2^r - w\right) \tag{31}$$

Among them, we also have: $V_1 = \dfrac{p_1^r - p_1^e}{1-\varepsilon}$; $V_2 = \dfrac{p_1^e - p_2^r}{\varepsilon - \delta}$; $V_3 = \dfrac{p_2^r - p_2^e}{\delta(1-\varepsilon)}$; $V_4 = \dfrac{p_2^e}{\delta\varepsilon}$. At this point, the meanings of V_1, V_2, V_3 and V_4 are consistent with those in the previous model of C.

In the discounted sales period, the retailer's decision model is expressed as follows:

$$\begin{aligned}&\max \pi_{R2}^{D} = (V_2-V_3)\left(p_2^r - w\right) \\ &\text{s.t. } 0 \le \frac{p_2^r - p_2^e}{\delta(1-\varepsilon)} \le \frac{p_1^e - p_2^r}{\varepsilon - \delta} \le 1\end{aligned} \tag{32}$$

Meanwhile, the supplier's decision model is expressed as follows:

$$\begin{aligned}&\max \pi_{M2}^{D} = (V_2-V_3)(w-c) + (V_3-V_4)\left(p_2^e - c\right) \\ &\text{s.t. } 0 \le \frac{p_2^e}{\delta\varepsilon} \le \frac{p_2^r - p_2^e}{\delta(1-\varepsilon)} \le \frac{p_1^e - p_2^r}{\varepsilon - \delta} \le 1\end{aligned} \tag{33}$$

In the full-price sales period, the retailer's decision model is expressed as follows:

$$\begin{aligned}&\max \pi_{R}^{D} = (1-V_1)\left(p_1^r - w\right) + (V_2-V_3)\left(p_2^r - w\right) \\ &\text{s.t. } 0 \le \frac{p_2^r - p_2^e}{\delta(1-\varepsilon)} \le \frac{p_1^e - p_2^r}{\varepsilon - \delta} \le \frac{p_1^r - p_1^e}{1-\varepsilon} \le 1\end{aligned} \tag{34}$$

Meanwhile, the supplier's decision model is expressed as follows:

$$\begin{aligned}&\max \pi_{M}^{D} = (1-V_1)(w-c) + (V_1-V_2)\left(p_1^e - c\right) + \\ &+(V_2-V_3)(w-c) + (V_3-V_4)\left(p_2^e - c\right) \\ &\text{s.t. } 0 \le \frac{p_2^e}{\delta\varepsilon} \le \frac{p_2^r - p_2^e}{\delta(1-\varepsilon)} \le \frac{p_1^e - p_2^r}{\varepsilon - \delta} \le \frac{p_1^r - p_1^e}{1-\varepsilon} \le 1\end{aligned} \tag{35}$$

Similarly, by using the same method used in Lemma 1, we obtain the conclusion of Lemma 3. The proof details of Lemma 3 are presented in Appendix G.

Lemma 3. *When the dual-channel system makes decentralized decisions without blockchain, the game equilibrium result of the model can be calculated by using backward induction as follows:*

$$p_1^{eD*} = \frac{(\delta-\varepsilon)(1-\varepsilon)\varepsilon(3\delta+\varepsilon-4)^2 - 2c(\delta-\varepsilon)(1-\varepsilon)^2(7\delta+\varepsilon-8) + w\varepsilon\left(27\delta^3 - 32 - 27\delta^2(4-\varepsilon) + \varepsilon\left((28-3\varepsilon)\varepsilon - 20\right) + \delta\left(116 - \varepsilon(16+19\varepsilon)\right)\right)}{27\delta^3\varepsilon - (4-\varepsilon)^3\varepsilon - \delta^2\left(28+\varepsilon(52+\varepsilon)\right) + \delta\left(32+\varepsilon\left(76-32\varepsilon+5\varepsilon^2\right)\right)} \tag{36}$$

$$p_1^{rD*} = \frac{H_1}{28\delta^2 - 32\delta + 64\varepsilon - 76\delta\varepsilon + 52\delta^2\varepsilon - 27\delta^3\varepsilon - 48\varepsilon^2 + 32\delta\varepsilon^2 + \delta^2\varepsilon^2 + 12\varepsilon^3 - 5\delta\varepsilon^3 - \varepsilon^4} \tag{37}$$

$$p_2^{eD*}=\frac{H_2}{27\delta^3\varepsilon-\left(4-\varepsilon\right)^3\varepsilon-\delta^2\left(28+\varepsilon\left(52+\varepsilon\right)\right)+\delta\left(32+\varepsilon\left(76-32\varepsilon+\varepsilon^2\right)\right)} \tag{38}$$

$$p_2^{rD*}=\frac{H_3}{27\delta^3\varepsilon-\left(4-\varepsilon\right)^3\varepsilon-\delta^2\left(28+\varepsilon\left(52+\varepsilon\right)\right)+\delta\left(32+\varepsilon\left(76-32\varepsilon+\varepsilon^2\right)\right)} \tag{39}$$

Similarly, H_1, H_2 and H_3 in the above expressions are very complicated, so they are presented in Appendix H.

Because the profit expressions of the dual-channel system members are complicated, they are presented in Appendix I in this paper instead.

Corollary 3. *When the dual-channel system makes decentralized decisions without blockchain, several conclusions can be calculated, regardless of the values of δ and ε, as the model assumes that 0 < ε < 1, 0 < δ < 1, and ε > δ is satisfied:*

$$\frac{\partial P_1^{eD*}}{\partial\delta}<0;\frac{\partial P_1^{rD*}}{\partial\delta}\left\langle 0;\frac{\partial P_2^{eD*}}{\partial\delta}\right\rangle 0\,;\frac{\partial P_2^{rD*}}{\partial\delta}>0.$$

Corollary 3 shows that, when the dual-channel system makes decentralized decisions without blockchain, regardless of the value of the channel preference coefficient ε, with the increase in consumers' strategic degree δ, the strategic consumers are more likely to delay their purchase. At this time, both entities of the system tend to reduce the prices of the two channels during the full-price sales period. Meanwhile, they both raise the prices of the two channels during the discounted sales period. Obviously, the purpose of the system is to attract more strategic consumers to buy in the full-price sales period in advance.

4.2.2 Decentralized Decision-Making with Blockchain (DB)

The total profits available to the supplier during two periods are expressed as follows:

$$\pi_{M1}^{DB}=\left(1-V_1\right)\left(w-c\right)+\left(V_1-V_2\right)\left(p_1^e-c\right) \tag{40}$$

$$\pi_{M2}^{DB}=\left(V_2-V_3\right)\left(w-\left(c-b\right)\right)+\left(V_3-V_4\right)\left(p_2^e-\left(c-b\right)\right)-\frac{1}{2}k_rb^2 \tag{41}$$

Therefore, the supplier's total profit during two periods is calculated as follows:

$$\begin{gathered}\pi_M^{DB}=\pi_{M1}^{DB}+\pi_{M2}^{DB}=\\ \left(1-V_1\right)\left(w-c\right)+\left(V_1-V_2\right)\left(p_1^e-c\right)+\left(V_2-V_3\right)\\ \left(w-\left(c-b\right)\right)+\left(V_3-V_4\right)\left(p_2^e-\left(c-b\right)\right)-\frac{1}{2}k_rb^2\end{gathered} \tag{42}$$

The total profits available to the retailer during two periods are expressed as follows:

$$\pi_{R1}^{DB}=\left(1-V_1\right)\left(p_1^r-w\right) \tag{43}$$

$$\pi_{R2}^{DB}=\left(V_2-V_3\right)\left(p_2^r-w\right) \tag{44}$$

Therefore, the retailer's total profit during two periods is calculated as follows:

$$\begin{gathered}\pi_R^{DB}=\pi_{R1}^{DB}+\pi_{R2}^{DB}=\\ \left(1-V_1\right)\left(p_1^r-w\right)+\left(V_2-V_3\right)\left(p_2^r-w\right)\end{gathered} \tag{45}$$

Among them, we also have: $V_1=\frac{p_1^r-p_1^e}{1-\varepsilon};V_2=\frac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta};V_3=\frac{p_2^r-p_2^e-\delta e\left(1-\varepsilon\right)}{\delta\left(1-\varepsilon\right)};$ $V_4=\frac{p_2^e}{\delta\varepsilon}-e$. At this point, the meanings of V_1, V_2, V_3 and V_4 are consistent with those in the previous model of *CB*.

In the discounted sales period, the retailer's decision model is expressed as follows:

$$\begin{aligned}&\max\pi_{R2}^{DB}=\left(V_2-V_3\right)\left(p_2^r-w\right)\\ &\text{s.t. } 0\le\frac{p_2^r-p_2^e-\delta e\left(1-\varepsilon\right)}{\delta\left(1-\varepsilon\right)}\le\frac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}\le 1\end{aligned} \tag{46}$$

Meanwhile, the supplier's decision model is expressed as follows:

$$\begin{aligned}&\max\pi_{M2}^{DB}=\\ &\left(V_2-V_3\right)\left(w-\left(c-b\right)\right)+\left(V_3-V_4\right)\left(p_2^e-\left(c-b\right)\right)\\ &\text{s.t. } 0\le\frac{p_2^e}{\delta\varepsilon}-e\le\frac{p_2^r-p_2^e-\delta e\left(1-\varepsilon\right)}{\delta\left(1-\varepsilon\right)}\le\frac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}\le 1\end{aligned} \tag{47}$$

In the full-price sales period, the retailer's decision model is expressed as follows:

$$\begin{aligned}&\max\pi_R^{DB}=\left(1-V_1\right)\left(p_1^r-w\right)+\left(V_2-V_3\right)\left(p_2^r-w\right)\\ &\text{s.t. } 0\le\frac{p_2^r-p_2^e-\delta e\left(1-\varepsilon\right)}{\delta\left(1-\varepsilon\right)}\le\frac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}\le\\ &\frac{p_1^r-p_1^e}{1-\varepsilon}\le 1\end{aligned} \tag{48}$$

Meanwhile, the supplier's decision model is expressed as follows:

$$\max\pi_M^{DB}=\left(1-V_1\right)\left(w-c\right)+\left(V_1-V_2\right)\left(p_1^e-c\right)+$$
$$+\left(V_2-V_3\right)\left(w-\left(c-b\right)\right)+\left(V_3-V_4\right)\left(p_2^e-\left(c-b\right)\right)-\frac{1}{2}k_rb^2$$
$$\text{s.t. } 0\le\frac{p_2^e}{\delta\varepsilon}-e\le\frac{p_2^r-p_2^e-\delta e\left(1-\varepsilon\right)}{\delta\left(1-\varepsilon\right)}\le\frac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}$$
$$\le\frac{p_1^r-p_1^e}{1-\varepsilon}\le1 \tag{49}$$

Similarly, by using the same method used in Lemma 2, we obtain the conclusion of Lemma 4. The proof details of Lemma 4 are presented in Appendix J.

