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Metaheuristic Algorithms for the Optimization of Integrated Production Scheduling and Vehicle Routing Problems in Supply Chains

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Abstract: This paper examines the challenge of integrated production and distribution, aiming to deliver products to customers precisely on time. Customers, situated within the transportation network, have predefined requirements regarding demand volume and time frames. In the first phase (F_1), the problem of planning and allocation of resources is presented as FJSP, while the second phase (F_2) addresses the vehicle routing problem as CVRPTW. The first phase, F_1, aims to optimize manufacturing processes by appropriately scheduling production tasks to maximize productivity and minimize the time of task execution on machines. Phase 2, F_2, encompasses the process of distribution to customers, seeking to minimize the number of vehicles, delivery time, and overall distance travelled. As both problems are among the most challenging in combinatorial optimization, integrating these phases into a single supply chain system poses a significant challenge in problem-solving. A mathematical formulation has been developed to include planning and task allocation in production, as well as vehicle routing, to obtain an optimal solution to the integrated problem. The input data used in the observed case study represent real data in both the first and second phases, forming one integrated supply chain system. Experimental results support the applied methodology.

Keywords: cross-dock; integrated production and distribution problems; scheduling; supply chain; vehicle routing problem

1 INTRODUCTION

Organization and management of material and information flows within supply chains play a crucial role in interconnecting activities. Supply chains encompass all activities from material sources to the end-user. Over the past few years, supply chains have garnered increasing interest both in scientific literature and in practice [1].

The procurement of basic raw materials for production and the functioning of production represent the first input material flow in supply chains. These are followed by production as the second step and the distribution of finished products as the final one [2]. Basic stages of supply chains such as procurement, production and distribution previously functioned independently of each other. However, today in the world of fast, complex, and numerous decisions it is necessary to connect all stages into one complex system. The coordination of the entire system and the integration of phases within supply chains are inevitable steps and prerequisites for successful operations [3]. Interconnecting sources of raw materials, production, and distributors leads to increased flow of information in all resource planning phases, aiming to reduce risks in the overall supply chain system. The implementation of a logistics system into the supply chain system contributes to an increase in service levels and the timely delivery of goods, ultimately aiming to enhance productivity and profit. The motivation in this work is the connection and integration of resource planning activities in production and vehicle routing. It is important to note that resource planning in production and resource planning in vehicle routing are two closely related problems within supply chains, forming a complex system. Production and the outbound distribution of finished products are crucial activities where the waiting time between two activities must be minimized, and service delivery must be at a maximum level [4, 5].

Production planning and vehicle routing are two well-studied problems in literature. Although these functions of the supply chain are interconnected, they are often solved separately. Resource planning is characterized by scheduling jobs in a production environment to increase productivity and reduce overall production time. Resource planning and scheduling consist of several models used depending on the type of production and machine layout, including Single Machine Scheduling Problem, Parallel Machine Scheduling Problem, Flow Shop Scheduling Problem, Job Shop Scheduling Problem, Open Shop Scheduling Problem [6]. All these models play a role in production planning, and in this study, the Flexible Job Shop Problem (FJSP) is applied. FJSP is an extension of the classical Job Shop Problem, allowing flexibility in executing operations on a set of alternative machines. The problem of resource planning and scheduling in this paper represents the second phase within supply chains, while the next phase involves the Vehicle Routing Problem (VRP). VRP is one of the most challenging problems in combinatorial optimization. It is increasingly applied to solve various problems and holds significant economic importance in reducing operational costs of distribution systems. To address real-world issues in VRP solutions, several constraints are typically introduced, such as a larger number of depots, different types of vehicles (homogeneous and heterogeneous), different types of customer demands (deterministic and stochastic), infrastructure constraints (one-way streets, restricted paths), types of services (transport, delivery, and mixed services), etc. [7, 8].

