Croatian Journal of Education
Vol.26; No.1, pages: 59-95
Original research paper
Paper submitted: 16th September 2022
Paper accepted: 12th January 2023
https://doi.org/10.15516/cje.v26i1.4984

Diversity and Coherence as Important Aspects of Curriculum: A Content Analysis of the Two Croatian Geometry Curricula for Primary Education

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Abstract

For years, school geometry has faced with many issues including losing its strong position in school mathematics and earning the reputation of being the "problem child" of mathematics teaching. These issues stem from the problems in the intended curriculum. The goal of this paper is to shed light on the two intended curricula for primary mathematics education in Croatia in terms of the diversity and coherence of geometry topics through the lens of fundamental ideas of geometry. Additionally, the analysis is associated with previous results on the attained and potentially implemented curriculum regarding textbook tasks and primary school students' images of geometry. Even though the new curriculum reflects more diversity regarding the fundamental ideas of geometry compared to the old curriculum, there is clear evidence that they still do not offer the learning opportunities in line with modern geometry education. Based on the data, the results are discussed with regard to their theoretical and practical implications with the goal of providing suggestions on how to improve the position of school geometry in mathematics education in Croatia conducive to modern geometry education, as emphasised in the literature.

Keywords: *fundamental ideas; geometry education; primary education; tripartite model of curriculum.*

Introduction

In line with the aims of education systems, the curriculum provides educational opportunities as configurations of "social, political and pedagogical conditions to provide pupils chances to acquire knowledge, to develop skills and to form attitudes concerning school subjects" (Valverde et al., 2002, p. 6). The term *curriculum*, which has been used for different purposes throughout the history of education and in different ways around the world, is more than just a list of topics and objectives to be covered (Remillard & Heck, 2014). The construct itself is complex, and there are different models that conceptualise its versatile nature by separating it into different curriculum layers. Schmidt et al. (1996) presented a tripartite model of curriculum. The authors distinguish between curriculum as system goals (*intended curriculum*), as instruction (*implemented curriculum*), and as student achievement (*attained curriculum*). Valverde et al. (2002) expanded this model with the fourth component, *potentially implemented curriculum,* having in mind the mediating role mathematics textbooks play between the intended and the implemented curriculum.

Being such an important discipline within the mathematics curriculum, geometry needs to be given more attention than it currently receives (Kuzle, 2022). First of all, there is a clear discrepancy between the significance of geometry as a mathematical discipline and as a part of mathematics education (Kuzle & Glasnović Gracin, 2020). Even though geometry drove the development of mathematics as a discipline, experiencing a growth in richness and diversity in the 19th and 20th centuries, geometry education did not undergo similar development within the mathematics curriculum (Jones, 2000). Secondly, despite the importance of learning geometry in school mathematics, over the past decades we have witnessed a significant reduction of geometry content in numerous mathematics curricula worldwide (e.g., Backe-Neuwald, 2000; Mammana & Villani, 1998), as a result of integration of new content into national curricula, such as statistics and probability (Jones, 2000). Thirdly, geometry curricula are often criticised for a lack of coherence between the geometry ideas and pedagogical approaches applied in various grades of primary school (e.g., Franke & Reinhold, 2016; Mammana & Villani, 1998; Van de Walle & Lovin, 2006). Fourthly, relevant international assessments, such as PISA (Programme for International Student Assessment) and TIMSS (Trends in International Mathematics and Science Study), place a greater emphasis on other "big ideas", such as numbers, rather than on space and shape (Glasnović Gracin, 2011; Mullis & Martin, 2013). This may lead to lower achievement in geometry items than in numeracy or data items, as was reported by Glasnović Gracin (2011) referring to the Croatian context. All of the four above-mentioned aspects refer to layers of the tripartite curriculum of Schmidt et al. (1996). The first two aspects refer to the intended curriculum, and the third refers to the intended and implemented geometry curriculum. The fourth aspect represents an issue recognised in the attained curriculum which is influenced by the intended curriculum and consequently by the implemented curriculum.

The above-mentioned list is certainly not complete, but these and other issues initiated an ongoing debate regarding the role and the importance of geometry within the mathematics curriculum (Mammana & Villani, 1998; Sinclair & Bruce, 2015). Jones

(2002), for instance, argues that "the study of geometry contributes to helping students develop the skills of visualisation, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof" (p. 125). Developing these skills, along with the strong tradition of geometry, provides a challenge for geometry education given that the questions of which topics should be included in the contemporary geometry curriculum (i.e., diversity), and how to connect them (i.e., coherence) are not trivial. The main goal of the inquiry presented in this paper was to provide an insight into the two intended curricula for primary mathematics education in Croatia regarding the diversity and the coherence of geometry topics through the lens of fundamental ideas of geometry, and to connect them to the two other layers of the tripartite curriculum model.

Theoretical background

In this section, the diversity and coherence constructs within the curriculum context will be explained, followed by the presentation of the construct of fundamental ideas, with a special focus on the model proposed by Kuzle and Glasnović Gracin (2020). In the last section, empirical research on the curriculum in Croatian geometry education is presented.

Diversity and coherence of the curriculum content

The literature on contemporary geometry education emphasises two key features, namely coherence and diversity. *Diversity* refers to different and various concepts within the geometry curriculum. It should be present at all levels of education, starting from the primary level (e.g., Battista & Clemens, 1988; Franke & Reinhold, 2016; Jones, 2000; Mammana & Villani, 1998; Van de Walle & Lovin, 2006), for which many foci in the context of geometry exist. For instance, Battista and Clemens (1988) suggest that the primary school geometry curriculum should contain the following ideas: physical shapes and their properties, development of intuition about spatial environment, the use of geometric modelling to solve problems, and development of arguments. Van de Walle and Lovin (2006) suggest: shapes and their properties, transformation, location, visualisation, and analysing and abstracting geometric concepts and relations, whereas Franke and Reinhold (2016) outline shapes in space and plane, symmetry, spatial visualisation and reasoning, and patterns and structure as the key ideas in the primary school geometry curriculum.

However, the diversity of topics does not necessarily imply a link between the learned objects. These ideas should be meaningfully related across the curriculum (Schweiger, 1992), which can be described by the construct of *coherence*. Piaget has argued for the importance of order in the learning of geometric ideas (Jones & Mooney, 2003), starting with topology, followed by the ideas of rectilinearity associated with projective geometry, and ending with students learning Euclidean notions of parallelism, angularity and distance. Such an idea of order supports a gradual development of geometric ideas over time "becoming increasingly integrated and synthesized as children progress" (Jones & Mooney, 2003, p. 11). However, Hansen (1998) criticised the lack of coherence in geometry curriculum because it often covered isolated fragments of geometry concepts such as "small bits of polygon classification, some formulas to measure various shapes, some incidence geometry, some mentioning of transformations, a few constructions, selected loci, introduction to vectors, and finally some analytic geometry" (p. 238). In this way, school geometry may reflect the notion of a "kind of inconsistent 'bazaar'" (Hansen, 1998, p. 238). Consequently, the much-needed diversity of topics in the intended curriculum may pose new challenges to geometry education because they should be mutually connected appropriately, both horizontally (within the same grade level) and vertically (within different grade levels).

Fundamental ideas of geometry

One of the contemporary approaches to improving geometry education and confronting the previously mentioned challenges focuses on the idea of a well-established and coherent curriculum based on fundamental ideas (Schweiger, 1992; Wittmann, 1999). *Fundamental ideas* may be described as a set of actions or strategies that are present in the historical development of mathematics, that appear vertically in the curriculum, that contribute to answering the question of what mathematics is, that make teaching more transparent and flexible, and that possess a corresponding linguistic or action-related archetype in everyday life (Schweiger, 1992). In such a way, fundamental ideas may be constantly developed at every level of education (Van de Walle & Lovin, 2006), and can be applied to all areas of mathematics, including geometry.

Wittmann (1999) proposed the organisation of school geometry around seven fundamental ideas which reflected the diversity and coherence of geometry topics: geometric forms and their construction, operations with forms, coordinates, measurement, geometric patterns, forms in the environment, and geometrization. The further development of Wittmann's (1999) model was conducted by Kuzle and Glasnović Gracin (2020). The modification includes an expansion of some fundamental ideas, and development of a finer classification of subcategories (see Table 1), as explained in Kuzle and Glasnović Gracin (2020).