Lemma 4. *When the dual-channel system makes decentralized decisions with blockchain, the equilibrium results of the game can be calculated by using backward induction as follows:*

$$p_1^{eDB*}=\frac{K_1}{27\delta^3\varepsilon-\left(4-\varepsilon\right)^3\varepsilon-\delta^2\left(16+76\varepsilon-11\varepsilon^2\right)+16\delta+\varepsilon\delta\left(112-56\varepsilon+9\varepsilon^2\right)} \tag{50}$$

$$p_1^{rDB*}=\frac{K_2}{16\delta^2-16\delta+64\varepsilon-112\delta\varepsilon+76\delta^2\varepsilon-27\delta^3\varepsilon-48\varepsilon^2+56\delta\varepsilon^2-11\delta^2\varepsilon^2+12\varepsilon^3-9\delta\varepsilon^3-\varepsilon^4} \tag{51}$$

$$p_2^{eDB*}=\frac{K_3}{27\delta^3\varepsilon-\left(4-\varepsilon\right)^3\varepsilon-\delta^2\left(16+76\varepsilon-11\varepsilon^2\right)+16\delta+\varepsilon\delta\left(112-56\varepsilon+9\varepsilon^2\right)} \tag{52}$$

$$p_2^{rDB*}=\frac{K_4}{27\delta^3\varepsilon^2-\left(4-\varepsilon\right)^3\varepsilon^2-\delta^2\varepsilon\left(16+76\varepsilon-11\varepsilon^2\right)+16\delta\varepsilon+\varepsilon^2\delta\left(112-56\varepsilon+9\varepsilon^2\right)} \tag{53}$$

K_1, K_2, K_3 and K_4 in the above expressions are very complicated, so they are presented in Appendix K.

Similarly, when the system makes decentralized decisions with blockchain, the respective profit expressions of the system members are complicated. Therefore, they are presented in Appendix L of this paper.

Corollary 4. *When the dual-channel system makes decentralized decisions with blockchain, several conclusions can be calculated, regardless of the values of δ and ε since the model assumes 0 < ε < 1, 0 < δ < 1, 0 < b < c, 0 < e < 1, and ε > δ is satisfied.*

(i) $\frac{\partial P_1^{eCB*}}{\partial\delta}\left\langle 0;\frac{\partial P_1^{eCB*}}{\partial b}\right\rangle 0;\frac{\partial P_1^{eCB*}}{\partial e}<0;$

(j) $\frac{\partial P_1^{rCB*}}{\partial\delta}\left\langle 0;\frac{\partial P_1^{rCB*}}{\partial b}\right\rangle 0;\frac{\partial P_1^{rCB*}}{\partial e}<0;$

(k) $\frac{\partial P_2^{eCB*}}{\partial\delta}>0;\frac{\partial P_2^{eCB*}}{\partial b}\left\langle 0;\frac{\partial P_2^{eCB*}}{\partial e}\right\rangle 0;$

(l) $\frac{\partial P_2^{rCB*}}{\partial\delta}<0;\frac{\partial P_2^{rCB*}}{\partial b}\left\langle 0;\frac{\partial P_2^{rCB*}}{\partial e}\right\rangle 0.$

Corollary 4 shows that, when the dual-channel system makes decentralized decisions with blockchain, no matter what the value of the channel preference coefficient ε is in the first period, with the increase in the consumers' strategic degree δ, both entities of the system tend to reduce the prices of the two channels during the full-price sales period to stimulate the strategic consumers into making a purchase in advance. As the cost reduction value per unit product b increases, the prices of the two channels will increase during the full-price sales period. This shows that the supplier's production cost will increase rapidly due to the blockchain input, and the investment cost of blockchain can only be recovered in advance by increasing the prices of the two channels during the full-price sales period. With the increase in e, the prices of the two channels will decrease during the full-price sales period. For the discounted sales period, if the system adopts blockchain, regardless of the value of ε, with the increase in δ, the supplier tends to raise the network channel prices during the discounted sales period. However, the retailer tends to reduce the physical channel price in the discounted sales period. As b increases, the prices of the two channels will decrease during the discounted sales period.

Corollary 4 show that when the system makes decentralized decisions with blockchain, the blockchain investment has led to a rapid increase in the supplier's production costs, and the supplier can only recover part of the blockchain investment costs in advance by raising the prices of the first two channels. In addition, blockchain can bring more benefits to strategic consumers, thus giving the system a strong incentive to raise the prices of the two channels during two sales periods.

5 NUMERICAL SIMULATION

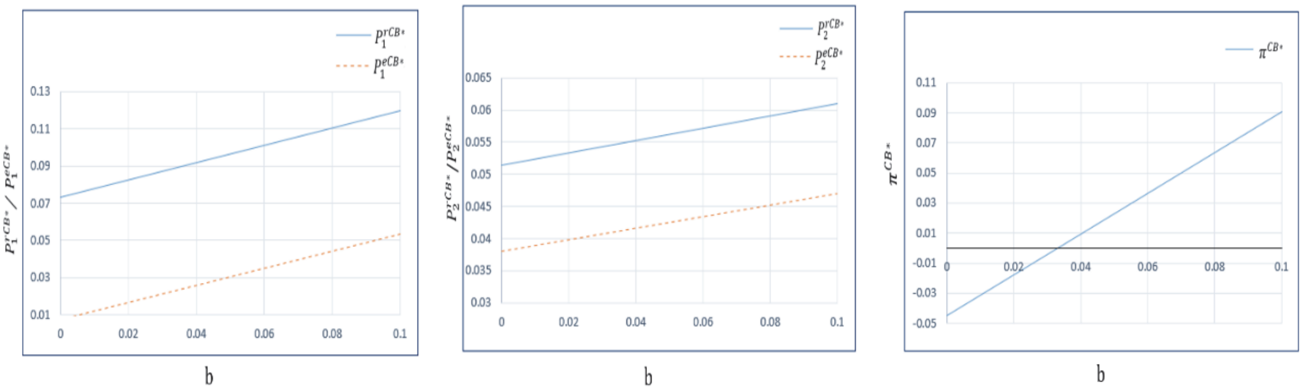
In order to intuitively investigate the impact of blockchain on the equilibrium of the model in the dual-channel system, we further give visual results and verification by means of numerical simulation to obtain more valuable management significance. We assume the following parameters: ε = 0.8, δ = 0.5, c = 0.1, w = 0.15, b = 0.02, e = 0.02 [22, 31, 33]. Limited by the assumptions in this model, when the channel preference coefficient ε is 0.8, the value of the consumers' strategic degree δ must be less than 0.8; otherwise, the model assumptions proposed above cannot be satisfied: $p_1^r > p_1^e > p_2^r > p_2^e > w > c$.

5.1 The Impacts of Blockchain under Centralized Decision-Making

When the dual-channel system makes centralized decisions, we analyse the impacts of blockchain on the optimal pricing and profits from two perspectives: the cost reduction value per unit product b and the utility increment per unit product e. The trend of changes in prices of the two channels and optimal profits in the dual-channel system is shown in the following function figures.

As can be seen in Fig. 4a and 4b, with the increase in b, the prices of the two channels show an increasing trend during two periods. In addition, the physical channel price is much higher than the network channel price. When the system makes centralized decisions, with the increase in b, the cost of the system's investment in blockchain also increases. Therefore, the system will increase the prices of the two channels during two periods.

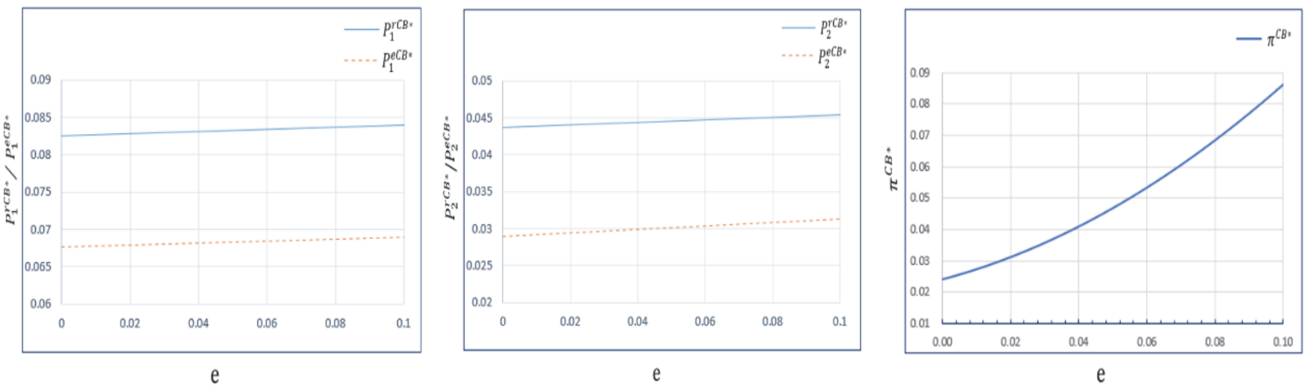
As can be seen in Fig. 4c, when b is at a low level, the total profit of the system is negative. This shows that the blockchain investment cost is too high, while the reduction in unit product cost due to blockchain is too small, making the total profit of the system negative. Subsequently, with the increase in b, the unit product cost is greatly reduced. However, the system also increases the prices of the two channels, leading to a rapid increase in the total profit. The above analysis shows that when the cost reduction value per unit product brought by blockchain is at a low level, the dual-channel system under the scenario of centralized decision-making has no strong incentive to adopt blockchain.



(a) Full-price-period price (b) Discounted-period price (c) Total profit of the system

Figure 4 Impacts of *b* on the prices and total profit in centralized model

As can be seen in Fig. 5a and 5b, with the increase in e, the prices of the two channels show a slow growth trend during two periods. In addition, the physical channel price is much higher than the network channel price. When the system makes centralized decisions, with the increase in e, the utility of each unit product to consumers increases. Therefore, the system will increase the prices of the two channels during two periods to expand their profits. As can be seen in Fig. 5c, when e increases gradually, the total profit of the system increases rapidly. This indicates that the greater the utility increment of each unit product e, the higher the prices of different channels during two periods, which leads to a rapid growth in the total profit. The above analysis shows that the greater the utility increment per unit product brought by blockchain, the stronger the motivation to adopt blockchain for the dual-channel system under the scenario of centralized decision-making.