Vehicle routing is a concept applied in various problems and holds significant economic importance in reducing operational costs within supply chains [9]. Establishing a sales network and routing vehicles for the distribution of finished products to retailers or end-users, in this case customers, is of great importance. Connecting phases within supply chains aims to increase productivity and reduce the overall delivery time of products to customers at the right time [10]. After assessing the entire production system, it is necessary to determine which mathematical model is most suitable for implementation to ensure the efficient functioning of the production system. It is important to note that FJSP and VRP are considered among the most challenging NP problems in their respective research areas, posing a significant challenge in integrating two separate optimization problems [11]. Integrating these two problems can significantly contribute to saving time and increased overall system productivity in the future. The advantage of merging these phases into a modern system within supply chains leads to average improvements ranging from 5% to 20%, as observed in the study [12]. Companies are increasingly encouraged to jointly optimize production scheduling and outbound distribution. Above all, companies are now more focused on key competencies than before [13]. The authors in the paper [13] provide an extensive literature review on integrated production planning and vehicle routing, considering 65 studies since 2010. In the paper [14], the focus is on integrated supply chain management, taking into account the perishability of goods. The research in the papers [15-17] revolves around the observation and integration of these phases from a strategic, tactical, and operational decision-making perspective. The authors of the paper [18] present a case study that focuses on the challenges of synchronous production and distribution planning, specifically on permutation flow planning in production operations and setup time dependent on the sequence and alternative routing of vehicles in distribution. In the paper [19], the authors emphasize the importance of considering a compromise between delays and delivery costs when making tactical decisions.

In the paper [20], the authors discuss the significant drawback of a lack of information exchange between key entities in supply chains. The authors propose a new concept for solving the problem of integrated production and routing with reverse logistics and remanufacturing. The subsequent sections of the paper present the work through several key components, as outlined below.

The paper consists of several key parts and is presented through five sections. In the introductory section, the basic motivation and advantages of the proposed idea in integrating two distinct research areas are presented. Chapter 2 illustrates the mathematical model, as well as the procedures for optimizing and integrating the Flexible Job Shop Problem (FJSP) and Vehicle Routing Problem (VRP) into an intelligent system in the form of supply chains. Chapter 3 outlines the applied methodology and the implementation of well-known metaheuristic algorithms. Chapter 4 presents a case study and the optimization of the problem on a specific example. In the final part of the paper, titled "Results," experimental research results are presented, along with the advantages of the integrated model as a planning system in the supply chain, representing the future and bringing significant benefits through the integration of these two research areas.

2 MATHEMATICAL MODEL

The mathematical model in this paper consists of two key phases within the supply chain. In the first phase, F_1, the supply chain problem is presented as the planning and scheduling of resources, known as the FJSP. In the second phase, F_2, VRP is presented. The primary motivation in the paper is the integration of FJSP and VRP into a unified system within the supply chain, aiming to enhance reliability, productivity, and minimize the overall time within and between phases. The graphical representation of the mathematical model is given in Fig. 1.

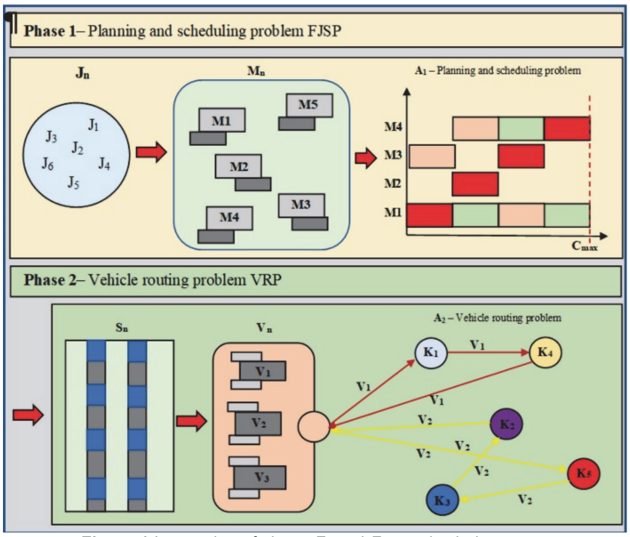

Figure 1 Integration of phase F_1 and F_2 supply chain system

2.1 Mathematical Formulation of the Model for Two Phase $F = F_1 + F_2$

Each job has its path through the machining center, and each operation within that job can be performed by any machine. It is necessary to arrange n products $J = \{J_1, J_2, \ldots, J_n\}$, where each job J_j ($j = 1,2, \ldots, n$) has a sequence of n_j operations ($O_{1,j}, O_{2,j}, \ldots, O_{nj,j}$) that need to be executed in a given order on m machines $M = \{M_1, M_2, \ldots, M_m\}$. For a fully flexible problem, each machine can perform only one operation at a given time, and the processing times of each operation depend on the available machines and are represented by $t_{,i,j,k}$ (processing time of operation $O_{i,j}$ on machine M_k). This activity in the supply chain plays a crucial role in job planning and scheduling to minimize time and increase production productivity, thereby minimizing product delays. The following notation [6] is used to define the mathematical formulation of the FJSP model:

For variable X_{ijk} it holds that it is 1 if machine k is selected to process operation O_{ij}, otherwise it is 0.