This framework reflects the multi-dimensional (diverse) and coherent view of geometry, which is in line with contemporary geometry education guidelines (Franke & Reinhold, 2016; Mammana & Villani, 1998; Sinclair & Bruce, 2015). Furthermore, it has already been used as a theoretical framework for analysing and studying various elements within the tripartite curriculum, namely, the potentially implemented curriculum in the study by Glasnović Gracin and Krišto (2022) and the attained curriculum in the studies by Glasnović Gracin and Kuzle (2021), Kuzle (2019), and Kuzle and Glasnović Gracin (2020), which will be mentioned in the following sections.

Table 1
Fundamental Ideas of Geometry (Kuzle & Glasnović Gracin, 2020, developed from Wittmann, 1999)

Code	Title	Description and subcategories
F1	Geometric forms and their construction	Basic and composite geometric forms of different dimensions, their properties and construction/creation. Subcategories: zero-dimensional objects (F1a), one-dimensional objects (F1b), two-dimensional objects (F1c), three-dimensional objects (F1d), geometric properties (F1e), drawing and drawing/construction tools (F1f), non-geometric tools for creating geometric objects (F1g), angles (F1h), composite figures (F1i).
F2	Operations with forms	Geometric mappings and other manipulations with forms, and the properties influenced or changed by these. Subcategories: translation (F2a), rotation (F2b), dilation (F2c), point symmetry (F2d), line symmetry (F2e), congruence (F2f), composing and decomposing (F2g), folding and unfolding (F2h), tessellation (F2i).
F3	Coordinates, spatial relationships and reasoning	Position and location of geometric forms in the plane or space as well as spatial reasoning about them. Subcategories: coordinate system (F3a), positional relationships (F3b), orientation and orientation tools (F3c), spatial visualisation, relation and orientation (F3d).
F4	Measurement	Qualitative and quantitative properties used to describe geometric forms as well as calculation of these using formulae. Subcategories: length (F4a), perimeter (F4b), surface area (F4c), volume (F4d), angle measure (F4e), measuring tools (F4f), estimation (F4g), conversion of measuring units (F4h), scaling (F4i).
F5	Geometric patterns	Geometric patterns created by using simple geometric forms (e.g., a frieze pattern, six-petal rosette).
F6	Geometric forms in the environment	Description of real-world objects, and operations on and with them by using geometric forms.
F7	Geometrization	Plane and spatial geometric theorems and problems, relationships between numbers (e.g., triangular numbers) and abstract relationships, which can be translated into the language of geometry and then translated again into practical solutions. Subcategories: geometrical facts (F7a), parallel projection (F7b), geometric problems (F7c), figurate numbers (F7d).

Empirical research on the attained and potentially implemented curriculum

Study on the attained curriculum. The study by Glasnović Gracin and Kuzle (2021) provided an insight into the images primary school students have of geometry by using the participant-produced drawings. Hence, the study involved the analysis of the attained curriculum, that is, of what the students had acquired. The participants (N = 249 Croatian students in the grades 2–4) were each given a sheet of paper with the assignment to draw what geometry is for them. The data were analysed using the framework of fundamental ideas of geometry (Kuzle & Glasnović Gracin, 2020) (see Table 1). The results revealed that the participants had a rather narrow understanding of geometry; most of the students' drawings (more than 90 % in each grade) contained either one or a maximum of two fundamental ideas of geometry (see Table 2). Fundamental idea F1 (geometric forms and their construction) dominated in the participant-produced drawings. Here, the students mainly represented one-, two-, and three-dimensional objects. Other fundamental ideas were significantly less frequently depicted, if at all.

Table 2
Distribution of the Seven Fundamental Ideas of Geometry in the Students' Drawings (Glasnović Gracin & Kuzle, 2021)

	F1	F2	F3	F4	F5	F6	F7
Proportion within all coded items	88 %	0 %	1 %	3 %	1 %	8 %	0 %

Study on the potentially implemented curriculum. Using the same framework (see Table 1), Glasnović Gracin and Krišto (2022) conducted a study on the potentially implemented geometry curriculum in Croatia. The study involved the analysis of all geometry tasks in the most frequently used Croatian mathematics textbook series for primary school. Since the textbooks are widely used in Croatian classrooms (Glasnović Gracin, 2011), the study focused on the analysis of the potentially implemented curriculum. The analysis of 267 tasks from the printed textbooks clearly indicated a lack of diversity in geometry task requirements, with the domination of the fundamental idea F1 (see Table 3). A deeper investigation of F1 subcategories showed that 35 % of all tasks referred to F1b (one-dimensional objects), and 37 % referred to F1c (two-dimensional objects). Textbook tasks with three-dimensional objects (F1d) were underrepresented with only 4 %, with no such tasks being present in Grades 3 and 4. Geometric patterns (F5) were covered in Grade 1 only, with the proportion of 19 %. Measurement (F4) was present in about 20 % of all geometry tasks in Grades 1–3, and 10 % in Grade 4. Hence, these results showed rather limited geometry learning opportunities for Croatian primary school students.

Table 3
Distribution of the Seven Fundamental Ideas of Geometry in Mathematics Textbooks (Glasnović Gracin & Krišto, 2022)

	F1	F2	F3	F4	F5	F6	F7
Proportion within all coded tasks	79 %	0 %	0 %	16 %	4 %	5 %	0 %

Two Croatian mathematics curricula

The Croatian education system is highly centralised. The intended curricula for all subjects in elementary education, including mathematics, are released by the education authorities and apply to all schools and grades at the state level (*Zakon o odgoju i obrazovanju u osnovnoj i srednjoj* školi, 2008).

The Teaching Plan and Programme (Ministry of Science, Education and Sports [MZOS], 2006), which was applied only in compulsory education in Croatia (Grades 1 to 8) and was structured around topics for each subject (including mathematics) and for each grade level, was valid since 2006. Each topic consisted of the name of the topic, its corresponding educational achievements, and keywords. There was no mention of the importance of the diversity of the mathematical content, nor was there a need for coherence among the topics. Rather, the authors of the document considered that "Mathematics is a school subject with a long tradition and well-defined content, and there is no need for major interventions in the content programmes; therefore, the starting point for designing the programme for mathematics was the existing programmes ..." (MZOS, 2006, p. 238, translation by the authors).

In 2019, a thorough curricular reform took place which included new curricula for all school types, covering the entire educational vertical line from Grades 1 to 12. The underlying idea of the new Mathematics curriculum (Ministry of Science and Education [MZO], 2019) was based on connecting mathematical processes and domains. Educational outcomes were set for each of the five content domains. They included the name and code of the educational outcome, an in-depth elaboration, educational outcomes for the mark "good" at the end of each grade, mathematical content related to the outcome, and recommendations for outcome achievement. Furthermore, the document emphasises the importance of the vertical content coherence in mathematics education: "Domains are gradually being developed and upgraded by the whole vertical line of learning and teaching mathematics" (p. 12, translation by the authors) as well as the coherence between the domains: "Although the domains connect the related concepts, their indivisibility is constantly noticed because the acquisition of the concepts of one domain is often a prerequisite for the acquisition of concepts in other domains" (MZO, 2019, p. 12, translation by the authors). This is applicable to geometry topics as well, which are covered within the domains Space and Shape, Measurement, and Algebra and Functions.

Research questions

The study presented in this paper refers to the content analysis of the two intended mathematics curricula for primary education in Croatia, through the lens of the fundamental ideas of geometry. Although the study by Glasnović Gracin and Kuzle (2021) highlighted the need for examining the intended curriculum since such research may contribute to "re-questioning the primary mathematics curriculum requirements concerning the multi-dimensional nature of geometry" (p. 23), an in-depth analysis of Croatian intended geometry curriculum has not yet been conducted. Furthermore, the analysis of the intended curricula – both the "old" (MZOS, 2006) and the "new" one (MZO, 2019), offers connection with the previously obtained findings on the rather narrow image of geometry in the attained and potentially implemented curriculum. The following research questions were posed:

1 What fundamental ideas of geometry are present in the old Croatian Teaching Plan and Programme for Mathematics (NPP) in primary education (MZOS, 2006)? How can these results be related to the previous research on students' images of geometry revealed via participant-produced drawings?
2 What fundamental ideas of geometry are present in the new Croatian curriculum (CUR) for primary education (MZO, 2019)? How can these results be related to the previous research on the geometry textbook tasks?
3 In what aspects do the two Croatian curricula reflect the coherence and diversity of fundamental ideas of geometry through their vertical perspective in primary education?