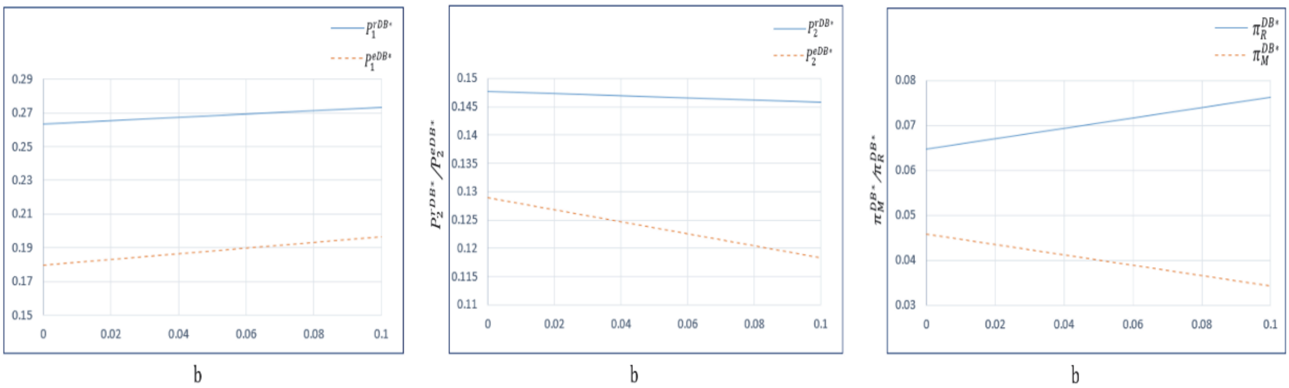
(a) Full-price-period price (b) Discounted-period price (c) Total profit of the system

Figure 5 Impacts of *e* on the prices and total profits in centralized model

5.2 The Impacts of Blockchain under Decentralized Decision-Making

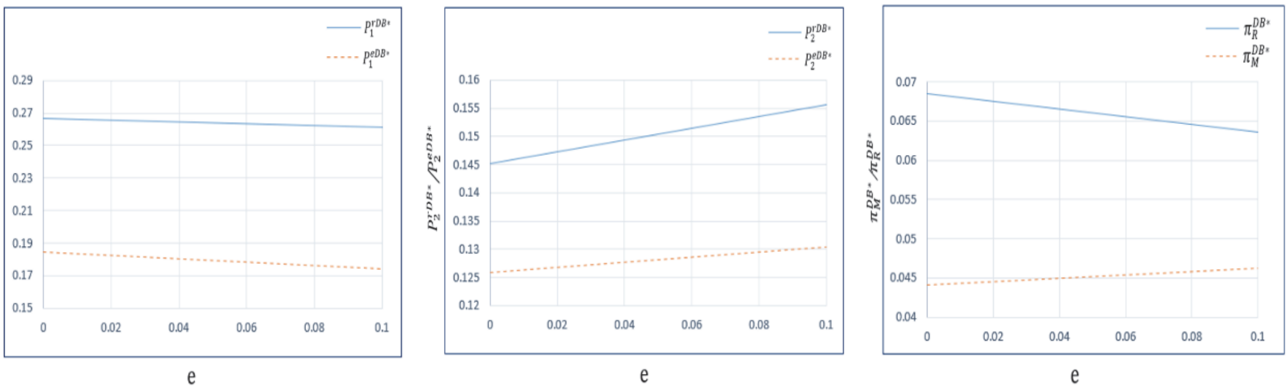
We continue to analyse the impacts of blockchain on the optimal pricing and profits of the system from two perspectives.

As can be seen in Fig. 6a and 6b, with the increase in b, the prices of the two channels show an increasing trend during the full-price sales period. The supplier's network channel price shows a rapid downward trend during the discounted sales period, while the retailer's physical channel price maintains a slow downward trend. In addition, the physical channel price is much higher than the network channel price. This indicates that the blockchain investment cost also increases with the increase in *b* under the scenario of decentralized decision-making. Therefore, the system will increase the prices of the two channels during the full-price sales period and attract the strategic consumers through the promotion of price reduction during the discounted sales period. As can be seen in Fig. 6c, when *b* gradually increases, the supplier's total profit rapidly decreases, but the retailer's total profit rapidly increases. This indicates that as *b* gradually increases, the total cost of the blockchain investment of the supplier increases, and the unit product cost also decreases significantly. However, the former has greater influence, which leads to a rapid decrease in the supplier's total profit. In contrast, the retailer does not directly bear the blockchain investment cost, and its discounted-period price reduction is extremely small, which then leads to the retailer's total profit maintaining a rapid growth trend. The above analysis shows that when making decentralized decisions, the dual-channel system has no strong incentive to adopt blockchain while the cost reduction value per unit product brought by blockchain is at a high level.



(a) Full-price-period price (b) Discounted-period price (c) Retailer's/Supplier's total profit

Figure 6 Impacts of *b* on the prices and total profits in decentralized model



(a) Full-price-period price (b) Discounted-period price (c) Retailer's/Supplier's total profit

Figure 7 Impacts of *e* on the prices and total profits in decentralized model

As can be seen in Fig. 7a and 7b, with the increase in *e*, the prices of the two channels show a slow downward trend during the full-price sales period, while they show a sustained upward trend during the discounted sales period. In addition, the physical channel price is much higher than the network channel price. When the system makes decentralized decisions, with the increase in *e*, the utility of each unit product to the consumers increases. Therefore, the system will increase the prices of the two channels during two periods to expand their profits. As can be seen in Fig. 7c, when *e* gradually increases, the supplier's total profit slowly increases, while the retailer's total profit rapidly decreases. This indicates that as *e* increases, the utility level of each unit product for the consumers increases. Therefore, the system is driven to increase the prices of the two channels during the discounted sales period. However, the retailer's price increase is much larger than that of the supplier, and the retailer's physical channel price is much higher than the supplier's network channel price. The above analysis shows that when making decentralized decisions, the dual-channel system has no strong incentive to adopt blockchain while the utility increment per unit product is at a high level.

6 CONCLUSION

This paper develops game-theoretic models to analyze dynamic pricing decisions in a dual-channel supply chain enabled with blockchain technology. The models account for strategic consumer purchasing behaviors across sales periods. Results reveal how blockchain adoption incentives channels to tactically raise and lower prices over time. The analysis also provides insights into how consumer strategic behavior influences optimal pricing policies. Findings contribute new knowledge on integrating emerging blockchain technologies into retail channel design and operations. Limitations of the current study present opportunities for future work, including exploring adaptive pricing strategies. The main conclusions we have drawn are presented below.

First, we find that when the dual-channel system makes centralized decisions with blockchain, as the cost reduction value per unit product increases, the prices of the two channels will both increase during the full-price sales period. Meanwhile, with the increase in the utility increment per unit product, the prices of the two channels will increase during the full-price sales period. For the discounted sales period, if the system adopts blockchain during this period, as the cost reduction value per unit product increases, the prices of the two channels will decrease during this period. However, with the increase in utility increment per unit product, the prices of the two channels will increase during this period.

Second, we also find that when the dual-channel system makes decentralized decisions with blockchain, as the cost reduction value per unit product increases, the prices of the two channels will increase during the full-price sales period. On the contrary, with the increase in utility increment per unit product, the prices of the two channels will decrease during the full-price sales period. For the discounted sales period, if the system adopts blockchain during this period, as the cost reduction value per unit product increases, the prices of the two channels will decrease during this period. However, with the increase in utility increment per unit product, the prices of the two channels will increase during this period.

Third, our research shows that in a blockchain-supported dual-channel system, the consumers' strategic degree has a great influence on the pricing of the system. When the dual-channel system makes decisions with blockchain, whether they are centralized or decentralized decision, with the increase in consumers' strategic degree, the system will lower the prices of the two channels in the full-price sales period, so as to stimulate strategic consumers to purchase in advance in the first period.

Finally, our research also shows that the attitude of a dual-channel supply chain system to blockchain depends on two key factors of blockchain and the decision-making conditions of the system. When the dual-channel system makes centralized decisions, if the cost reduction value or the utility increment per unit product is at a low level, the system has no strong incentive to invest in blockchain. However, when the dual-channel system makes decentralized decisions, if the cost reduction value or the utility increment per unit product is at a high level, the system has no strong incentive to invest in blockchain.

Therefore, we propose promoting the use of blockchain in the dual-channel system and improving the transparency of products through blockchain so as to enhance the utility level of consumers and to expand the market scale. Furthermore, we propose introducing blockchain into the dual-channel system to enhance trust among the system members, to reduce transaction costs, and to provide more value to consumers. We recommend that when a dual-channel system introduces blockchain, it needs to consider the balance of interests among its members, in order to build a sustainable supply-chain system based on mutual benefit and cost sharing.

Obviously, with the background of low-carbon and sustainable development for the global economy, the digital economy supported by blockchain and other technologies will experience further rapid growth in the future. In this paper, we only consider the case that the supplier sells products through two channels, but we do not consider the case that the retailer also sells products through two channels. With the extensive application of the digital economy, the dual-channel model will evolve into the full-channel model. Therefore, future research can further analyse dynamic pricing under the omni-channel marketing environment. In addition, this paper is only limited to the case of the supplier adopting blockchain in the discounted sales period. Our research does not consider the case that the supplier adopts blockchain in the first period, nor does it consider the case that the retailer adopts blockchain during two periods. Therefore, future research can be extended to the case of the supplier or the retailer adopting blockchain during two periods.

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Appendix

Appendix A

From the utility boundary under different channels without blockchain, we obtain the following equations, and then solve them respectively. By solving the equation: $V - p_1^r = \varepsilon V - p_1^e$, we obtain: $V_1 = \dfrac{p_1^r - p_1^e}{1-\varepsilon}$; by solving the equation: $\varepsilon V - p_1^e = \delta V - p_2^r$, we obtain: $V_2 = \dfrac{p_1^e - p_2^r}{\varepsilon - \delta}$; by solving the equation: $\delta V - p_2^r = \delta\varepsilon V - p_2^e$, we obtain: $V_3 = \dfrac{p_2^r - p_2^e}{\delta(1-\varepsilon)}$; by solving the equation: $\delta\varepsilon V - p_2^e = 0$, we obtain: $V_4 = \dfrac{p_2^e}{\delta\varepsilon}$.

Appendix B

Proof for Lemma 1

1. The Decision Model and Solution in the Discounted Sales Period

According to the Eq. (8) in the paper, we construct the Lagrange function as follows:

$$\begin{aligned} L = \pi_2^C + k_1\left(-\frac{p_2^e}{\delta\varepsilon}\right) + k_2\left(\frac{p_2^e}{\delta\varepsilon} - \frac{p_2^r - p_2^e}{\delta(1-\varepsilon)}\right) + \\ + k_3\left(\frac{p_2^r - p_2^e}{\delta(1-\varepsilon)} - \frac{p_1^e - p_2^r}{\varepsilon - \delta}\right) + k_4\left(\frac{p_1^e - p_2^r}{\varepsilon-\delta} - 1\right) \end{aligned} \tag{B1}$$

The KKT conditions are expressed as follows:

$$\begin{cases} \dfrac{\partial L}{\partial p_2^r} = 0 \\ \dfrac{\partial L}{\partial p_2^e} = 0 \\ k_1\left(-\dfrac{p_2^e}{\delta\varepsilon}\right) = k_2\left(\dfrac{p_2^e}{\delta\varepsilon} - \dfrac{p_2^r - p_2^e}{\delta(1-\varepsilon)}\right) = 0 \\ k_3\left(\dfrac{p_2^r - p_2^e}{\delta(1-\varepsilon)} - \dfrac{p_1^e - p_2^r}{\varepsilon-\delta}\right) = 0 \\ k_4\left(\dfrac{p_1^e - p_2^r}{\varepsilon - \delta} - 1\right) = 0 \\ k_i \ge 0, i = 1,2,3,4. \end{cases} \tag{B2}$$