For variable $Y_{ijk'j'k}$ the condition is that it is 1 if operation O_{ij} precedes operation $O_{i'j'}$ on machine k, otherwise it is *0*.

The minimization of the objective function, [21, 22]:

$$F_1 = \min c_{\max} \tag{1}$$

with the following restrictions:

$$\sum_{k \in M_j} X_{ijk} = 1; \quad \forall j \in J, \forall j \in O_i, \tag{2}$$

$$S_{ijk} + C_{ijk} \le \left(X_{ijk} \right) \cdot L, \quad \forall i \in J, \forall j \in O_i, \forall k \in M_j, \tag{3}$$

$$C_{ijk} \ge S_{ijk} + t_{ijk} - \left(1 - X_{ijk}\right) \cdot L, \quad \forall i \in J, \forall j \in O_i, \forall k \in M_j, \tag{4}$$

$$S_{ijk} \geq C_{i'j'k} - Y_{iji'j'k} \cdot L, \quad \forall i < i', \forall j \in O_i, \forall j' \in O_{i'}, \forall M_j \cap M_{j'}, \tag{5}$$

$$S_{i'j'k} \geq C_{ijk} - \left(1 - Y_{iji'j'k}\right) \cdot L, \quad \forall i < i', \forall j \in O_i, \forall j' \in O_{i'}, \forall M_j \cap M_{j'}, \tag{6}$$

$$\sum_{k \in M_j} S_{ijk} \geq \sum_{k \in M_j} C_{i,j-1,k}, \quad \forall i \in J, \forall j \in O_i - \left\{O_{if(i)}\right\}, \tag{7}$$

$$C_i \geq \sum_{k \in M_j} C_{i,oil(i),k}, \quad \forall i \in J, \tag{8}$$

$$C_{\max} \geq C_i, \quad \forall i \in J, \tag{9}$$

Eq. (1) represents the objective function, aiming to minimize the total time during the planning and scheduling of operations on machines. Eq. (2) represents the constraint of assigning one operation to a machine. Eq. (3) represents the temporal constraint for performing operations on machine k; if operation O_{ij} is not scheduled on machine k, the operation's time is zero. Eq. (4) represents the difference between the start and completion times, ensuring it is at least equal to the processing time on machine k. Eqs. (5) and (6) represent the impossibility of simultaneously performing operations O_{ij} and $O_{i'j'}$ on the same set of machines $M_j \cap M_{j'}$. Eq. (7) represents the constraint on the order and ensures the timely execution of priority jobs compared to other less important ones. Constraint (8) determines the completion time of jobs on machines. Constraint (9) determines the span of job completion.

The primary goal of the study is to minimize the total time for $F = F_1 + F_2$. The methodology for the VRP with constraints on capacity and time will be presented in continuation. This means that, in addition to vehicle capacity constraints, constraints related to the start of service and vehicle waiting time for customers are introduced. This problem is defined in the literature as the Capacitated Vehicle Routing Problem with Time Windows. Additionally, this problem can be represented as a combination of routing and scheduling problems, which is a common scenario in real life [23].

Servicing each customer with the k-th vehicle must start within a time window $[e_{ik}, l_{ik}]$, where e_{ik} is the earliest and l_{ik} is the latest arrival time at customer $i (i \in V)$. If a vehicle arrives at a customer before the defined time, the vehicle is typically allowed to wait until time e_{ik} before starting the service. Let t_i be the arrival time of the vehicle at customer i (the time elapsed from the vehicle leaving the depot until arriving at customer i), and w_i be the waiting time at customer i before starting the service, where t_i, $w_i \in R$, $t_i \geq 0$ and $w_i \geq 0$. Let t'_{ij} be the travel time from customer i to customer j, and t_{oi} be the service time at customer i. It is evident that the arrival time of the vehicle at customer j consists of the arrival time at customer i, t_i, the waiting time before starting the service at customer i, w_i, the service time at customer i, t_{oi}, and the travel time between customer i and j, t'_{ij}.