The reason for connecting the NPP with the study on students' drawings (Glasnović Gracin & Kuzle, 2021) lies in the fact that this study was conducted before the introduction of the CUR. The study by Glasnović Gracin and Krišto (2022) focused on the analysis of the textbook tasks which were aligned with the CUR.

Methodology

This study focused on the document review of the two Croatian intended mathematics curricula (MZOS, 2006; MZO, 2019) for primary education (Grades 1 to 4). This period is crucial for the development of geometric thinking and for creating a basis for later acquisition of geometry content (Franke & Reinhold, 2016; Mamanna & Villani, 1998). Furthermore, the previous research on the potentially implemented and attained curriculum focused on the primary grades (Glasnović Gracin & Krišto, 2022; Glasnović Gracin & Kuzle, 2021), so the analysis of the results of the intended curriculum for the same grade levels may explain some of the previous findings.

The analysis encompassed the parts of the curriculum covering geometry ideas for primary schools (Grades 1 to 4) using the qualitative content analysis method (Mayring, 2000). Following the category application, both authors analysed each curriculum independently. The analysis was based on a slight modification of the tool presented in Table 1. The following subcategories emerged from the analysis and were added to the inventory: F1j (plane and space), F3d (visual perception), F3e (spatial visualisation, relation and orientation), and F4j (calculation with measurable attributes). The Plane and Space category refers to explicitly mentioning plane and three-dimensional space as geometric ideas, and also covers the term "flat surface" regarding a plane. Visual perception refers to perceiving visible objects and concretely operating with them. Calculation with measurable attributes encompasses calculation activities with measurement units, for example, "Calculate 3 m + 8 m".

Both researchers assigned the codes of the fundamental ideas (F1 to F7) and their subdimensions (e.g., F1a, F1b) to each topic or outcome across both curricula. Here, the analysis included all the texts that belonged to the particular topic in the NPP (keywords and educational achievements within the particular topic), and to the particular educational outcome in the CUR (its elaboration, content block and recommendations). In the next step, the authors compared the assigned codes and subcodes. Any inconsistencies that occurred were discussed, which led to adjustments in the inventory, and consequently joint decisions about the final codes were made. The final interrater reliability was high (100 % agreement). The exemplary coding of the topic "Right angle" in Grade 4 (MZOS, 2006, p. 243) is given in Table 4.

Table 4
Coding of the Topic "Right angle" in Grade 4 in the NPP (MZOS, 2006)

Topic, keywords and educational achievements	Codes of fundamental ideas or their subcategories
Topic: 6. Right angle Keyword: right angle Achievements: to draw and mark the right angle	F1f; F1h

In this study, the coherence of fundamental ideas and their subcategories is present if a particular topic appears across consecutive grades. Diversity includes a wide range of fundamental ideas found within a particular grade. After the coding process, the descriptive statistics was presented on the basis of the obtained final codes and subcodes according to their occurrence in each grade. If the code was present in a particular grade, it was assigned a "+" sign, otherwise the cell was left empty. In this way, both the diversity of fundamental ideas as well as the coherence of geometric topics across grades could be clearly visible in each analysed curriculum. Further on, the study sought to connect the obtained data with the results of previous studies: the data obtained from the CUR were compared with the findings from Glasnović Gracin and Krišto (2022), while the data obtained from the NPP were compared with the findings from Glasnović Gracin and Kuzle (2021). The results obtained for a particular fundamental idea or its subcategory in each intended curriculum were compared to the corresponding category in the previous study. If a mark in the intended curriculum results was assigned a "+" sign and the percentage in the corresponding study was at least 8 %, the connection was designated as a "match". If the cell in the CUR or NPP results was empty and the proportion in the assigned previous study was lower than 8 %, the connection was also designated as a "match". Otherwise, the connection was designated as "no match".

Results

The results are divided into three sections, in line with the three research questions.

Fundamental ideas in the intended (NPP) and attained curriculum

Diversity in the intended curriculum (NPP). The distribution of fundamental ideas in the NPP intended curriculum (MZOS, 2006) for Grades 1 to 4 is presented in Table 5. The results indicate that the fundamental idea F1 (geometric forms and their construction) is present in each grade, F3 (coordinates, spatial relationships, and reasoning) in Grades 1 to 3, and F4 (measurement) in Grades 1, 3 and 4. The fundamental idea F6 (geometric forms in the environment) is only mentioned explicitly in Grade 1, although the general aims and tasks of mathematics education within the NPP state that students should be able to apply mathematics knowledge in everyday life (MZOS, 2006, p. 238). Fundamental ideas F2 (operations with forms), F5 (geometric patterns), and F7 (geometrization) are not present at all. Thus, the results indicate a lack of diversity in geometry within the NPP, which is examined in more depth through the subcodes of the fundamental ideas (see Tables 6 – 8).

Table 5
Distribution of the Fundamental Ideas of Geometry in the NPP

	F1	F2	F3	F4	F5	F6	F7
Grade 1	+		+	+		+	
Grade 2	+		+				
Grade 3	+		+	+			
Grade 4	+			+			

Note. F1 - geometric forms and their construction, F2 - operations with forms, F3 - coordinates, spatial relationships, and reasoning, F4 - measurement, F5 - geometric patterns, F6 - geometric forms in the environment, F7 - geometrization

With respect to F1, the subcategories F1a (zero-dimensional objects), F1b (one-dimensional objects), and F1f (drawing and drawing/construction tools) are found in Grades 1 to 4 (see Table 6). The subcategories F1c (two-dimensional objects) and F1e (geometric properties) are found in Grades 1, 3 and 4, whereas F1d (three-dimensional objects) is found in Grades 1 and 4 only. From the perspective of grade levels, the results show the diversity of F1 subcategories in Grades 1, 3 and 4. The NPP (MZOS, 2006) in Grade 2 outlines only two topics (out of 31) devoted to geometry, which may explain the reduced diversity of F1 subcategories.

Table 6
Distribution of the Subcategories within the Fundamental Idea Geometric Forms and Their Construction (F1) in the NPP

	F1a	F1b	F1c	F1d	F1e	F1f	F1g	F1h	F1i	F1j
Grade 1	+	+	+	+	+	+				+
Grade 2	+	+				+				
Grade 3	+	+	+		+	+				+
Grade 4	+	+	+	+	+	+		+		

Note. F1a - zero-dimensional objects, F1b - one-dimensional objects, F1c - two-dimensional objects, F1d - three-dimensional objects, F1e - geometric properties, F1f - drawing and drawing/construction tools, F1g - non-geometric tools for creating geometric objects, F1h - angles, F1i - composite figures, F1j - plane and space

With respect to F3, only one (F3b) out of five subcategories is covered from Grades 1 to 3 (see Table 7). Thus, only positional relationships, such as within/outside, belongs to/does not belong to, parallel to, orthogonal to are dealt with. These results reflect the lack of diversity within the subcategories of F3.

Table 7
Distribution of the Subcategories within the Fundamental Idea Coordinates, Spatial Relationships, and Reasoning (F3) in the NPP

	F3a	F3b	F3c	F3d	F3e
Grade 1		+			
Grade 2		+			
Grade 3		+			
Grade 4					

Note. F3a - coordinate system, F3b - positional relationships, F3c - orientation and orientation tools, F3d - visual perception, F3e - spatial visualization, relation and orientation

Concerning the fundamental idea F4, a diversity of subcategories is only present in Grade 4, with six out of 10 subcategories being covered (see Table 8).