The prices of two channels during the discounted sales period are calculated as follows:

$$p_2^{eC} = \frac{1}{2}\left(c + \delta p_1^e\right) \tag{B3}$$

$$p_2^{rC} = \frac{c\varepsilon + \delta p_1^e}{2\varepsilon} \tag{B4}$$

2. The Decision Model and Solution in the Full-Price Sales Period

According to the Eq. (9) in the paper, we construct the Lagrange function as follows:

$$\begin{aligned} L = \pi^C + k_1\left(-\frac{p_2^e}{\delta\varepsilon}\right) + k_2\left(\frac{p_2^e}{\delta\varepsilon} - \frac{p_2^r - p_2^e}{\delta(1-\varepsilon)}\right) + \\ + k_3\left(\frac{p_2^r - p_2^e}{\delta(1-\varepsilon)} - \frac{p_1^e - p_2^r}{\varepsilon-\delta}\right) + k_4\left(\frac{p_1^e - p_2^r}{\varepsilon - \delta} - \frac{p_1^r - p_1^e}{1-\varepsilon}\right) + \\ + k_5\left(\frac{p_1^r - p_1^e}{1-\varepsilon} - 1\right) \end{aligned} \tag{B5}$$

The KKT conditions are expressed as follows:

$$\begin{cases} \dfrac{\partial L}{\partial p_1^r} = 0 \\ \dfrac{\partial L}{\partial p_1^e} = 0 \\ k_1\left(-\dfrac{p_2^e}{\delta\varepsilon}\right) = k_2\left(\dfrac{p_2^e}{\delta\varepsilon} - \dfrac{p_2^r - p_2^e}{\delta(1-\varepsilon)}\right) = 0 \\ k_3\left(\dfrac{p_2^r - p_2^e}{\delta(1-\varepsilon)} - \dfrac{p_1^e - p_2^r}{\varepsilon-\delta}\right) = k_4\left(\dfrac{p_1^e - p_2^r}{\varepsilon-\delta} - \dfrac{p_1^r - p_1^e}{1-\varepsilon}\right) \\ = k_5\left(\dfrac{p_1^r - p_1^e}{1-\varepsilon} - 1\right) = 0 \\ k_i \ge 0, i = 1,2,3,4,5. \end{cases} \tag{B6}$$

By substituting p_2^{eC}, p_2^{rC} in (B3) and (B4) into (B5), Lemma 1 can be obtained.

Appendix C

From the utility boundary under different channels with blockchain, we obtain the following equations, and then solve them respectively. By solving the equation: $V - p_1^r = \varepsilon V - p_1^e$, we obtain: $V_1 = \dfrac{p_1^r - p_1^e}{1-\varepsilon}$; by solving the equation: $\varepsilon V - p_1^e = \delta(V + e) - p_2^r$, we obtain: $V_2 = \dfrac{p_1^e - p_2^r + \delta\varepsilon}{\varepsilon - \delta}$; by solving the equation: $\delta(V+e) - p_2^r = \delta\varepsilon(V+e) - p_2^e$, we obtain: $V_3 = \dfrac{p_2^r - p_2^e - \delta\varepsilon(1-\varepsilon)}{\delta(1-\varepsilon)}$; by solving the equation: $\delta\varepsilon(V+e) - p_2^e = 0$, we obtain: $V_4 = \dfrac{p_2^e}{\delta\varepsilon} - e$.

Appendix D

Proof for Lemma 2

1. The Decision Model and Solution in the Discounted Sales Period

According to the expression (20) in the paper, we construct the Lagrange function as follows:

$$\begin{aligned} L = {} & \pi_2^{CB} + k_1\left(e - \frac{p_2^e}{\delta\varepsilon}\right) + \\ & + k_2\left(\frac{p_2^e}{\delta\varepsilon} - e - \frac{p_2^r - p_2^e - \delta e(1-\varepsilon)}{\delta(1-\varepsilon)}\right) + \\ & + k_3\left(\frac{p_2^r - p_2^e - \delta e(1-\varepsilon)}{\delta(1-\varepsilon)} - \frac{p_1^e - p_2^r + \delta e}{\varepsilon - \delta}\right) + \\ & + k_4\left(\frac{p_1^e - p_2^r + \delta e}{\varepsilon - \delta} - 1\right) \end{aligned} \tag{D1}$$

The KKT conditions are expressed as follows:

$$\begin{cases} \dfrac{\partial L}{\partial p_2^r} = 0 \\ \dfrac{\partial L}{\partial p_2^e} = 0 \\ k_1\left(e - \dfrac{p_2^e}{\delta\varepsilon}\right) = k_2\left(\dfrac{p_2^e}{\delta\varepsilon} - e - \dfrac{p_2^r - p_2^e - \delta e(1-e)}{\delta(1-\varepsilon)}\right) \\ = k_3\left(\dfrac{p_2^r - p_2^e - \delta e(1-e)}{\delta(1-\varepsilon)} - \dfrac{p_1^e - p_2^r + \delta e}{\varepsilon - \delta}\right) \\ = k_4\left(\dfrac{p_1^e - p_2^r + \delta e}{\varepsilon - \delta} - 1\right) = 0 \\ k_i \geq 0, i = 1,2,3,4. \end{cases} \tag{D2}$$

The prices of the two channels during the discounted sales period are calculated as follows:

$$\begin{aligned} p_2^{eCB} = {} & \frac{1}{2(2\delta - \varepsilon - 1)} \\ & \left(\begin{aligned} & b - c - 2b\delta + 2c\delta - b\varepsilon + c\varepsilon + 2b\delta\varepsilon - 2c\delta\varepsilon - e\delta\varepsilon + \\ & + e\delta^2\varepsilon + 2\delta\varepsilon^2 + e\delta\varepsilon^2 - \delta^2\varepsilon^2 - e\delta^2\varepsilon^2 - 2\delta\varepsilon^3 + \\ & + \delta^2\varepsilon^3 + \delta p_1^e - \delta\varepsilon p_1^e \end{aligned}\right) \end{aligned} \tag{D3}$$

$$\begin{aligned} & p_2^{rCB} = \\ & \frac{(1-\varepsilon)\left(\varepsilon\left(c - b + \delta\varepsilon + \delta(e-\varepsilon)(\varepsilon-\delta)\right) + \delta p_1^e\right)}{2\varepsilon(2\delta - \varepsilon - 1)} \end{aligned} \tag{D4}$$

2. The Decision Model and Solution in the Full-Price Sales Period

According to the expression (21) in the paper, we construct the Lagrange function as follows:

$$\begin{aligned} & L = \\ & \pi^{CB} + k_1\left(e - \frac{p_2^e}{\delta\varepsilon}\right) + k_2\left(\frac{p_2^e}{\delta\varepsilon} - e - \frac{p_2^r - p_2^e - \delta e(1-\varepsilon)}{\delta(1-\varepsilon)}\right) + \\ & + k_3\left(\frac{p_2^r - p_2^e - \delta e(1-\varepsilon)}{\delta(1-\varepsilon)} - \frac{p_1^e - p_2^r + \delta e}{\varepsilon - \delta}\right) + \\ & + k_4\left(\frac{p_1^e - p_2^r + \delta e}{\varepsilon - \delta} - \frac{p_1^r - p_1^e}{1-\varepsilon}\right) + k_5\left(\frac{p_1^r - p_1^e}{1-\varepsilon} - 1\right) \end{aligned} \tag{D5}$$

The KKT conditions are expressed as follows:

$$\begin{cases} \dfrac{\partial L}{\partial p_1^r} = 0 \\ \dfrac{\partial L}{\partial p_1^e} = 0 \\ k_1\left(e - \dfrac{p_2^e}{\delta\varepsilon}\right) = k_2\left(\dfrac{p_2^e}{\delta\varepsilon} - e - \dfrac{p_2^r - p_2^e - \delta e(1-\varepsilon)}{\delta(1-\varepsilon)}\right) \\ = k_3\left(\dfrac{p_2^r - p_2^e - \delta e(1-\varepsilon)}{\delta(1-\varepsilon)} - \dfrac{p_1^e - p_2^r + \delta e}{\varepsilon - \delta}\right) \\ = k_4\left(\dfrac{p_1^e - p_2^r + \delta e}{\varepsilon - \delta} - \dfrac{p_1^r - p_1^e}{1-\varepsilon}\right) = k_5\left(\dfrac{p_1^r - p_1^e}{1-\varepsilon} - 1\right) = 0 \\ k_i \geq 0, i = 1,2,3,4,5. \end{cases} \tag{D6}$$

By substituting p_2^{eCB} and p_2^{rCB} in (D3) and (D4) into (D5), Lemma 2 can be obtained.

Appendix E

$$\begin{aligned} E_1 = {} & (1-\varepsilon)\left(16\delta^3\varepsilon - 4\varepsilon(1+\varepsilon)^2 - 4\delta^2(1+\varepsilon)(1+5\varepsilon) + 3\delta + 5\varepsilon\delta\left(3 + 5\varepsilon + \varepsilon^2\right)\right) + \\ & + 2(1-\varepsilon)\left(c(1-\varepsilon)\left(\delta - 2\delta^2 - 2\varepsilon + 3\delta\varepsilon\right) - 2\varepsilon(\delta - \varepsilon)\left(b(4\delta - 3 - \varepsilon) + \delta\varepsilon\left(4 - 5\delta + \varepsilon^2\right)\right) + e\delta(\delta + \varepsilon - 1 - \delta\varepsilon)\right) \end{aligned}$$

$$\begin{aligned} E_2 = {} & c\delta - 2c\delta^2 - 2c\varepsilon + 2\delta\varepsilon + 6b\delta\varepsilon + 2c\delta\varepsilon - 8\delta^2\varepsilon - 8b\delta^2\varepsilon + 2c\delta^2\varepsilon + 2e\delta^2\varepsilon + 8\delta^3\varepsilon - 2e\delta^3\varepsilon - 2\varepsilon^2 - 6b\varepsilon^3 + 2c\varepsilon^2 + \\ & + 12\delta\varepsilon^2 + 10b\delta\varepsilon^2 - 3c\delta\varepsilon^2 - 2e\delta\varepsilon^2 - 24\delta^2\varepsilon^2 + 10\delta^3\varepsilon^2 + 2e\delta^3\varepsilon^2 - 4\varepsilon^3 - 2b\varepsilon^3 + 18\delta\varepsilon^3 + 2e\delta\varepsilon^3 - 10\delta^2\varepsilon^3 - 2e\delta^2\varepsilon^3 - \\ & - 2\delta^3\varepsilon^3 - 2\varepsilon^4 + 2\delta^2\varepsilon^4 \end{aligned}$$

$$E_3 = (1-\varepsilon)\left(\begin{aligned} & c\delta^2(1-\varepsilon) - 4c\varepsilon^2(1+\varepsilon) + c\delta\varepsilon(1+7\varepsilon) + b\varepsilon\left(4\delta^2 + 4\varepsilon(1+\varepsilon) - 3\delta(1+3\varepsilon)\right) + \\ & \delta\varepsilon\left(-\delta^2\left(4 + e + 7e\varepsilon + (3-7\varepsilon)\varepsilon\right) - 2\varepsilon\left(1 + \varepsilon\left(3 - 2\varepsilon^2 + 2e(1+\varepsilon)\right)\right)\right) + \delta\left(2 + \varepsilon\left(9 + (4-11\varepsilon)\varepsilon + 5e + 55e\varepsilon\right)\right) \end{aligned}\right)$$

$$E_4=(1-\varepsilon)(b(4\varepsilon(1+\varepsilon)-\delta(3+2\varepsilon)(1+3\varepsilon)+\delta^2(4+8\varepsilon))+c(-4\varepsilon(1+\varepsilon)-\delta^2(5+11\varepsilon)+\delta(3+\varepsilon(13+8\varepsilon))))+$$
$$+\delta\varepsilon(2\varepsilon(1-2e+4\varepsilon)(1+\varepsilon)+\delta^2(4-e(3+5\varepsilon)+\varepsilon(7+5\varepsilon))-2\delta(1+2\varepsilon)(1+\varepsilon(4+\varepsilon))+\delta e(3+9\varepsilon+4\varepsilon^2))$$