In the model of the Capacitated Vehicle Routing Problem with Time Windows (CVRPTW), it is necessary to introduce an additional constraint regarding the maximum duration of vehicle engagement on a single route. For this purpose, the variable T_{rk} is introduced, representing the maximum allowed travel time for vehicle k on route r. It is customary for the weight matrix in this model to consist of a travel time matrix, assuming that all vehicles depart from the depot at time zero. Before formulating the mathematical objective function F_2, certain assumptions need to be introduced. We assume that there are K vehicles at the depot, each with a known capacity Q_k. To ensure the feasibility of the solution, it is necessary that $q \leq Q_k$ for each node i ($i = 1, \dots, n$) and each vehicle k ($k = 1, \dots, K$).

If we assume that each vehicle can perform at most one route and that K is not less than $K_{\min}$, where $K_{\min}$ represents the minimum required number of vehicles to serve all customers, the value of $K_{\min}$ can be determined by solving a new optimization problem. This problem involves determining the minimum number of vehicles (with capacity Q_k) needed to serve all demands q_i. The minimum number of vehicles affects traffic safety in urban areas [24]. For each i, j, $k \in V$, the following decision variable can be defined: The variable x_{ijk} is equal to 1 if vehicle k, after visiting customer i, proceeds to visit customer j; otherwise, it is 0. Now, the mathematical model for the CVRPTW, for n customers and k vehicles, can be represented as follows:

$$F_2 = \min \sum_{k=1}^{K} \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ijk} x_{ijk} \tag{10}$$

with the following restrictions:

$$\sum_{i=0}^{n} x_{i0k} = 1, \quad k = 1, 2, \dots, K, \tag{11}$$

$$\sum_{j=0}^{n} x_{0jk} = 1, \quad k = 1, 2, \dots, K, \tag{12}$$

$$\sum_{k=1}^{K} \sum_{i=0}^{n} x_{ijk} = 1, \quad j = 1, 2, \dots, n, \tag{13}$$

$$\sum_{i=1}^{n} q_i x_{ijk} \leq Q_k, \quad k = 1, 2, \dots, K, \tag{14}$$

$$\sum_{i=1}^{n} x_{ijk} = \sum_{i=1}^{n} x_{jik}, \quad i = 1, 2, \dots, n; k = 1, 2, \dots, K, \tag{15}$$

$$\sum_{i,j \in S} x_{ijk} \leq |S| - 1, \quad \forall S \subseteq \{2, \dots n\}, k = 1, 2, \dots, K, \tag{16}$$

$$x_{ijk} = \{0, 1\}, \quad i, j = 1, 2, \dots, n; k = 1, 2, \dots, K, \tag{17}$$

$$\sum_{k=1}^{K} \sum_{i=0}^{n} x_{ijk} \left(t_i + w_i + t_{oi} + t'_{ij}\right) \leq T_{rk}, \quad j = 1, 2, \dots, n \tag{18}$$

$$t_0 = t_{o0} = w_0 = 0 \tag{19}$$

$$\sum_{k=1}^{K}\sum_{i=0}^{n} x_{ijk}\left(t_i + w_i + t_{oi} + t'_{ij}\right) \le t_j, \quad j = 1, 2, \ldots, n, \tag{20}$$

$$e_i \le \left(t_i + w_i\right) \le l_i, \quad i = 0, 1, \ldots, n, \tag{21}$$

Constraint (11) and (12) imply that each vehicle leaving the depot must return to the same depot. Constraint (13) ensures that each customer must be visited exactly once by exactly one vehicle. Constraint (14) ensures that the total capacity of the customers served by a single vehicle does not exceed the capacity of that vehicle. Constraint (15) allows the preservation of the vehicle flow, i.e., it ensures that vehicle k must leave customer j after serving customer j. Constraint (16) prevents the occurrence of cycles that do not represent a complete route. Constraint (17) defines the passage through the edge between customers i and j by vehicle k and can take a value of 0 or 1. Constraint (18) ensures that the total travel time of vehicle k on a route is less than or equal to the maximum allowed engagement time for that vehicle. Constraint (19) sets the departure time of each vehicle from the depot, the waiting time, and the service time in the depot to zero. Constraint (20) ensures that the arrival time of vehicle k at customer j is less than the predicted arrival time at that node, while constraint (21) ensures that the sum of the arrival time and waiting time for each vehicle is greater than or equal to the earliest allowed arrival time at customer i and less than or equal to the latest allowed arrival time at customer i.