Table 8
Distribution of the Subcategories within the Fundamental Idea Measurement (F4) in the NPP

	F4a	F4b	F4c	F4d	F4e	F4f	F4g	F4h	F4i	F4j
Grade 1							+			
Grade 2										
Grade 3	+					+		+		
Grade 4	+	+	+	+		+	+			

Note. F4a - length, F4b - perimeter, F4c - surface area, F4d - volume, F4e - angle measure, F4f - measuring tools, F4g - estimation, F4h - conversion of measuring units, F4i - scaling, F4j - calculating with quantities

Connecting the intended and attained curriculum. Comparison of Tables 2 and 5 allows for the examination of possible connections between the intended NPP (MZOS, 2006) and the attained curriculum (Glasnović Gracin & Kuzle, 2021). Both tables reflect a rather narrow image of geometry with an emphasis on geometric forms and their construction (F1). The lack of the fundamental ideas F2, F5 and F7 in the intended curriculum (see Table 5) may explain their absence in the attained curriculum (see Table 2). The fundamental idea F6 is mentioned in Grade 1 only, and it was present in 8 % of drawn items (8 % in Grade 2, 9 % in Grade 3, and 6 % in Grade 4) (Glasnović Gracin & Kuzle, 2021). Even though F6 was not mentioned in the specific topics, it was emphasised in the general aims and tasks of mathematics education (MZOS, 2006, p. 238), and for that reason it was expected in the attained curriculum. On the other hand, the results obtained for the fundamental ideas F3 and F4 do not match - while F3 and F4 are found in the intended curriculum in three out of four grades, they were barely identified in the participants' drawings. This result may be due to the fact that geometric forms (F1) are easier to draw than spatial relationships or measurement (Kuzle & Glasnović Gracin, 2020). The results on the relation between the intended and attained curriculum are presented in Table 9.

Table 9
Relation between the NPP and the Attained Curriculum (Glasnović Gracin & Kuzle, 2021)

	Matching results	Explanation
F1	Match	Found in NPP across all grades. High occurrence in the attained curriculum.
F2	Match	Not found in NPP. Not found in the attained curriculum.
F3	No match	Found in NPP in three grades. Barely found in the attained curriculum (1 %).
F4	No match	Found in NPP in three grades. Barely found in the attained curriculum (3 %).
F5	Match	Not found in NPP. Not found in the attained curriculum.
F6	Match	Found in Grade 1 only, but also in the general aims of NPP. Moderately found in the attained curriculum (8 %).
F7	Match	Not found in NPP. Not found in the attained curriculum.

With respect to the subcategories of F1, the results obtained from the drawings showed that the participants identified F1b (19 % of items), F1c (38 % of items) and F1d (17 %) as key elements of their image of geometry (Glasnović Gracin & Kuzle, 2021). Among them, only F1b was found in all grade levels of the NPP (see Table 6) and it reflects a matching connection between the two curricula. Subcategory F1c, although the most frequently used one in drawings, was not found in Grade 2 of the NPP. Interestingly, three-dimensional objects (F1d) were found in 17 % of all drawn items (Glasnović Gracin & Kuzle, 2021), although they are not well covered within the NPP (see Table 6). Even though subcategory F1a (zero-dimensional objects) was presented in the children's drawings in only 2 % of items, it is found in all four grades in the NPP. Similarly, F1e (geometric properties) and F1f (drawing and drawing/construction tools) are presented in the NPP in three or four grade levels, but were presented in approximately 4-5 % of drawn items. Also, some subcategories which were not found in the NPP were likewise not illustrated in the students' drawings (e.g., F1g, F1h, F1i).

Fundamental ideas in the intended (CUR) and the potentially implemented curriculum

Diversity in the intended curriculum CUR. Distribution of the diversity of fundamental ideas in the intended curriculum CUR (MZO, 2019) for Grades 1 to 4 is presented in Table 10. The results indicate that the fundamental ideas F1 (geometric forms and their construction), F3 (coordinates, spatial relationships, and reasoning), and F4 (measurement) are found in each grade of primary school. The fundamental idea F2 (operations with forms), however, is only found in the first two grades, and F7 (geometrization) only in Grades 3 and 4. Fundamental idea F5 (geometric patterns) is found in Grade 1 only, but F6 (geometric forms in the environment) is found in Grades 1, 2 and 4. Thus, all seven fundamental ideas of geometry are found in the

CUR. These results indicate a greater diversity of fundamental ideas in the CUR in comparison with the NPP (see Tables 5 and 10). Still, some fundamental ideas (e.g., operations with forms (F2), geometric patterns (F5)) are not represented across the majority of grade levels.

Table 10
Distribution of the Seven Fundamental Ideas of Geometry in the CUR

	F1	F2	F3	F4	F5	F6	F7
Grade 1	+	+	+	+	+	+	
Grade 2	+	+	+	+		+	
Grade 3	+		+	+			+
Grade 4	+		+	+		+	+

Note. F1 - geometric forms and their construction, F2 - operations with forms, F3 - coordinates, spatial relationships, and reasoning, F4 - measurement, F5 - geometric patterns, F6 - geometric forms in the environment, F7 - geometrization

From the perspective of grade levels, the CUR covers six fundamental ideas in Grade 1 (all except F7), five fundamental ideas in Grades 2 and 4, and four in Grade 3. Again, the greatest diversity of ideas is presented in Grade 1. Geometrization (F7) is found only in Grades 3 and 4, which is to be expected due to its abstract nature (Kuzle & Glasnović Gracin, 2020). These results reveal a diversity of geometry ideas in the CUR, especially in comparison with the NPP.

With respect to the fundamental idea F1, five subcategories are found in the CUR in each grade, namely F1a, F1b, F1c, F1e and F1f (see Table 12). The subcategories F1d and F1i are found in Grades 1, 2 and 4. These findings indicate a diversity of subcategories within F1 in the CUR. Subcategory F1h (angles) is found in Grade 4 only. From the perspective of grade levels, the results show the diversity of F1 subcategories in primary education, however, this diversity is greater in Grades 1, 2 and 4 than in Grade 3.

Table 11
Distribution of the Subcategories within the Fundamental Idea Geometric Forms and Their Construction (F1) in the CUR

	F1a	F1b	F1c	F1d	F1e	F1f	F1g	F1h	F1i	F1j
Grade 1	+	+	+	+	+	+	+		+	+
Grade 2	+	+	+	+	+	+	+		+	
Grade 3	+	+	+		+	+				
Grade 4	+	+	+	+	+	+		+	+	

Note. F1a - zero-dimensional objects, F1b - one-dimensional objects, F1c - two-dimensional objects, F1d - three-dimensional objects, F1e - geometric properties, F1f - drawing and drawing/construction tools, F1g - non-geometric tools for creating geometric objects, F1h - angles, F1i - composite figures, F1j - plane and space

Concerning the fundamental idea F2, only one out of nine subcategories is covered, namely F2g (see Table 12), and only in Grades 1 and 2. Here the curricular recommendations involve using tangrams for composing and decomposing various plane figures (MZO, 2019).

Table 12
Distribution of the Subcategories within the Fundamental Idea Operations with Forms (F2) in the CUR

	F2a	F2b	F2c	F2d	F2e	F2f	F2g	F2h	F2i
Grade 1							+		
Grade 2							+		
Grade 3									
Grade 4									

Note. F2a - translation, F2b - rotation, F2c - dilation, F2d - point symmetry, F2e - line symmetry, F2f - congruence, F2g - composing and decomposing, F2h - folding and unfolding, F2i - tessellation

With respect to the fundamental idea F3, three out of five subcategories, namely F3b, F3c and F3d are covered (see Table 13). Here, F3b spans from Grades 2 to 4.

Table 13
Distribution of the Subcategories within the Fundamental Idea Coordinates, Spatial Relationships, and Reasoning (F3) in the CUR

	F3a	F3b	F3c	F3d	F3e
Grade 1				+	
Grade 2		+			
Grade 3		+			
Grade 4		+	+	+	

Note. F3a - coordinate system, F3b - positional relationships, F3c - orientation and orientation tools, F3d - visual perception, F3e - spatial visualization, relation and orientation

Concerning the fundamental idea F4, seven out of 10 subcategories are covered, namely F4a–F4c, F4f–F4h, and F4j (see Table 14). Still, F4d, F4e and F4i are not covered at all in the CUR.[1] While F4g was found in all grade levels of the CUR, F4a, F4f and F4h were addressed in Grades 2 to 4.

Table 14
Distribution of the Subcategories within the Fundamental Idea Measurement (F4) in the CUR

	F4a	F4b	F4c	F4d	F4e	F4f	F4g	F4h	F4i	F4j
Grade 1							+			
Grade 2	+					+	+	+		+
Grade 3	+	+				+	+	+		+
Grade 4	+		+			+	+	+		

Note. F4a - length, F4b - perimeter, F4c - surface area, F4d - volume, F4e - angle measure, F4f - measuring tools, F4g - estimation, F4h - conversion of measuring units, F4i - scaling, F4j - calculating with quantities

The fundamental idea F7 (geometrization) is found in Grades 3 and 4 in the CUR (see Table 10) covering only the subcategory geometrical problems (see Table 15).