Appendix F

$$\pi_R^{CB*}=\frac{F_1}{4(1-\varepsilon)\left(4\varepsilon(1+\varepsilon)^2+4\delta^2(1+3\varepsilon)-\delta(1+3\varepsilon)(3+5\varepsilon)\right)^2}$$

$$F_1=(\varepsilon-1)((4\varepsilon(1+\varepsilon)^2+4\delta^2(1+3\varepsilon)-\delta(1+3\varepsilon)(3+5\varepsilon))(-4\delta^3\varepsilon(e-(5+e)\varepsilon+\varepsilon^2-4)+$$
$$+4\delta^2(c-1+7c\varepsilon+\varepsilon(e-4b-9\varepsilon-(5+e)\varepsilon^2+\varepsilon^3-6))-4\varepsilon(1+\varepsilon(2+\varepsilon+b(3+\varepsilon))-c(1+\varepsilon(5+2\varepsilon)))+$$
$$+\delta(3-4c(1+3\varepsilon)^2+\varepsilon(15+4b(3+5\varepsilon)+\varepsilon(25-4e(1-\varepsilon)+21\varepsilon))))+\frac{1}{(\delta-\varepsilon)\varepsilon}(\varepsilon(2\delta((4-e)(1-\delta)\delta-1)+$$
$$(2+\delta(2e+28\delta-15-2(5+e)\delta^2))\varepsilon-2(1-\delta)(\delta(12+e+\delta)-4)\varepsilon^2+(10-\delta(15+2\delta))\varepsilon^3+4\varepsilon^4)-$$
$$-2c(\delta-\varepsilon)(\delta+7\delta\varepsilon-\varepsilon(5+2\varepsilon)-1)+b(2\varepsilon(2+\varepsilon)(1+3\varepsilon)+4\delta^2(1+5\varepsilon)-\delta(1+5\varepsilon)(3+5\varepsilon)))(c(\varepsilon-\delta)(\delta-4\varepsilon(1+\varepsilon)(1+3\varepsilon)+$$
$$+\delta\varepsilon(6+25\varepsilon))+\varepsilon(b(4\varepsilon(1+\varepsilon)(1+3\varepsilon)+4\delta^2(1+7\varepsilon)-\delta(1+3\varepsilon)(3+13\varepsilon))+\delta(1-\varepsilon)(\delta^2(4+e+7e\varepsilon+(3-7\varepsilon)\varepsilon)+$$
$$+2\varepsilon(1+\varepsilon(3-2\varepsilon^2+2e(1+\varepsilon)))-\delta(2-\varepsilon(\varepsilon(-4+11\varepsilon)-9-5e-55\varepsilon))))))$$

$$\pi_M^{CB*}=\frac{F_2}{4\delta\varepsilon\left(4\varepsilon(1+\varepsilon)^2+4\delta^2(1+3\varepsilon)-\delta(1+3\varepsilon)(3+5\varepsilon)\right)^2}$$

$$F_2=(b(4\varepsilon(1+\varepsilon)(1+3\varepsilon)+4\delta^2(1+\varepsilon(5+2\varepsilon))-\delta(1+3\varepsilon)(3+\varepsilon(11+2\varepsilon)))-\delta(\varepsilon-1)\varepsilon(2(1-2e+4\varepsilon)\varepsilon(1+\varepsilon)+$$
$$+\delta^2(4-e(3+5\varepsilon)+\varepsilon(7+5\varepsilon))+\delta(-2(1+2\varepsilon)(1+\varepsilon(4+\varepsilon))+e(3+\varepsilon(9+4\varepsilon))))+c(-4\varepsilon(1+\varepsilon)(1+3\varepsilon)-$$
$$-\delta^2(3+\varepsilon(18+11\varepsilon))+\delta(3+\varepsilon(18+\varepsilon(35+8\varepsilon)))))(b(4\varepsilon-4\varepsilon^3+4\delta^2(1+\varepsilon)+\delta(-3+\varepsilon)(1+3\varepsilon))+$$
$$+c(-2\delta^2(3+5\varepsilon)+4\varepsilon(\varepsilon^2-1)+\delta(3+\varepsilon(12+\varepsilon)))+\delta\varepsilon(4\varepsilon(1+\varepsilon)(1+e+\varepsilon+3e\varepsilon-3\varepsilon^2)+2\delta^2(4+\varepsilon-13\varepsilon^2+e(3+13\varepsilon))-$$
$$-\delta(4+\varepsilon(15-\varepsilon(6+37\varepsilon))+e(3+\varepsilon(24+37\varepsilon)))))+4\delta(c(\delta+7\delta\varepsilon-1-\varepsilon(5+2\varepsilon))+\varepsilon((1+\varepsilon)^2+b(3-4\delta+\varepsilon)+$$
$$+\delta(e-4-(8+e)\varepsilon)+\delta^2(4-e(1-\varepsilon)+(5-\varepsilon)\varepsilon)))(c(\delta-\varepsilon)(\varepsilon-1)(\delta+3\delta\varepsilon+4\varepsilon^2)+b\varepsilon(-4\delta^2(1+3\varepsilon)-4\varepsilon(1+3\varepsilon)+$$
$$+\delta(3+\varepsilon(18+11\varepsilon)))+\delta\varepsilon(\delta+\delta^2(1+3\varepsilon)(e-(5+e)\varepsilon+\varepsilon^2)+\delta\varepsilon(1-e(1-\varepsilon)(1+7\varepsilon)+(4-\varepsilon)\varepsilon(3+7\varepsilon))+$$
$$+\varepsilon(-1+\varepsilon(-2+\varepsilon(-5-4e(-1+\varepsilon)+4(-3+\varepsilon)\varepsilon)))))-2b^2\delta\varepsilon(4\varepsilon(1+\varepsilon)^2+4\delta^2(1+3\varepsilon)-\delta(1+3\varepsilon)(3+5\varepsilon))^2k_r$$

$$\pi^{CB*}=\frac{F_3}{4\delta(\delta-\varepsilon)\varepsilon\left(4\varepsilon(1+\varepsilon)^2+4\delta^2(1+3\varepsilon)-\delta(1+3\varepsilon)(3+5\varepsilon)\right)}$$

$$F_3=-c^2(\delta-\varepsilon)(-1+\varepsilon)(3(-1+\delta)\delta+(-1+\delta)(-4+13\delta)?+4(3-7\delta)\varepsilon^2)+b^2(4(-1+\varepsilon)\varepsilon^2(1+3\varepsilon)+\delta^3(4+4(2-7\varepsilon)\varepsilon)-$$
$$-\delta\varepsilon(1+3\varepsilon)(-7+\varepsilon(3+8\varepsilon))+\delta^2(1+3\varepsilon)(-3+\varepsilon(-6+17\varepsilon)))-2c\delta(\delta-\varepsilon)\varepsilon(4\varepsilon(1+e+e\varepsilon(2+5\varepsilon)+\varepsilon(5+6\varepsilon-4\varepsilon^2))-$$
$$-\delta(4+e(3+5\varepsilon)(1+\varepsilon(3+4\varepsilon))-4(-2+\varepsilon)\varepsilon(3+\varepsilon(7+5\varepsilon)))+\delta^2(4+\varepsilon(29+(18-19\varepsilon)\varepsilon)+e(3+\varepsilon(10+19\varepsilon))))+$$
$$+2b(c(\delta-\varepsilon)(-1+\varepsilon)(4\varepsilon(1+3\varepsilon)+4\delta^2(1+4\varepsilon)-\delta(1+3\varepsilon)(3+8\varepsilon))+\delta\varepsilon(e(\delta-\varepsilon)(4\delta^2(1+\varepsilon(3+4\varepsilon))+4\varepsilon(1+\varepsilon(2+5\varepsilon))-$$
$$-\delta(3+\varepsilon(14+\varepsilon(31+16\varepsilon))))+\varepsilon(4\delta^3(3+\varepsilon-4\varepsilon^2)+4\varepsilon^2(-3+\varepsilon(-5+4\varepsilon))+4\delta\varepsilon(5+\varepsilon(11-4\varepsilon(1+\varepsilon)))+$$
$$+\delta^2(-9+\varepsilon(-34+\varepsilon(-5+32\varepsilon))))))+\delta\varepsilon(-4\varepsilon^2(1+\varepsilon)^2-\delta^4\varepsilon(16+e^2(-1+\varepsilon)(3+13\varepsilon)+e(-8+2\varepsilon(3+(14-13\varepsilon)\varepsilon))+$$
$$+\varepsilon(40+\varepsilon(13+\varepsilon(-18+13\varepsilon))))+\delta^3(4+\varepsilon(24+e^2(-1+\varepsilon)(1+5\varepsilon)(3+5\varepsilon)-2e(-1+\varepsilon)(2+5\varepsilon)(-2+\varepsilon+5\varepsilon^2)+$$
$$+\varepsilon(68+\varepsilon(7+5\varepsilon)(13+5(-2+\varepsilon)\varepsilon))))+\delta\varepsilon(7+\varepsilon(23+\varepsilon(29-8e(-1+\varepsilon)(-1+2\varepsilon)(1+2\varepsilon)+4e^2(-1+\varepsilon)(1+3\varepsilon)+$$
$$+\varepsilon(33+4\varepsilon(3+\varepsilon(-7+5\varepsilon))))))-\delta^2(3+\varepsilon(19+\varepsilon(49+e^2(-1+\varepsilon)(7+\varepsilon(29+12\varepsilon))-8e(-1+\varepsilon)(-2+\varepsilon(-1+\varepsilon(8+3\varepsilon)))+$$
$$+\varepsilon(86+\varepsilon(61+\varepsilon(-45+\varepsilon(23+12\varepsilon))))))))-2b^2\delta(\delta-\varepsilon)\varepsilon(4?\varepsilon(1+\varepsilon)^2+4\delta^2(1+3\varepsilon)-\delta(1+3\varepsilon)(3+5\varepsilon))k_r$$