3 METHODOLOGY

The methodology applied in the paper is based on the implementation of two well-established algorithms for solving job scheduling and vehicle routing problems: Genetic Algorithm and Simulated Annealing Algorithm. It should be noted that the proposed algorithms belong to the group of metaheuristic algorithms. A metaheuristic represents a higher-level heuristic, and the main goal is to find the optimal solution and converge the solution based on the desired objective function through a series of iterations. Depending on the problem, the goal is to minimize or maximize the objective function. In this paper, the objective is to minimize the total delivery time, minimize the waiting time between the two integrated phases, and maximize the quality of services through an integrated system within the supply chains. The concept of metaheuristic was first mentioned by the renowned scientist Fred Glover in 1986 [6]. These algorithms or methods are now recognized as some of the most practical approaches to solving many complex problems, which is particularly relevant for addressing various real-world combinatorial optimization problems [7, 8]. Fig. 2 provides a real graphical representation of serviced routes in the Republic of Serbia.

The two mentioned metaheuristic methods are presented in the continuation of the paper. They have already been applied to solve and optimize the integrated production planning and vehicle routing problem in supply chains [9, 25].

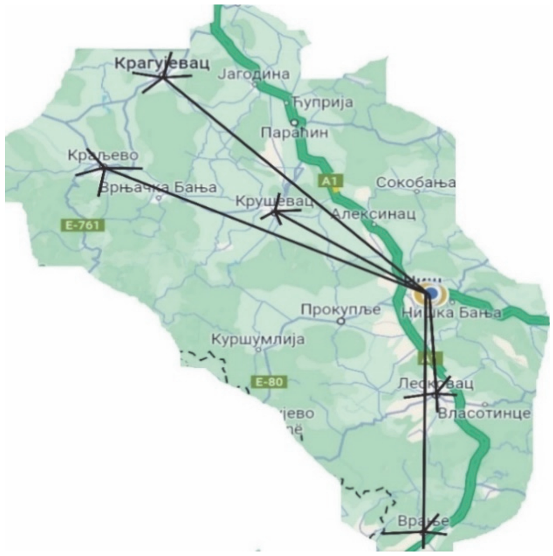

Figure 2 Supply chain system and real graphic representation of routes

3.1 Genetic Algorithm

The Genetic Algorithm (GA), as mentioned earlier, belongs to the category of metaheuristic algorithms, drawing inspiration from the evolutionary process in nature. The GA was initially introduced by the well-known scientist John Holland and his collaborators in the 1960s and 1970s. In this paper, a GA is employed to solve the job scheduling and routing problem in a supply chain, where the first phase involves planning and scheduling tasks, and the second phase deals with delivering products precisely on time through vehicle routing [21].

Genetic algorithm

Beginning
Objective function $f(x)$, $x = (x_1, \ldots, x_n)^T$
Initialization of the initial population in the form of chromosomes (binary form or real form of chromosomes)
Generation of initial population with a sequential schedule of jobs, which represents a number of possible solutions.
Defining the crossover probability p_c
Defining mutation probability p_m
while (t < defining the maximum number of iterations)
Generation of new sequential schedule of jobs
If p_c > *rand*, Crossover;
end if
If p_m > rand, Mutation;
end if
Accepting the new sequential work schedule if it is better
Selection of the optimal solution based on the objective function
end while
Printing of the optimal work schedule and visualization on Ganttogram.
End

Figure 3 General procedure of the genetic algorithm

The fundamental modification and complexity of the presented problem lie in integrating these two phases into a single supply chain system. It is worth noting that each of these phases is already quite complex [16]. The setup of the GA involves several stages characteristic of the algorithm itself: defining initial optimization conditions and problem setup, the selection process, crossover, mutation, and evaluating the optimal solution based on the specified criteria and the objective function for the observed problem. All stages within the GA - selection, crossover, mutation - are repeated through a series of predefined iterations until the initial optimization conditions achieve a sufficiently good solution or an optimal solution. It is important to mention that the GA has a wide range of applications in various research fields [26, 27]. Fig. 3 outlines the general procedure of the genetic algorithm [28].

3.2 Simulated Annealing

Simulated Annealing (SA) belongs to the category of metaheuristic algorithms and mimics the natural process based on the tempering of materials, a technique commonly used in the metal tempering process in metallurgy. The SA algorithm was first proposed by the renowned scientist Kirkpatrick and his collaborators back in 1983 [7, 8]. The simulation of the material tempering process involves gradually decreasing the temperature until the initial optimization conditions are met, aiming to achieve an optimal solution based on the specified objective function [6, 9]. Fig. 4 outlines the general procedure of simulated annealing [28].