[1] It is important to note that measuring liquids (in litres and decilitres) is included in the curriculum (MZO, 2019), but this topic was not included in this research data. The reason for this exclusion is that measuring the volume of liquid in the CUR places a strong emphasis on physical features rather than on geometry.

Table 15
Distribution of the Subcategories within the Fundamental Idea Geometrization (F7) in the CUR

	F7a	F7b	F7c	F7d
Grade 1				
Grade 2				
Grade 3			+	
Grade 4			+	

Note. F7a - geometric facts, F7b - parallel projection, F7c – geometric problems, F7d - figurate numbers

Connecting the intended and implemented curriculum. Comparison of Tables 3 and 10 reveals some discrepancies between the CUR (MZO, 2019) and the textbook features (Glasnović Gracin & Krišto, 2022) regarding the diversity of fundamental ideas. F1 is found in Grades 1 to 4 in the CUR (see Table 10) and was likewise dominant in textbook tasks: 79 % of all geometry tasks in the examined textbooks referred to F1 (see Table 3). F4 is found in all primary grades in the CUR (see Table 10), and it was found in 16 % of geometry tasks in the examined textbooks (see Table 3). F2, F3 and F7 were, however, not represented at all in the examined textbooks, but were found in the CUR in Grades 3 and 4. Interestingly, F3 is found in the CUR in Grades 1 to 4, but not at all in the textbooks. F5 and F6 are found with only 4-5 %, although their presence was greater in the CUR in comparison to the NPP. These findings show that opportunities for geometry learning in the new textbook series are rather narrow, which is not in line with the curriculum requirements. Thus, the results may imply that, in spite of the diversity of the fundamental ideas in the CUR, the examined textbook series is still closer to the NPP requirements regarding the diversity of fundamental ideas. The results on the relation between the intended and potentially implemented curriculum are presented in Table 16.

Table 16
Relation between the CUR (see Table 10) and the Textbook Tasks (Glasnović Gracin & Krišto, 2022)

	Matching results	Explanation
F1	Match	Found in CUR across all grades. High occurrence in the textbooks.
F2	No match	Found in CUR in Grades 1 and 2. Not found in the textbooks at all.
F3	No match	Found in CUR across all grades. Not found in the textbooks at all.
F4	Match	Found in CUR across all grades. Moderately found in the textbooks (16 %).
F5	Match	Found in CUR in Grade 1 only. Found in the textbooks for Grade 1 only (19 % in Grade 1).
F6	Match/No match	Found in CUR in Grades 1, 2 and 4. Found in the textbooks with 16 % (Grade 1), 11 % (Grade 2), 1 % (Grade 3) and 0 % (Grade 4).
F7	No match	Found in CUR. Not found in the textbooks at all.

Closer consideration of the results presented in Glasnović Gracin and Krišto (2022) regarding grade levels shows that F5 was found only in the Grade 1 textbook (as much as 19 % of all Grade 1 tasks referred to F5), but it was completely absent in other grades. This finding corresponds to the content of Table 10, which indicates that geometric patterns are found only in Grade 1 in both layers of curriculum. Also, the proportion of textbook tasks with the idea geometric forms in the environment (F6) decreases from Grades 1 to 4 (Glasnović Gracin & Krišto, 2022). In fact, in Grade 4, there are no textbook tasks dealing with F6, even though it is present in the CUR (see Table 10).

With respect to subcategories of F1, the results show that one- and two-dimensional objects (F1b and F1c) are found in all grades in the CUR (see Table 11), and were found in about a third of the examined textbook tasks, including angles as well (Glasnović Gracin & Krišto, 2022), reflecting a match between both curricula. However, subcategories F1a (zero-dimensional objects) and F1d (three-dimensional objects) are found in Grade 1 and 2 textbooks only, which is not in line with the CUR requirements for Grades 3 and 4.

Coherence and diversity across the vertical perspective

Coherence. In the NPP, the vertical coherence of fundamental ideas can be identified for F1 and F3. The fundamental idea F1 is found in all four grade levels of primary school (see Table 5). Coherence is of particular interest in the subcategories. Table 6 shows a lack of vertical coherence for F1d (three-dimensional objects) because this subcategory is only found in Grades 1 and 4. Also, subcategory F1e (geometric properties) is found in all primary school grades except in Grade 2. A closer consideration of the NPP (MZOS, 2006) shows that geometry topics are represented to a minimal extent in Grade 2, which does not contribute to the vertical coherence of geometric topics. F3 is found in the NPP in Grades 1 to 3 (see Table 7), and for one of its subcategories, namely F3b (positional relationships). F4 is found in Grades 1, 3 and 4, so its vertical coherence is interrupted in Grade 2 (see Table 5). Examination of the subcategories of F4 (see Table 8) indicates a lack of coherence regarding measurement features. For example, subcategory F4g (estimation) is only found in Grades 1 and 4 in the NPP. The fundamental idea F6 (geometric forms in the environment) is explicitly mentioned only in Grade 1, although it is mentioned in the general outlines of the mathematics curriculum (MZOS, 2006). Thus, it may be understood as applicable for all grade levels, and therefore coherent over the grades. For the fundamental ideas which are not found in the NPP (i.e., F2, F5, F7) one cannot talk about coherence at all.

In the CUR, the vertical coherence is clearly visible in F1, F3 and F4, which are found in all primary school grades (see Table 10). The examination of the subcategories of F1 (see Table 11) reveals that F1a, F1b, F1c, F1e, and F1f are found in all primary school grades. The interruption of coherence in Grade 3 is visible in F1d (three-dimensional objects) and F1i (composite figures). A closer consideration of the CUR (MZO, 2019) shows that in Grade 3, emphasis is placed on one- and two-dimensional objects and their measurement (i.e., length, perimeter). Geometric solids (F1d) are not included in

Grade 3 of the CUR. In comparison to the NPP, where the gap in coherence regarding three-dimensional objects (F1d) is even greater (see Table 6), we may notice that the CUR took a step forward (see Table 11), but still lacked vertical coherence in F1d. Although the fundamental idea F3 (see Table 10) is found in all primary levels, the insight into its subcategories reveals inconsistence in its internal coherence (see Table 13). For example, subcategory F3d (visual perception) is only found in Grades 1 and 4, which implies a lack of coherence of visual perception requirements.

In summary, the results indicate that some fundamental ideas of geometry and their subcategories are coherent over the primary school grades, some of them are more coherent in the CUR than in the NPP, and some are still to be revised and improved in the future.

Diversity. The results showed that the CUR offers a greater diversity of fundamental ideas in comparison to the NPP (see Tables 5 and 10). Interestingly, the poorest grade diversity in CUR (four fundamental ideas) corresponds to the greatest grade diversity of NPP. According to grade levels, the NPP has the greatest diversity in Grade 1 with four different fundamental ideas, and the poorest diversity in Grades 2 and 4, with only two fundamental ideas being covered. Similarly, in the CUR, the greatest diversity is offered in Grade 1, with 6 fundamental ideas of geometry. The poorest diversity in the CUR is found in Grade 3, with four fundamental ideas.

The subcategories of F1 also reflect a greater diversity in the CUR than in the NPP (see Tables 6 and 11). Unlike the CUR, the NPP does not cover all subcategories of F1 in primary school grades. The greatest diversity in the NPP within F1 is found in Grades 1 and 4 with seven subcategories, and the least in Grade 2 with three subcategories. In the CUR, the greatest diversity is again in Grade 1 (as many as nine subdimensions), and the least in Grade 3 (five of them). The subcategories of F3 point to the lack of diversity in both intended curricula (see Tables 7 and 13). Only in Grade 4 of the CUR, three (out of five) F3 subcategories are found. Regarding the subcategories of F4, the NPP reflects diversity only in Grade 4, while the CUR reflects diversity in all grades except Grade 1 (see Tables 8 and 14). This may imply that measurement in Grade 1 is not considered important in the CUR, but it increases in importance in the following grades.

In summary, the findings showed a greater diversity in the CUR than in the NPP, but the results also pointed to some issues and challenges found in the CUR, especially concerning Grade 3.

Discussion and conclusions

The study presented in this paper focused on the analysis of the two intended curricula and explored their connection to the results of previous studies on the attained and potentially implemented curricula (Glasnović Gracin & Krišto, 2022; Glasnović Gracin & Kuzle, 2021) regarding the diversity and coherence of geometry topics through the lens of the fundamental ideas of geometry.