Appendix G

Proof for Lemma 3

1. The Decision Model and Solution in the Discounted Sales Period

According to the expression (32) in the paper, we construct the Lagrange function as follows:

$$L = \pi_{R2}^{D} + k_1\left(-\frac{p_2^r - p_2^e}{\delta(1-\varepsilon)}\right) + k_2\left(\frac{p_2^r - p_2^e}{\delta(1-\varepsilon)} - \frac{p_1^e - p_2^r}{\varepsilon-\delta}\right) + k_3\left(\frac{p_1^e - p_2^r}{\varepsilon-\delta} - 1\right) \tag{G1}$$

The KKT conditions are expressed as follows:

$$\begin{cases} \dfrac{\partial L}{\partial p_2^r} = 0 \\ k_1\left(-\dfrac{p_2^r - p_2^e}{\delta(1-\varepsilon)}\right) = k_2\left(\dfrac{p_2^r - p_2^e}{\delta(1-\varepsilon)} - \dfrac{p_1^e - p_2^r}{\varepsilon-\delta}\right) \\ = k_3\left(\dfrac{p_1^e - p_2^r}{\varepsilon-\delta} - 1\right) = 0 \\ k_i \ge 0, i = 1,2,3. \end{cases} \tag{G2}$$

According to the expression (33) in the paper, we construct the Lagrange function as follows:

$$L = \pi_{M2}^{D} + k_1\left(-\frac{p_2^e}{\delta\varepsilon}\right) + k_2\left(\frac{p_2^e}{\delta\varepsilon} - \frac{p_2^r - p_2^e}{\delta(1-\varepsilon)}\right) + k_3\left(\frac{p_2^r - p_2^e}{\delta(1-\varepsilon)} - \frac{p_1^e - p_2^r}{\varepsilon-\delta}\right) + k_4\left(\frac{p_1^e - p_2^r}{\varepsilon-\delta} - 1\right) \tag{G3}$$

The KKT conditions are expressed as follows:

$$\begin{cases} \dfrac{\partial L}{\partial p_2^e} = 0 \\ k_1\left(-\dfrac{p_2^e}{\delta\varepsilon}\right) = k_2\left(\dfrac{p_2^e}{\delta\varepsilon} - \dfrac{p_2^r - p_2^e}{\delta(1-\varepsilon)}\right) \\ = k_3\left(\dfrac{p_2^r - p_2^e}{\delta(1-\varepsilon)} - \dfrac{p_1^e - p_2^r}{\varepsilon-\delta}\right) = k_4\left(\dfrac{p_1^e - p_2^r}{\varepsilon-\delta} - 1\right) = 0 \\ k_i \ge 0, i = 1,2,3,4. \end{cases} \tag{G4}$$

By solving the above model, the prices of the two channels during the discounted sales period are respectively calculated as follows:

$$p_2^{eD} = \frac{2c - 2c\delta - 2c\varepsilon + 3w\varepsilon + 2c\delta\varepsilon - 3w\delta\varepsilon + \delta p_1^e - \delta\varepsilon p_1^e}{4 - 3\delta - \varepsilon} \tag{G5}$$

$$p_2^{rD} = \frac{c(\varepsilon-\delta) + 2w\varepsilon + c\delta\varepsilon - 3w\delta\varepsilon - c\varepsilon^2 + w\varepsilon^2 + 2\delta p_1^e - 2\delta\varepsilon p_1^e}{\varepsilon(4 - 3\delta - \varepsilon)} \tag{G6}$$

2. The Decision Model and Solution in the Full-Price Sales Period

According to the expression (34) in the paper, we construct the Lagrange function as follows:

$$L = \pi_{R}^{D} + k_1\left(-\frac{p_2^r - p_2^e}{\delta(1-\varepsilon)}\right) + k_2\left(\frac{p_2^r - p_2^e}{\delta(1-\varepsilon)} - \frac{p_1^e - p_2^r}{\varepsilon-\delta}\right) + k_3\left(\frac{p_1^e - p_2^r}{\varepsilon-\delta} - \frac{p_1^r - p_1^e}{1-\varepsilon}\right) + k_4\left(\frac{p_1^r - p_1^e}{1-\varepsilon} - 1\right) \tag{G7}$$

The KKT conditions are expressed as follows:

$$\begin{cases} \dfrac{\partial L}{\partial p_1^r} = 0 \\ k_1\left(-\dfrac{p_2^r - p_2^e}{\delta(1-\varepsilon)}\right) = k_2\left(\dfrac{p_2^r - p_2^e}{\delta(1-\varepsilon)} - \dfrac{p_1^e - p_2^r}{\varepsilon-\delta}\right) \\ k_3\left(\dfrac{p_1^e - p_2^r}{\varepsilon-\delta} - \dfrac{p_1^r - p_1^e}{1-\varepsilon}\right) = k_4\left(\dfrac{p_1^r - p_1^e}{1-\varepsilon} - 1\right) = 0 \\ k_i \ge 0, i = 1,2,3,4. \end{cases} \tag{G8}$$

According to the expression (35) in the paper, we construct the Lagrange function as follows:

$$L = \pi_{M}^{D} + k_1\left(-\frac{p_2^e}{\delta\varepsilon}\right) + k_2\left(\frac{p_2^e}{\delta\varepsilon} - \frac{p_2^r - p_2^e}{\delta(1-\varepsilon)}\right) + k_3\left(\frac{p_2^r - p_2^e}{\delta(1-\varepsilon)} - \frac{p_1^e - p_2^r}{\varepsilon-\delta}\right) + k_4\left(\frac{p_1^e - p_2^r}{\varepsilon-\delta} - \frac{p_1^r - p_1^e}{1-\varepsilon}\right) + k_5\left(\frac{p_1^r - p_1^e}{1-\varepsilon} - 1\right) \tag{G9}$$

The KKT conditions are expressed as follows:

$$\begin{cases} \dfrac{\partial L}{\partial p_1^e} = 0 \\ k_1\left(-\dfrac{p_2^e}{\delta\varepsilon}\right) = k_2\left(\dfrac{p_2^e}{\delta\varepsilon} - \dfrac{p_2^r - p_2^e}{\delta(1-\varepsilon)}\right) \\ = k_3\left(\dfrac{p_2^r - p_2^e}{\delta(1-\varepsilon)} - \dfrac{p_1^e - p_2^r}{\varepsilon-\delta}\right) \\ = k_4\left(\dfrac{p_1^e - p_2^r}{\varepsilon-\delta} - \dfrac{p_1^r - p_1^e}{1-\varepsilon}\right) = k_5\left(\dfrac{p_1^r - p_1^e}{1-\varepsilon} - 1\right) = 0 \\ k_i \ge 0, i = 1,2,3,4,5. \end{cases} \tag{G10}$$

By substituting p_2^{eD}, p_2^{rD} in expressions (G5) and (G6) into expressions (G7) and (G9) respectively, Lemma 3 can be obtained.

Appendix H

$$H_1=14\delta^2+7c\delta^2+14w\delta^2+32\varepsilon+8c\varepsilon+48w\varepsilon-16\delta-8c\delta-16w\delta-30\delta\varepsilon+10c\delta\varepsilon-96w\delta\varepsilon+24\delta^2\varepsilon-14c\delta^2\varepsilon+$$
$$+80w\delta^2\varepsilon-18\delta^3\varepsilon-27w\delta^3\varepsilon-48\varepsilon^2-17c\varepsilon^2-14w\varepsilon^2+54\delta\varepsilon^2+4c\delta\varepsilon^2+24w\delta\varepsilon^2-36\delta^2\varepsilon^2+7c\delta^2\varepsilon^2-13w\delta^2\varepsilon^2+$$
$$+18\delta^3\varepsilon^2+18\varepsilon^3+10c\varepsilon^3-8w\varepsilon^3-8\delta\varepsilon^3-6c\delta\varepsilon^3+7w\delta\varepsilon^3-2\delta^2\varepsilon^3-2\varepsilon^4-c\varepsilon^4+w\varepsilon^4$$

$$H_2=2c(\varepsilon-1)\Big((4-\varepsilon)^2\varepsilon-9\delta^3\varepsilon+\delta^2\left(7+\varepsilon(19+\varepsilon)\right)+\delta\left(\varepsilon(4\varepsilon-23)-8\right)\Big)+\varepsilon(\delta(\varepsilon-\delta)(1-\varepsilon)^2(3\delta+\varepsilon-4)+$$
$$+w\Big(9\delta^3(4\varepsilon-1)-3\varepsilon(4-\varepsilon)^2+\delta\left(16+(76-11\varepsilon)\varepsilon\right)-4\delta^2-73\varepsilon\delta^2-4\delta^2\varepsilon^2\Big)$$

$$H_3=3\delta^3\Big(3w(5\varepsilon-2)-2(1-\varepsilon)^2\Big)-w\varepsilon(4-\varepsilon)^2(2+\varepsilon)-c(\varepsilon-\delta)(1-\varepsilon)(3\delta+\varepsilon-4)^2+\delta\varepsilon(w(76+5\varepsilon(2-\varepsilon)-$$
$$-2(4-\varepsilon)(1-\varepsilon)^2))+\delta^2\Big(4\left(2-3\varepsilon+\varepsilon^3\right)+w\left(20-94\varepsilon+7\varepsilon^2\right)\Big)$$

Appendix I

$$\pi_R^{D*}=\frac{I_1}{4\Big(4\delta^3\varepsilon-4(-2+\varepsilon)^2\varepsilon+\delta^2(-7+\varepsilon)(1+\varepsilon)+\delta\big(8+(-3+\varepsilon)\varepsilon(-5+3\varepsilon)\big)\Big)^2}$$

$$I_1=(1-\varepsilon)(\delta(c(4-3\delta)+(1-w)(8-7\delta))-16\varepsilon+(c(1+\delta)(3\delta-4)+\delta(19-6(2-\delta)\delta)+w(8+\delta-7\delta^2))\varepsilon+(12+c(4-3\delta)-$$
$$-(10-\delta)\delta-w(8-7\delta))\varepsilon^2-(2-\delta)\varepsilon^3)^2-\frac{1}{(\delta\varepsilon(-2+\delta+\varepsilon)^2)}((1-\delta)(\varepsilon-\delta)(c(2\delta^3\varepsilon-4(-2+\varepsilon)^2\varepsilon+\delta\varepsilon(25+3(-6+\varepsilon)\varepsilon)+$$
$$+\delta^2(1+\varepsilon(-14+5\varepsilon)))+\varepsilon(\delta(-1+\varepsilon)(-2+\delta+\varepsilon)(-4+3\delta+\varepsilon)-2w(-1+\delta)(3\delta^2+4(-2+\varepsilon)^2+\delta(-13+6\varepsilon))))^2))$$

$$\pi_M^{D*}=\frac{I_2}{2\delta\varepsilon(-2+\delta+\varepsilon)\Big(4\delta^3\varepsilon-4(-2+\varepsilon)^2\varepsilon+\delta^2(-7+\varepsilon)(1+\varepsilon)+\delta\big(8+(-3+\varepsilon)\varepsilon(-5+3\varepsilon)\big)\Big)}$$