Simulated Annealing

Beginning
Objective function $f(x)$, $x = (x_1, \dots x_p)^T$
Defining the beginning temperature T_0 and beginning iterations $x^{(0)}$
Defining the final temperature T_f and the total number of iterations observed problem N
Defining cooling $T - \alpha T$, $(0 < \alpha < 1)$
While ($T > T_f$ and $n < N$)
Random research of new locations: $x_{n+1} = x_n + rand$
Research and calculation of new solutions, in case it is better to remember and accept the new solution $\Delta f = f_{n+1}(x_{n+1}) - f_n(x_n)$,
If otherwise generate a new solution or number j r
Accepting if condition is met $p = exp\ [-\Delta f/T] > r$
end if
Updating the new best solution x_* i f_*
$n = n+1$
end while
Printing of the optimal work schedule and visualization on Ganttogram
End

Figure 4 General procedure of simulated annealing

A distinctive feature of the SA algorithm is that it reduces the temperature of the material to an admissible state corresponding to the lowest temperature, whereas the material tempering process involves pre-defining temperatures at which the material will satisfy defined criteria.

During the optimization process using the SA algorithm, it starts with a high temperature that allows for diversity in the proposed solutions. Through a series of predefined iterations, the algorithm gradually reduces the temperature, narrowing down the selection of potential solutions.

4 CASE STUDY AND RESULTS

This part of the paper presents the interconnection between the two key activities, F_1 - planning and scheduling of resources and F_2 - vehicle route planning and product delivery to customers. Based on the previously introduced mathematical models for both activities, the supply chain system will be considered as the combination of activities $F_1 + F_2$. In the current literature, the problems of resource planning and vehicle routing are often treated as separate issues due to the complexity of each problem. The motivation for integrating these activities is to form a unified supply chain where a single production system operates as both a production and on-time product delivery system. One of the goals is to minimize the time between these two activities, leading to significant time savings in product delivery to customers and in the planning process.

To address the problems within the supply chain, metaheuristic algorithms will be applied, specifically the Genetic Algorithm and Simulated Annealing. Comparing these two algorithms and implementing them with specific procedures represents a significant challenge in solving this type of problem. The application of artificial intelligence in the realm of planning, considering processing times on machines, combined with fleet planning and routing, incorporating delivery times while minimizing the objective function for both activities, is the main motivation and objective of this study. It is important to note that real data were used as input parameters for optimization in both the first phase (F_1) and the second phase (F_2) of the research.The input parameters for activities in both F_1 and F_2 are presented in Tabs. 3 and 4.

Table 3 Input parameters of phase F_1 of the supply chain system

F_1 - Optimization of the first phase of the problem of resource planning and allocation using GA and SA									
Jobs	Operations	Processing times							
		M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
$J_1 \times 100$	O_{11}	6	8	5	4	7	9	8	6
	O_{12}	9	13	11	15	12	12	16	15
$J_2 \times 150$	O_{21}	2	5	4	6	8	6	5	8
	O_{22}	6	10	4	8	9	11	3	4
$J_3 \times 100$	O_{31}	4	2	3	4	7	9	8	5
	O_{32}	4	4	1	8	4	8	10	7
	O_{33}	3	7	3	2	1	4	6	3
	O_{34}	8	9	11	12	6	6	8	7
$J_4 \times 100$	O_{41}	5	6	4	2	6	4	5	6
	O_{42}	7	2	8	7	4	5	3	5
	O_{43}	3	13	5	4	5	6	5	2
$J_5 \times 130$	O_{51}	8	12	6	9	11	8	7	6
	O_{52}	7	6	5	5	4	8	9	11
	O_{53}	9	8	12	11	6	8	8	6

Table 4 Input parameters of phase F_2 of the supply chain system

F_2 - planning the vehicle route and delivering the product to customers							
Customer	Cities	Latitude	Longitude	Total number of products	Number of products	Results of the first phase F_1	
Products	Niš	43.32646	21.94676	580	580	$GA - C_{max}$	$SA - C_{max}$
Job_1	Kragujevac	44.015758	20.920489	100	18	C_{1max} = 1300 min	C_{1max} = 1300 min
		44.013289	20.900105		26		
		44.027207	20.921991		29		
		44.002825	20.892723		19		
		44.010912	20.868218		8		
Job_2	Kraljevo	43.730766	20.657172	150	32	C_{2max} = 750 min	C_{2max} = 750 min
		43.734053	20.675540		25		
		43.716624	20.683265		16		
		43.721835	20.704121		42		
		43.728471	20.700345		22		
		43.733991	20.721717		13		
Job_3	Kruševac	43.565504	21.326410	100	19	C_{3max} = 1000 min	C_{3max} = 1100 min
		43.566997	21.342718		16		
		43.573713	21.328298		12		
		43.578874	21.314994		28		
		43.586148	21.339628		25		
Job_4	Leskovac	42.985985	21.949674	100	9	C_{4max} = 1000 min	C_{4max} = 750 min
		42.982531	21.970445		13		
		42.994837	21.963407		19		
		42.993644	21.953193		21		
		42.996532	21.925041		27		
		43.014546	21.950447		11		
Job_5	Vranje	42.531655	21.906158	130	14	C_{5max} = 1950 min	C_{5max} = 2080 min
		42.545821	21.880752		29		
		42.551132	21.898948		26		
		42.561375	21.901781		19		
		42.561052	21.982429		42		