The analysis of the NPP revealed a lack of diversity in geometry topics. Although F1, F3, F4 and F6 were covered, only F1 was found throughout all of the analysed grades. Both the NPP and the children's drawings (Glasnović Gracin & Kuzle, 2021) gave a rather narrow picture of geometry, with emphasis on geometric forms and their construction (F1). Also, the results revealed a lack of fundamental ideas F2, F5 and F7 in both the intended (NPP) and the attained curriculum (Glasnović Gracin & Kuzle, 2021). The results of the comparison of the CUR and the NPP showed that the CUR offers a greater diversity of fundamental ideas. The CUR covered all seven fundamental ideas of geometry, with F1, F3 and F4 being covered in all primary school grades. Regarding the grade levels, the NPP showed the greatest diversity in Grade 1 with four different fundamental ideas, and the poorest diversity in Grades 2 and 4, with only two fundamental ideas being addressed. Likewise, in the CUR, the greatest diversity was found in Grade 1, with six fundamental ideas of geometry. The poorest diversity in the CUR was found in Grade 3, with four fundamental ideas. The comparison of the results from the CUR and the textbook tasks (Glasnović Gracin & Krišto, 2022) showed a discrepancy regarding the diversity of the fundamental ideas F2, F3, F6, and F7. While the vertical coherence of some fundamental ideas over the grades (F1 and F3 in NPP, and F1, F3 and F4 in CUR) was observable, other fundamental ideas and their subcategories lacked coherence, such as three-dimensional objects in both curricula.

These results are concerning in terms of the benefits that geometry education should bring, such as critical thinking and intuition, and helping students develop skills of visualisation, problem solving and logical argument (Jones, 2002; Mammana & Villani, 1998). This study contributes to drawing attention to the geometry requirements in the intended curriculum, particularly regarding content diversity and coherence, and relating them to the attained and potentially implemented curriculum. Valverde et al. (2002) highlighted that the tripartite curriculum model presents the starting point for providing well-established learning opportunities for students. The intended curriculum plays a vital role here, since it influences both the implemented and attained curriculum, especially in countries with highly centralised education systems, such as Croatia. In view of the "problem child" status of geometry in school mathematics (Backe-Neuwald, 2000), placing emphasis on the diversity and coherence of geometry topics may be of interest to curriculum developers and researchers from the international community.

The lack of some fundamental ideas in Croatian curriculum for primary school grades

The discrepancy between the opportunities which should be provided through geometry education (Jones, 2002; Mammana & Villani, 1998) and the results showing poor diversity and coherence obtained in this study has led to a number of observations concerning the importance of covering certain fundamental ideas in the intended curriculum. In the following lines, the focus is put on Operations with forms (F2), Coordinates, spatial relationships, and reasoning (F3), and Geometric patterns (F5) because these ideas are particularly important to be promoted in primary education.

Operations with forms. When talking about the geometry curriculum for primary education, the ideas of symmetry, composing, folding and tessellation are of great importance. Numerous authors (e.g., Franke & Reinhold, 2016; Moor & Van den Brink, 1997; Winter, 1976) highlighted the indispensable role of symmetry in primary geometry education because it helps develop visual and spatial ability and encourages creativity and fantasy. Similarly, Jones (2002) pointed out that the key ideas of geometry include invariance, symmetry and transformation, so they should be included in all parts of the geometry curriculum. Grade 1 students can already bring experiences with symmetry from everyday life (Franke & Reinhold, 2016). Furthermore, the idea of composing and decomposing is essential for early-grade geometry because it provides the basis for measurement in higher grades, such as finding the area, surface area or volume of irregular shapes (Van de Walle et al., 2019). Yet, although folding and unfolding are very important in primary school grades because they increase children's motivation, provoke geometric reasoning, develop visual abilities, problem solving and communication, and refine motor skills (Schipper et al., 2015), these activities were not addressed at all in either of the Croatian geometry curricula. These aspects, however, are included in many primary school geometry curricula worldwide (e.g., in Germany, Senatsverwaltung für Bildung, Jugend und Wissenschaft Berlin, 2015). Unlike the participants in Germany (Kuzle & Glasnović Gracin, 2020), the Croatian students did not present this fundamental idea in their drawings at all. Therefore, the future geometry curriculum in Croatia should consider including subcategories of F2 in their content and outcomes.

Coordinates, spatial relationships, and reasoning. The literature provides strong arguments why visual perception and spatial visualisation and orientation should be included in primary school curriculum. For instance, Franke and Reinhold (2016) highlighted them as the most important goals of the primary education geometry curriculum, and likewise pointed out the importance of visual perception and coordinates as a prerequisite for spatial visualisation. These topics support the development of geometric perception and make the basis for mathematical thinking (Shipper et al., 2015). The construct of spatial visualisation is in itself complex since it encompasses different mental activities (i.e., mental rotation, spatial relationships and orientation in the space) (Franke & Reinhold, 2016), and likewise all these components should be visible in the curriculum.

Geometric patterns. Geometric patterns are perceived as an important part of early geometry curriculum given that this fundamental idea supports content competencies in geometry (recognising identical shapes and their positions, implementation of symmetry, experience with angles), process competencies (problem solving and language development through describing of regularities and principles), general competencies (fine motor skills and perception abilities), artistic competencies (fantasy and creativity, aesthetic sense for colours and shapes), social competencies (enjoying mathematics education through group work and communication), and spatial relationships (Franke & Reinhold, 2016; Schipper et al., 2015). Besides that, Winter (1976) highlighted the algebraic aspect of geometric patterns because they provide an important connection between geometry and algebra. Therefore, the future designers of the (Croatian) geometry curriculum should consider including geometric patterns throughout primary education.

Connecting research, limitations of the study and future research

The study presented in this paper focused on linking the analysis of two Croatian curricula with two recent studies (Glasnović Gracin & Krišto, 2022; Glasnović Gracin & Kuzle, 2021) that focused on the potentially implemented and attained curriculum. The "missing link" was the intended curriculum, which may have influenced other layers of the curriculum. The study shed light on the old and new curriculum by analysing the presence or absence of particular fundamental ideas and their subcategories. A comparison of the NPP and the CUR using absolute or relative frequencies was not possible because the two documents are different in their structure and complexity. Also, the results by Glasnović Gracin and Krišto (2022) did not address all subcategories of fundamental ideas, so the comparison was not complete.

The analytical tool used and modified within this study enables insight into different curricula worldwide, making it a viable tool for researchers who seek access to this research area. Hence, future research may encompass the application of this analytical tool to the curricula of other countries, including coherence and diversity issues. In the future, the obtained results should be supported by qualitative data such as what fundamental ideas are being taught in the classroom and how, from the perspective of diversity and coherence. Research encompassing all layers of the curriculum may contribute to a deeper understanding of the geometry curriculum and how to improve the position of school geometry within mathematics education.

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Raznolikost i koherentnost kao važni aspekti kurikula: analiza geometrijskih sadržaja dvaju hrvatskih kurikula u razrednoj nastavi

Sažetak

Školska geometrija tijekom godina suočavala se s mnogim problemima, u što ulazi i gubitak nekad snažne pozicije unutar školske matematike te dobivanje reputacije „problematičnog djeteta" nastave Matematike. Ovi problemi proizlaze iz planiranog kurikula. Cilj ovoga rada je analiza dvaju planiranih kurikula za razrednu nastavu Matematike u Hrvatskoj s fokusom na raznolikost i koherentnost geometrijskih sadržaja i kroz objektiv fundamentalnih ideja geometrije. Uz to, rezultati su povezani s prethodnim nalazima o postignutom (slike učenika o geometriji) i potencijalno primijenjenom kurikulu (udžbenički zadatci). Iako novi kurikul odražava više raznolikosti u pogledu fundamentalnih ideja geometrije u odnosu na stari, rezultati pokazuju da ti kurikuli još uvijek ne nude značajke suvremene metodike geometrije u punom potencijalu. Na temelju dobivenih rezultata provedena je rasprava s prijedlozima kako poboljšati položaj školske geometrije u matematičkom obrazovanju u Hrvatskoj prema preporukama iz literature.