$$I_2=c^2(\varepsilon-1)(-3\delta^4 ?-4(-2+\varepsilon)^2\varepsilon+\delta^3(3+(14-3\varepsilon)\varepsilon)+\delta^2(-11+\varepsilon(-23+11\varepsilon))+\delta(8+\varepsilon(27+\varepsilon(-22+3\varepsilon))))+$$
$$c\varepsilon(-8\delta^5\varepsilon+8w(-2+\varepsilon)^2\varepsilon+\delta^4(11+(34-13\varepsilon)\varepsilon+2w(-3+7\varepsilon))+2\delta\varepsilon(w(-39+(26-3\varepsilon)\varepsilon)+2(-2+\varepsilon)(7-(6-\varepsilon)\varepsilon))+$$
$$+2\delta^3(-17+\varepsilon(-30+(27-4\varepsilon)\varepsilon)+w(9+\varepsilon(-32+7\varepsilon)))+\delta^2(24-2w(7+\varepsilon(-51+20\varepsilon))+\varepsilon(86-3\varepsilon(37-(12-\varepsilon)\varepsilon))))+$$
$$+\varepsilon((1-\delta)\delta(\delta-\varepsilon)(-1+\varepsilon)\varepsilon(-2+\delta+\varepsilon)^2+w\delta(-2+\delta+\varepsilon)(8\delta^3\varepsilon+8\varepsilon(-3+2\varepsilon)+\delta^2(\varepsilon(9\varepsilon-26)-7)-$$
$$+\delta(-8+\varepsilon(\varepsilon(22+\varepsilon)-39)))+w^2(\delta^4(7-15\varepsilon)-8(-2+\varepsilon)^2\varepsilon+2\delta\varepsilon(39+\varepsilon(-26+3\varepsilon))-2\delta^3(11-\varepsilon(35-8\varepsilon))+$$
$$+\delta^2(16-\varepsilon(104-\varepsilon(39+\varepsilon)))))$$

Appendix J

Proof for Lemma 4

1. The Decision Model and Solution in the Discounted Sales Period

According to the expression (46) in the paper, we construct the Lagrange function as follows:

$$L=\pi_{R2}^{DB}+k_1\left(-\frac{p_2^r-p_2^e-\delta e(1-\varepsilon)}{\delta(1-\varepsilon)}\right)+$$
$$+k_2\left(\frac{p_2^r-p_2^e-\delta e(1-\varepsilon)}{\delta(1-\varepsilon)}-\frac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}\right)+ \qquad (J1)$$
$$+k_3\left(\frac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}-1\right)$$

The KKT conditions are expressed as follows:

$$\begin{cases}\dfrac{\partial L}{\partial p_2^r}=0\\ k_1\left(-\dfrac{p_2^r-p_2^e-\delta e(1-\varepsilon)}{\delta(1-\varepsilon)}\right)=0\\ k_2\left(\dfrac{p_2^r-p_2^e-\delta e(1-\varepsilon)}{\delta(1-\varepsilon)}-\dfrac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}\right)=0\\ k_3\left(\dfrac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}-1\right)=0\\ k_i\ge 0, i=1,2,3.\end{cases} \qquad (J2)$$

According to the expression (47) in the paper, we construct the Lagrange function as follows:

$$\begin{aligned} L=&\pi_{M2}^{DB}+k_1\left(e-\frac{p_2^e}{\delta\varepsilon}\right)+\\ &+k_2\left(\frac{p_2^e}{\delta\varepsilon}-e-\frac{p_2^r-p_2^e-\delta e(1-\varepsilon)}{\delta(1-\varepsilon)}\right)+\\ &+k_3\left(\frac{p_2^r-p_2^e-\delta e(1-\varepsilon)}{\delta(1-\varepsilon)}-\frac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}\right)+\\ &+k_4\left(\frac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}-1\right)\end{aligned} \tag{J3}$$

The KKT conditions are expressed as follows:

$$\begin{cases} \dfrac{\partial L}{\partial p_2^e}=0\\ k_1\left(e-\dfrac{p_2^e}{\delta\varepsilon}\right)=0\\ k_2\left(\dfrac{p_2^e}{\delta\varepsilon}-e-\dfrac{p_2^r-p_2^e-\delta e(1-\varepsilon)}{\delta(1-\varepsilon)}\right)=0\\ +k_3\left(\dfrac{p_2^r-p_2^e-\delta e(1-\varepsilon)}{\delta(1-\varepsilon)}-\dfrac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}\right)+=0\\ k_4\left(\dfrac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}-1\right)=0\\ k_i\ge 0, i=1,2,3,4.\end{cases} \tag{J4}$$

By solving the above model, the prices of the two channels during the discounted sales period are respectively obtained as follows:

$$p_2^{eDB}=\frac{1}{4-3\delta-\varepsilon}\left(2c-2b+2b\delta-2c\delta+2b\varepsilon-2c\varepsilon+3w\varepsilon-2b\delta\varepsilon+2c\delta\varepsilon+e\delta\varepsilon-3w\delta\varepsilon-e\delta\varepsilon^2+\delta p_1^e-\delta\varepsilon p_1^e\right) \tag{J5}$$

$$p_2^{rDB}=\frac{1}{\varepsilon(4-3\delta-\varepsilon)}\left(b\delta-c\delta-b\varepsilon+c\varepsilon+2w\varepsilon-b\delta\varepsilon+c\delta\varepsilon+2e\delta\varepsilon-3w\delta\varepsilon+b\varepsilon^2-c\varepsilon^2+w\varepsilon^2-2e\delta\varepsilon^2+2\delta p_1^e-2\delta\varepsilon p_1^e\right) \tag{J6}$$

2. The Decision Model and Solution in the Full-Price Sales Period

According to the expression (48) in the paper, we construct the Lagrange function as follows:

$$\begin{aligned} L=&\pi_R^{DB}+k_1\left(-\frac{p_2^r-p_2^e-\delta e(1-\varepsilon)}{\delta(1-\varepsilon)}\right)+\\ &+k_2\left(\frac{p_2^r-p_2^e-\delta e(1-\varepsilon)}{\delta(1-\varepsilon)}-\frac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}\right)+\\ &+k_3\left(\frac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}-\frac{p_1^r-p_1^e}{1-\varepsilon}\right)+k_4\left(\frac{p_1^r-p_1^e}{1-\varepsilon}-1\right)\end{aligned} \tag{J7}$$

The KKT conditions are expressed as follows:

$$\begin{cases} \dfrac{\partial L}{\partial p_1^r}=0\\ k_1\left(-\dfrac{p_2^r-p_2^e-\delta e(1-\varepsilon)}{\delta(1-\varepsilon)}\right)=0\\ k_2\left(\dfrac{p_2^r-p_2^e-\delta e(1-\varepsilon)}{\delta(1-\varepsilon)}-\dfrac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}\right)=0\\ k_3\left(\dfrac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}-\dfrac{p_1^r-p_1^e}{1-\varepsilon}\right)=0\\ k_4\left(\dfrac{p_1^r-p_1^e}{1-\varepsilon}-1\right)\\ k_i\ge 0, i=1,2,3,4.\end{cases} \tag{J8}$$

According to the expression (49) in the paper, we construct the Lagrange function as follows:

$$\begin{aligned} L=&\pi_M^{DB}+k_1\left(e-\frac{p_2^e}{\delta\varepsilon}\right)+\\ &+k_2\left(\frac{p_2^e}{\delta\varepsilon}-e-\frac{p_2^r-p_2^e-\delta e(1-\varepsilon)}{\delta(1-\varepsilon)}\right)+\\ &+k_3\left(\frac{p_2^r-p_2^e-\delta e(1-\varepsilon)}{\delta(1-\varepsilon)}-\frac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}\right)+\\ &+k_4\left(\frac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}-\frac{p_1^r-p_1^e}{1-\varepsilon}\right)+k_5\left(\frac{p_1^r-p_1^e}{1-\varepsilon}-1\right)\end{aligned} \tag{J9}$$

The KKT conditions are expressed as follows:

$$\begin{cases} \dfrac{\partial L}{\partial p_1^e}=0\\ k_1\left(e-\dfrac{p_2^e}{\delta\varepsilon}\right)=0\\ k_2\left(\dfrac{p_2^e}{\delta\varepsilon}-e-\dfrac{p_2^r-p_2^e-\delta e(1-\varepsilon)}{\delta(1-\varepsilon)}\right)=0\\ k_3\left(\dfrac{p_2^r-p_2^e-\delta e(1-\varepsilon)}{\delta(1-\varepsilon)}-\dfrac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}\right)=0\\ k_4\left(\dfrac{p_1^e-p_2^r+\delta e}{\varepsilon-\delta}-\dfrac{p_1^r-p_1^e}{1-\varepsilon}\right)=0\\ k_5\left(\dfrac{p_1^r-p_1^e}{1-\varepsilon}-1\right)=0\\ k_i\ge 0, i=1,2,3,4,5.\end{cases} \tag{J10}$$

By substituting p_2^{eDB}, p_2^{rDB} in expressions (J5) and (J6) into expressions (J7) and (J9) respectively, Lemma 4 can be obtained.

Appendix K

Among the expressions from (50) to (53), we have:

$$K_1=(2c(1-\varepsilon)^2(8\delta\varepsilon-\delta^2-(8-\varepsilon)\varepsilon)-2b(1-\varepsilon)(\varepsilon(8+\varepsilon)-2\delta\varepsilon(8+\varepsilon)-\delta^2(1-10\varepsilon))+\varepsilon((1-\varepsilon)(9(1+2e)\delta^3-(4-\varepsilon)^2\varepsilon-$$
$$-(3+4e)\delta^2(8+\varepsilon)+\delta(4-\varepsilon)(4+5\varepsilon)+2e\delta(8+\varepsilon^2))+w(27\delta^3-32-27\delta^2(4-\varepsilon)+\varepsilon((28-3\varepsilon)\varepsilon-20)+116\delta-\varepsilon\delta(16+19\varepsilon))))$$

$$K_2=(8\delta^2-8\delta-8w\delta-b\delta^2+c\delta^2+8w\delta^2+32\varepsilon+8b\varepsilon+8c\varepsilon+48w\varepsilon-56\delta\varepsilon-16b\delta\varepsilon-8c\delta\varepsilon-8e\delta\varepsilon-114w\delta\varepsilon+$$
$$+42\delta^2\varepsilon+11b\delta^2\varepsilon-2c\delta^2\varepsilon+16e\delta^2\varepsilon+92w\delta^2\varepsilon-18\delta^3\varepsilon-9e\delta^3\varepsilon-27w\delta^3\varepsilon-48\varepsilon^2-7b\varepsilon^2-17c\varepsilon^2-14w\varepsilon^2+$$
$$+84\delta\varepsilon^2+14b\delta\varepsilon^2+16c\delta\varepsilon^2+8e\delta\varepsilon^2+36w\delta\varepsilon^2-54\delta^2\varepsilon^2-10b\delta^2\varepsilon^2+c\delta^2\varepsilon^2-14e\delta^2\varepsilon^2-19w\delta^2\varepsilon^2+18\delta^3\varepsilon^2+9e\delta^3\varepsilon^2+$$
$$+18\varepsilon^3-b\varepsilon^3+10c\varepsilon^3-8w\varepsilon^3-22\delta\varepsilon^3+2b\delta\varepsilon^3-8c\delta\varepsilon^3-e\delta\varepsilon^3+5w\delta\varepsilon^3+4\delta^2\varepsilon^3-2e\delta^2\varepsilon^3-2\varepsilon^4-c\varepsilon^4+w\varepsilon^4+2\delta\varepsilon^4+e\delta\varepsilon^4$$