Within the first phase (F_1), the production of cables in Niš is observed. The cables are categorized into five different product types: Job_1, Job_2, Job_3, Job_4, and Job_5. The goal of the first phase is to optimize production, thereby increasing productivity and minimizing the makespan (C_{max}) of the output function.

The second phase (F_2) aims to take the finished products from the first phase and optimize the vehicle routing problem. The primary objective is to minimize the delivery time of the finished products to all customers who have requested the service. Additionally, it involves optimizing the vehicle capacity during the product delivery process.

The total time required for the distribution of ordered products to end-users was predicted by optimizing the production process and product delivery to the specified address. The overall time required to complete all products in production and ready for further delivery for each product was: $Job_1 = C_{1max}$ = 1300 min, $Job_2 = C_{2max}$ = 750 min, $Job_3 = C_{3max}$ = 1000 min, $Job_4 = C_{4max}$ = 750 min, $Job_5 = C_{5max}$ = 1950 min, *All Jobs* = C_{Umax}= 5750 min. Meanwhile, the total time required for the distribution of these products to end-users was: Kragujevac: Job_1 = 190 min, Job_1 = 210 min, Kraljevo: Job_2 = 280 min, Job_2 = 266 min, Job_2 = 261 min, Kruševac: Job_3 = 158 min, Job_3 = 146 min, Leskovac: Job_4 = 109 min, Job_4 = 100 min, Vranje: Job_5 = 175 min, Job_5 = 166 min, Job_5 = 158 min. The total time of the objective function when integrating all phases in the supply chain $F = F_1 + F_2$ was: $Job_1 = F_{11}$ = 1490 min, $Job_1 = F_{12}$ = 1319 min, $Job_2 = F_{21}$ = 1030 min, $Job_2 = F_{22}$ = 1016 min, $Job_2 = F_{23}$ = 1011 min, $Job_3 = F_{31}$ = 1158 min, $Job_3 = F_{32}$ = 1146 min, $Job_4 = F_{41}$ = 859 min, $Job_4 = F_{42}$ = 850 min, $Job_5 = F_{51}$ = 2125 min, $Job_5 = F_{52}$ = 2116 min, $Job_5 = F_{53}$ = 2108 min, while the total delivery time of all products to the exact locations was F = 15958 min.

Based on the obtained results, it can be concluded that the proposed methodology is successful. The main advantages and benefits of this approach lie in the time savings between observed phases, as well as the accurate delivery and planning of product delivery on time. The study represents a significant contribution to the field of supply chain optimization through the integration of key activities and the application of advanced metaheuristic algorithms, offering potential solutions for improving the efficiency of production and delivery processes.

Table 5 Optimization results $F = F_1 + F_2$

Customer	Cities	Total delivery time / min	Total time of the objective function $F = F_1 + F_2$ / min
Products	Niš	Rute, delivery time F_2	F
Job_1	Kragujevac	0 - 1 - 3 - 0, 190	1490
		0 - 2 - 5 - 4 - 0, 219	1319
Job_2	Kraljevo	0 - 4 - 6 - 0, 280	1030
		0 - 5 - 2 - 0, 266	1016
		0 - 1 - 3 - 0, 261	1011
Job_3	Kruševac	0 - 4 - 5 - 0, 158	1158
		0 - 2 - 1 - 3 - 0, 146	1146
Job_4	Leskovac	0 - 5 - 4 - 0, 109	859
		0 - 2 - 3 - 6 - 0, 100	850
Job_5	Vranje	0 - 1 - 3 - 0, 175	2125
		0 - 2 - 4 - 0, 166	2116
		0 - 5 - 0, 158	2108

The results presented in Tab. 5, as well as graphically in Fig. 3, demonstrate the success of the proposed methodology. A future direction of research may include introducing some logistics centre in the supply chain between production and customers [29].