Ključne riječi: *fundamentalne ideje; nastava geometrije; razredna nastava; trodijelni model kurikula.*

Uvod

U skladu s ciljevima obrazovnih sustava, kurikul treba pružiti obrazovne mogućnosti u smislu formiranja društvenih, političkih i pedagoških uvjeta koji će učenicima omogućiti stjecanje znanja, razvijanje vještina i oblikovanje stavova kroz školske predmete (Valverde i sur., 2002). Pojam *kurikul*, koji se tijekom povijesti obrazovanja koristio u različite svrhe i na različite načine diljem svijeta, više je od samog popisa tema i ciljeva koje treba pokriti (Remillard i Heck, 2014). Radi se o složenom konstruktu i stoga postoje različiti modeli koji konceptualiziraju njegovu višestruku prirodu, raslojavajući ga na različite dijelove. Schmidt i sur. (1996) predstavili su trodijelni model kurikula, razlikujući pritom kurikul kao skup ciljeva sustava (*planirani kurikul*),

kurikul kao nastavu (*primijenjeni kurikul*) i kurikul kao postignuće učenika (*postignuti kurikul*). Valverde i sur. (2002) proširili su ovaj model četvrtom komponentom, odnosno *potencijalno primijenjenim kurikulom*, imajući pritom na umu posredničku ulogu između planiranoga i primijenjenoga kurikula koju imaju udžbenici matematike.

S obzirom na njezinu važnost, geometriji bi trebalo posvetiti više pažnje nego što joj se trenutačno posvećuje (Kuzle, 2022), iz više razloga. Prvo, velika je razlika u značaju između geometrije kao matematičke discipline i kao dijela matematičkoga obrazovanja (Kuzle i Glasnović Gracin, 2020). Naime, iako je geometrija od davnina potaknula razvoj matematike te doživjela bogatstvo i raznolikost svojih tema u 19. i 20. stoljeću, geometrijsko obrazovanje nije prošlo sličan razvoj unutar matematičkoga kurikula (Jones, 2000). Drugo, unatoč važnosti geometrije, u posljednjim desetljećima događa se značajno reduciranje njezina sadržaja u brojnim kurikulima diljem svijeta (npr. Backe-Neuwald, 2000; Mammana i Villani, 1998), što je povezano s uključivanjem novih sadržaja u nacionalne kurikule, poput statistike i vjerojatnosti (Jones, 2000). Treće, kurikuli geometrije često su kritizirani zbog nedostatka koherentnosti sadržaja i pristupa kroz razrede (npr. Franke i Reinhold, 2016; Mammana i Villani, 1998; Van de Walle i Lovin, 2006). Četvrto, velika međunarodna ispitivanja, poput PISA-e i TIMSS-a, stavljaju naglasak na neke druge „velike ideje", npr. na brojeve u odnosu na oblik i prostor (Glasnović Gracin, 2011; Mullis i Martin, 2013). To može dovesti do nižih postignuća u geometrijskim zadatcima u odnosu na brojeve ili podatke, kako izvještava Glasnović Gracin (2011) u hrvatskom kontekstu. Sva četiri gore navedena aspekta odnose se na slojeve trodijelnoga kurikula (Schmidt i sur., 1996). Prva dva aspekta odnose se na planirani kurikul, a treći se odnosi na planirani i primijenjeni kurikul geometrije. Četvrti aspekt predstavlja problem prepoznat u postignutom kurikulu na koji utječe planirani te posljedično i primijenjeni kurikul.

Ovaj popis aspekata nije potpun, ali svakako pokreće tekuću raspravu o ulozi i važnosti geometrije unutar matematičkoga kurikula (Mammana i Villani, 1998; Sinclair i Bruce, 2015). Primjerice, Jones (2002) tvrdi da nastava geometrije pomaže učenicima u razvoju vizualizacije, kritičkoga mišljenja, intuicije, perspektive, rješavanja problema, naslućivanja, deduktivnoga zaključivanja, logičkoga argumentiranja te dokazivanja. Razvoj ovih vještina s jedne strane i snažna tradicija s druge, predstavljaju izazov za nastavu geometrije jer pitanja koji bi sadržaji trebali biti uključeni u suvremeni kurikul geometrije (tj. raznolikost) te kako ih povezati (tj. koherentnost) nisu trivijalna. Cilj istraživanja prikazanoga u ovom radu jest analiza dvaju planiranih osnovnoškolskih kurikula u Hrvatskoj s obzirom na raznolikost i koherentnost njihovih geometrijskih sadržaja i kroz objektiv fundamentalnih ideja geometrije te ih povezati s ostalim dijelovima trodijelnoga modela.

Teorijska pozadina

Ovo poglavlje donosi objašnjenje termina raznolikosti i koherentnosti unutar kurikulskoga konteksta. Nakon toga predstavljen je konstrukt fundamentalnih ideja s posebnim fokusom na model od Kuzle i Glasnović Gracin (2020). Slijede prikazane prethodne studije o nastavi geometrije u Hrvatskoj.

Raznolikost i koherentnost kurikulskoga sadržaja

Literatura o suvremenoj nastavi geometrije stavlja naglasak na njezine dvije glavne karakteristike, a to su koherentnost i raznolikost. *Raznolikost* se odnosi na različite koncepte unutar kurikula. Ona treba biti prisutna na svim razinama obrazovanja, počevši od primarnoga (npr., Battista i Clemens, 1988; Franke i Reinhold, 2016; Jones, 2000; Mammana i Villani, 1998; Van de Walle i Lovin, 2006), i to s različitim sadržajnim fokusima. Primjerice, Battista i Clemens (1988) sugeriraju da geometrija u primarnom obrazovanju treba sadržavati oblike i njihova svojstva, razvoj intuicije o prostoru, korištenje geometrijskih modela s ciljem rješavanja problema te razvoj argumentiranja. Van de Walle i Lovin (2006) pak preporučuju oblike i njihova svojstva, transformacije, lokaciju, vizualizaciju te analizu i apstrahiranje geometrijskih koncepata i odnosa. Franke i Reinhold (2016) ističu oblike u prostoru i ravnini, simetriju, prostornu vizualizaciju i mišljenje te uzorke i strukturu kao osnovne koncepte u kurukulu za primarno geometrijsko obrazovanje.

Međutim, raznolikost sadržaja ne garantira nužno i njihovo povezivanje. Naučene ideje trebaju biti smisleno povezane kroz cijeli kurikul (Schweiger, 1992), što se opisuje pojmom *koherentnosti*. Sam Piaget je naglasio važnost redoslijeda u učenju geometrijskih ideja (Jones i Mooney, 2003), počevši s topologijom, preko pravocrtnosti povezane s projektivnom geometrijom, do učenja euklidskih ideja paralelnosti, kuta i udaljenosti. Takav redoslijed podržava postupan razvoj geometrijskih ideja tijekom vremena i sve se više integrira i sintetizira kako djeca napreduju (Jones i Mooney, 2003). Unatoč važnosti koherentnosti, Hansen (1998) kritizira njezin nedostatak u kurikulu jer programi često sadrže izolirane fragmente geometrijskih koncepata poput „komadića klasifikacije mnogokuta, nekoliko formula za mjerenje različitih likova, malo incidencije, malo spominjanja transformacija, nekoliko konstrukcija, odabranih geometrijskih mjesta točaka, uvoda u vektore i, konačno, nešto analitičke geometrije" (str. 238). Kao posljedica toga, prijeko potrebna raznolikost u planiranom kurikulu donosi nove izazove u nastavu geometrije jer ti sadržaji svakako trebaju biti međusobno povezani, kako horizontalno (unutar istoga razreda), tako i vertikalno (kroz različite razrede).

Fundamentalne ideje geometrije

Jedan od suvremenih pristupa kako unaprijediti nastavu geometrije i odgovoriti na spomenute izazove odnosi se na uspostavu koherentnoga kurikula baziranoga na fundamentalnim idejama (Schweiger, 1992; Wittmann, 1999). *Fundamentalne ideje* mogu se opisati kao skup aktivnosti ili strategija koje su prisutne u povijesnom razvoju matematike, koje se pojavljuju u kurikulu po vertikali, koje doprinose odgovoru na pitanje što je matematika, koje čine poučavanje transparentnijim i fleksibilnijim te koje posjeduju odgovarajući jezični i akcijski arhetip u svakodnevnom životu (Schweiger, 1992). Na taj se način fundamentalne ideje mogu neprestano razvijati na svakom stupnju obrazovanja (Van de Walle i Lovin, 2006) i biti primijenjene u svakom matematičkom području, uključujući i geometriju.