$$K_3=2c(1-\varepsilon)(9\delta^3\varepsilon-(4-\varepsilon)^2\varepsilon+\delta(4-\varepsilon)(1+8\varepsilon)-\delta^2(5+(23-\varepsilon)\varepsilon))-2b(1-\varepsilon)(9\delta^3\varepsilon-(4-\varepsilon)^2\varepsilon-\delta(4-\varepsilon)(\varepsilon^2-9\varepsilon-1)+$$
$$+\delta^2(4\varepsilon^2-26\varepsilon-5))+\varepsilon(\delta(1-\varepsilon)(4-3\delta-\varepsilon)(\varepsilon(\varepsilon-1-e(4-\varepsilon))+\delta(1+2e-(1-e)\varepsilon))+w(9\delta^3(4\varepsilon-1)-3\varepsilon(4-\varepsilon)^2+$$
$$+\delta(4+(100-23\varepsilon)\varepsilon)+\delta^2(8-97\varepsilon+8\varepsilon^2)))$$

$$K_4=b(1-\varepsilon)((4-\varepsilon)^2\varepsilon^2-9\delta^3\varepsilon-\delta\varepsilon(2+\varepsilon)(14-5\varepsilon)+\delta^2(4+(28-5\varepsilon)\varepsilon))+c(1-\varepsilon)(9\delta^3\varepsilon-(4-\varepsilon)^2\varepsilon^2-\delta^2(2+\varepsilon)(2+7\varepsilon)-$$
$$-3\delta\varepsilon(3\varepsilon^2-8\varepsilon-4))+\varepsilon(2\delta(1-\varepsilon)(4-3\delta-\varepsilon)(\varepsilon(\varepsilon-1-e(4-\varepsilon))+\delta(1+2e-(1-e)\varepsilon))+w(9\delta^3(5\varepsilon-2)-(4-\varepsilon)^2\varepsilon(2+\varepsilon)+$$
$$+\delta^2(32-\varepsilon(118-5\varepsilon))-8\delta+\varepsilon\delta(88+10\varepsilon-9\varepsilon^2)))$$

Appendix L

$$\pi_R^{DB*}=\frac{L_1}{\left(27\delta^3\varepsilon-\left(4-\varepsilon\right)^3\varepsilon+\delta^2\left(-16+\varepsilon\left(-76+11\varepsilon\right)\right)+\delta\left(16+\varepsilon\left(112-56\varepsilon+9\varepsilon^2\right)\right)\right)^2}$$

$$L_1=(1-\varepsilon)(\frac{1}{\delta(\varepsilon-\delta)\varepsilon}(\delta-1)(c((4-\varepsilon)^2\varepsilon^2-9\delta^3\varepsilon+\delta^2(2+\varepsilon)(2+7\varepsilon)+3\delta\varepsilon(-4+\varepsilon(-8+3\varepsilon)))+b(9\delta^3\varepsilon-(4-\varepsilon)^2\varepsilon^2+$$
$$+\delta\varepsilon(28+(4-5\varepsilon)\varepsilon)+\delta^2(-4+\varepsilon(-28+5\varepsilon)))+2\varepsilon(\delta(-4+3\delta+\varepsilon)(\varepsilon(\varepsilon-e(4-\varepsilon)-1)+\delta(1+2e+(-1+e)\varepsilon))+$$
$$+w(9\delta^3-(4-\varepsilon)^2\varepsilon-3\delta^2(8+\varepsilon)+3\delta(4+(8-3\varepsilon)\varepsilon))))^2-(9(2+e)\delta^3\varepsilon-\varepsilon(8(4+b+c-2w)+(b-16-9c+18w)\varepsilon+$$
$$+(2+c-2w)\varepsilon^2)+\delta(8+8(8+2b+c+e)\varepsilon-2w(1-\varepsilon)(4+7\varepsilon)+\varepsilon^2(2b-8c-20+(2+e)\varepsilon))+\delta^2(b-c(1-\varepsilon)-$$
$$-10b\varepsilon-2(4-4w(1-\varepsilon)+\varepsilon(25-2\varepsilon+e(8+\varepsilon)))))^2)$$

$$\pi_M^{(DB*)}=\frac{L_2}{27\delta^3\varepsilon+?(-4+\varepsilon)^3\varepsilon+\delta^2(-16+\varepsilon(-76+11\varepsilon))+\delta(16+\varepsilon(112+\varepsilon(-56+9\varepsilon)))}-L_3-\frac{L_4}{L_5}-\frac{L_6}{L_7}-\frac{b^2k_r}{2}L_2$$
$$=\left(c-w\right)(32\varepsilon-9(2+e)\delta^3\varepsilon+\delta^2(8-b+c-8w+(50+10b-c+16e+8w)\varepsilon-2(2-e)\varepsilon^2)+\delta(-8+8w-8(8+2b+c+e)\varepsilon+$$
$$+6w\varepsilon-2(b-4c+7w-10)\varepsilon^2-(2+e)\varepsilon^3)+\varepsilon(b(8+\varepsilon)-(8-\varepsilon)(2(w+\varepsilon-w\varepsilon)-c(1-\varepsilon))))$$

$$L_3=((2\delta(c(8-7\delta)-b\delta)+(16(b-3c+2w)-4(4+8b-24c+4e+29w)\delta+2(12+11b-40c+16e+54w)\delta^2+$$
$$+9(-1+3c-2e-3w)\delta^3)\varepsilon+(2(8-7b+7c+10w)+4(7b-6c+4(e+w))\delta-(21+20b-13c+28e+27w)\delta^2+$$
$$+9(1+2e)\delta^3)\varepsilon^2-(24-8c+28w+b(2-4\delta)+(7c+2e-21-19w)\delta+(3+4e)\delta^2)\varepsilon^3+$$
$$+(9-c+3w-5\delta+2e\delta)\varepsilon^4-\varepsilon^5)(4(c-b)\delta^2+2\delta(14b-6c+4w+(5c-11b-12w)\delta+(4b-4c-2e+5w-1)\delta^2)\varepsilon+$$
$$+(4\delta(4b+3c-4e+w-4)-32b+2(22-2b-7c+30e+12w)\delta^2+(b+8c-29-48e-10w)\delta^3+9(1+2e)\delta^3)\varepsilon^2+$$
$$+(16+2c-2w(2+7\delta)+b(30-\delta(20-3\delta))-\delta(34+2e(2+\delta)^2-\delta(11-5\delta)))\varepsilon^3+(2w(2+\delta)-8-2c-b(8-3\delta)+$$
$$+\delta(17-\delta+2e(4+\delta)))\varepsilon^4+(1+b-(3+2e)\delta)\varepsilon^5)))/(((\delta-\varepsilon)\varepsilon((27\delta^3\varepsilon+(4-\varepsilon)^3\varepsilon-\delta^2(16+76\varepsilon-11\varepsilon^2))+$$
$$+\delta(16+\varepsilon(112-56\varepsilon+9\delta^2))))^5))$$

$$L_4 = (c-b-w)(1-\delta)(c(-9\delta^3\varepsilon + (4-\varepsilon)^2\varepsilon^2 + \delta^2(2+\varepsilon)(2+7\varepsilon) + 3\delta\varepsilon(\varepsilon(3\varepsilon-8)-4)) + b(9\delta^3\varepsilon - (4-\varepsilon)^2\varepsilon^2 +$$
$$+\delta\varepsilon(28+(4-5\varepsilon)\varepsilon) + \delta^2(\varepsilon(5\varepsilon-28)-4)) + 2\varepsilon(\delta(3\delta+\varepsilon-4)(\varepsilon(e(-4+\varepsilon)+\varepsilon-1) + \delta(1+2e+(-1+e)\varepsilon)) +$$
$$+w(9\delta^3 - (4-\varepsilon)^2\varepsilon - 3\delta^2(8+\delta) + 3\delta(4+(8-3\varepsilon)\varepsilon))))$$

$$L_5 = \delta(\delta-\varepsilon)\left(27\delta^3\varepsilon + (-4+\varepsilon)^3\varepsilon + \delta^2\left(-16+\varepsilon(-76+11\varepsilon)\right) + \delta\left(16+\varepsilon\left(112+\varepsilon(-56+9\varepsilon)\right)\right)\right)$$

$$L_6 = (\varepsilon(\delta(-4+3\delta+\varepsilon)(\varepsilon(-1+e(-4+\varepsilon)+\varepsilon) + \delta(1+2e+(-1+e)\varepsilon)) + w(9\delta^3 - (-4+\varepsilon)^2\varepsilon - 3\delta^2(8+\varepsilon) + 3\delta(4+(8-3\varepsilon)\varepsilon))) -$$
$$-b(9\delta^3\varepsilon - (4-\varepsilon)^2(2-\varepsilon)\varepsilon + 3\delta^2(-2+(-8+\varepsilon)\varepsilon) + \delta(8+\varepsilon(42+\varepsilon(-30+7\varepsilon)))) + c(9\delta^3\varepsilon - (4-\varepsilon)^2(2-\varepsilon)\varepsilon +$$
$$\delta^2(9\varepsilon^2 - 6 - 30\varepsilon) + \delta(8+\varepsilon(50-\varepsilon(40-9\varepsilon)))))(c((4-\varepsilon)^2\varepsilon(2+\varepsilon) - 9\delta^3\varepsilon(1+2\varepsilon) + \delta^2(6+\varepsilon(40+(37-2\varepsilon)\varepsilon)) +$$
$$+\delta(-8+\varepsilon(-58+\varepsilon(-22+7\varepsilon)))) + \varepsilon(\delta(-1+\varepsilon)(-4+3\delta+\varepsilon)(\varepsilon(-1+e(-4+\varepsilon)+\varepsilon) + \delta(1+2e+(-1+e)\varepsilon)) + w(-3(4-\varepsilon)^2\varepsilon +$$
$$+9\delta^3(4\varepsilon-1) + \delta(4+(100-23\varepsilon)\varepsilon) + \delta^2(8+\varepsilon(-97+8\varepsilon)))) + b(-(-4+\varepsilon)^2\varepsilon(2+\varepsilon) + 9\delta^3\varepsilon(1+2\varepsilon) +$$
$$+\delta^2(\varepsilon(-34-\varepsilon(49-8\varepsilon))-6) + \delta(8+\varepsilon(50+\varepsilon(40-\varepsilon(19-2\varepsilon))))))$$

$$L_7 = \delta\varepsilon\left(27\delta^3\varepsilon - (4-\varepsilon)^3\varepsilon + \delta^2\left(\varepsilon(-76+11\varepsilon)-16\right) + \delta\left(16+\varepsilon\left(112-\varepsilon(56-9\varepsilon)\right)\right)\right)^2$$