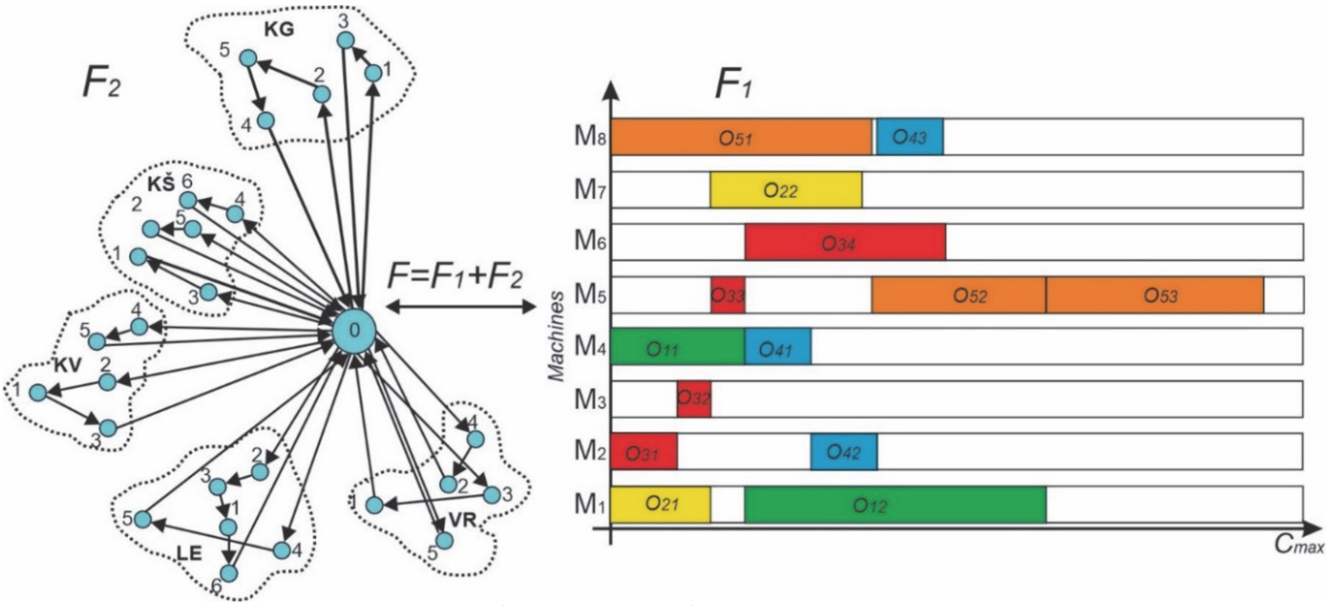

Figure 3 Graphic representation of the proposed methodology

5 CONCLUSION

In this study, the interconnection and optimization of key activities in the supply chain were analyzed. These activities encompass the planning and allocation of F_1 resources, as well as the planning of vehicle routes and product delivery to customers (F_2). The researchers present these activities as part of a unified supply chain ($F_1 + F_2$) with the goal of creating a system where the production process and product delivery operate synchronously and accurately according to plan.

The significant contribution of the research lies in connecting the resource planning and vehicle routing problems, traditionally treated as separate issues due to their complexity. The integration of these activities aims to minimize the time between them, resulting in more efficient product delivery to customers and optimization of planning. The genetic algorithm and simulated annealing are used as fundamental tools to solve this problem, with comparing and implementing these algorithms being challenging aspects of the research. The main motivation of the work is the application of artificial intelligence in the domain of time planning for machine operations and vehicle fleet routing to minimize the objective function, covering both activities. Real data were used as input parameters for optimization in both phases of the research. In the first phase (F_1), the focus was on optimizing cable production in Niš, with distribution across five different product types. The goal was to increase productivity and minimize the completion time function C_{max}. In the second phase (F_2), the task was to optimize the vehicle routing problem with an emphasis on minimizing the delivery time of products to all customers and optimizing the use of vehicle capacity.

Future direction of research, which would improve the work, could be that in the first phase production is considered as a stochastic process. Also, it should be considered in the second phase of customer requests as stochastic demand and in real time. Such a study would correspond to the real situation and significantly improve the processes in both the first and second phases. Considering the increasing role of smart technologies in everyday life, the application of smart transport and logistics technologies can also be a challenge for future research directions.

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