Wittmann (1999) je ponudio organizaciju školske geometrije oko sedam fundamentalnih ideja koje reflektiraju raznolikost i koherentnost geometrijskih sadržaja: geometrijske oblike i njihovu konstrukciju, operacije s oblicima, koordinate, mjerenje, geometrijske uzorke, oblike u okolini i geometrizaciju. Daljnju razradu Wittmanova (1999) modela provele su Kuzle i Glasnović Gracin (2020), a modifikacija se odnosi na proširenje nekih fundamentalnih ideja te razvoj detaljnije klasifikacije u potkategorijama (Tablica 1), kako je objašnjeno u Kuzle i Glasnović Gracin (2020).

Tablica 1

Ovaj okvir odražava višedimenzionalni (raznoliki) i koherentni pogled na geometriju, što je u skladu sa suvremenim obrazovnim smjernicama (Franke i Reinhold, 2016; Mammana i Villani, 1998; Sinclair i Bruce, 2015). Nadalje, model se već koristio kao teorijski okvir za analizu i ispitivanje raznih elemenata unutar trodijelnoga kurikula: potencijalno primijenjenoga kurikula u Glasnović Gracin i Krišto (2022) te postignutoga kurikula u Glasnović Gracin i Kuzle (2021), Kuzle (2019) i Kuzle i Glasnović Gracin (2020), o čemu će biti govora u daljnjem tekstu.

Empirijsko istraživanje postignutog i potencijalno primijenjenog kurikuluma

Istraživanje postignutoga kurikula. Istraživanje koje su provele Glasnović Gracin i Kuzle (2021) donosi uvid u predodžbe koje učenici u primarnom obrazovanju imaju o geometriji, uz korištenje svojih ilustracija. Dakle, istraživanje je uključilo analizu postignutoga kurikula, to jest, ono što su učenici usvojili. Ispitanicima (N = 249 hrvatskih učenika od 2. do 4. razreda) dan je papir sa zadatkom da nacrtaju što je za njih geometrija. Podatci su analizirani uz korištenje okvira fundamentalnih ideja u geometriji (Kuzle i Glasnović Gracin, 2020) prikazanoga u Tablici 1. Rezultati su pokazali da ispitanici imaju prilično usko shvaćanje geometrije; većina učeničkih ilustracija (više od 90 % u svakom razredu) sadržavala je jednu ili najviše dvije fundamentalne ideje (Tablica 2). Fundamentalna ideja F1 (geometrijski oblici i njihova konstrukcija) dominirala je na ilustracijama, pri čemu su učenici uglavnom prikazivali jedno-, dvo- i trodimenzionalne objekte. Ostale fundamentalne ideje prikazane su u mnogo manjoj mjeri ili uopće nisu prikazane.

Tablica 2

Istraživanje potencijalno primijenjenoga kurikula. Isti okvir fundamentalnih ideja u geometriji korišten je i u studiji potencijalno primijenjenoga kurikula geometrije opisanoj u Glasnović Gracin i Krišto (2022). Ondje su analizirani svi zadatci iz geometrije u najkorištenijem udžbeniku matematike u Hrvatskoj u razrednoj nastavi.

Budući da se u Hrvatskoj matematički udžbenici koriste u velikoj mjeri (Glasnović Gracin, 2011), istraživanje se odnosilo na potencijalno primijenjeni kurikul. Analiza 267 zadataka iz tiskanih udžbenika jasno je ukazala na nedostatak raznolikosti u zahtjevima geometrijskih zadataka te na dominaciju fundamentalne ideje F1 (Tablica 3). Dublja analiza potkategorija ideje F1 pokazala je da se 35 % zadataka odnosilo na F1b (jednodimenzionalne objekte) te 37 % na F1c (dvodimenzionalne objekte). Udžbenički zadatci s trodimenzionalnim objektima (F1d) bili su slabo zastupljeni, sa samo 4 %, a uopće nisu bili prisutni u trećem i četvrtom razredu. Geometrijski uzorci (F5) prisutni su samo u prvom razredu s udjelom od 19 %. Mjerenje (F4) je prisutno u oko 20 % svih geometrijskih zadataka od prvoga do trećega razreda te s 10 % u četvrtom razredu. Stoga ovi rezultati ukazuju na prilično ograničene mogućnosti učenja geometrije.

Tablica 3

Dva hrvatska matematička kurikuluma

Hrvatski obrazovni sustav je vrlo centraliziran; planirani kurikul za sve predmete pa tako i za Matematiku donose obrazovne vlasti i primjenjuje se na sve škole i razrede na razini države (*Zakon o odgoju i obrazovanju u osnovnoj i srednjoj školi*, 2008).

Nastavni plan i program (Ministry of science, education and sports [MZOS], 2006), koji se odnosio samo na obvezno obrazovanje u Hrvatskoj (1. do 8. razred) i koji je strukturiran prema nastavnim temama za svaki predmet i razred, uključujući Matematiku, primjenjivao se od 2006. godine. Svaka tema sastojala se od naziva teme, pripadajućih obrazovnih postignuća te ključnih riječi. U njemu se ne spominje važnost raznolikosti matematičkih sadržaja ni potreba za koherentnosti između tema. Umjesto toga, autori dokumenta smatraju da je Matematika „predmet s dugotrajnom tradicijom i dobro definiranim sadržajima te nisu potrebni veliki zahvati u sadašnjim programima, stoga su polaznu osnovu pri izradbi Programa za matematiku tvorili postojeći programi" (MZOS, 2006, str. 238).

U 2019. godini provedena je opsežna obrazovna reforma koja je obuhvaćala novi kurikul za sve tipove škola kroz cijelu obrazovnu vertikalu. Temeljna ideja novoga kurikula za Matematiku (Ministry of science and education [MZO], 2019) oslanjala se na povezivanju matematičkih procesa i domena. Za svaku od pet sadržajnih domena dani su odgojno-obrazovni ishodi koji sadrže naziv i šifru ishoda, razradu, ishode na razini usvojenosti „dobar" na kraju pojedinoga razreda, sadržaj povezan s ishodom te preporuke za ostvarivanje ishoda. Nadalje, ovaj dokument naglašava važnost vertikalne sadržajne koherentnosti u nastavi Matematike: „Domene se postupno razvijaju i nadograđuju cijelom vertikalom učenja i poučavanja matematike" (MZO, 2019, str. 12), kao i koherentnosti među domenama: „Premda domene povezuju srodne koncepte, njihova se nedjeljivost stalno primjećuje jer je usvojenost koncepata jedne domene često pretpostavka usvajanju koncepata u drugim domenama" (MZO, 2019, str. 12). Ovo se može primijeniti i na geometrijske sadržaje koji su pokriveni domenama Oblik i prostor, Mjerenje te Algebra i funkcije.

Istraživačka pitanja

Studija prikazana u ovom radu odnosi se na sadržajnu analizu dvaju hrvatskih matematičkih kurikula za razrednu nastavu iz perspektive fundamentalnih ideja geometrije. Iako je istraživanje Glasnović Gracin i Kuzle (2021) naglasilo potrebu ispitivanja planiranoga kurikula (jer takvo istraživanje može doprinijeti propitivanju kurikulskih zahtjeva vezanih uz višedimenzionalnu prirodu geometrije), detaljna analiza hrvatskoga planiranog kurikula za geometriju do sada još nije provedena. Nadalje, analiza obaju planiranih kurikula – „starog" (MZOS, 2006) i „novog" (MZO, 2019) nudi povezivanje s prije dobivenim nalazima o prilično uskoj predodžbi geometrije u postignutom i potencijalno primijenjenom kurikulu. Stoga su postavljena sljedeća istraživačka pitanja:

1. Koje su fundamentalne ideje geometrije prisutne u Nastavnom planu i programu (NPP) za razrednu nastavu (MZOS, 2006)? Kako se ti rezultati mogu povezati s prethodnim istraživanjem o učeničkim predodžbama o geometriji dobivenih pomoću ilustracija ispitanika?
2. Koje su fundamentalne ideje geometrije prisutne u novom kurikulu (CUR) za razrednu nastavu (MZO, 2019)? Kako se ti rezultati mogu povezati s prethodnim istraživanjem o geometrijskim zadatcima iz udžbenika?
3. U kojim se aspektima dvaju hrvatskih kurikula za razrednu nastavu odražavaju koherentnost i raznolikost fundamentalnih ideja geometrije kroz svoju vertikalnu perspektivu?

Razlog za povezivanje NPP-a s istraživanjem o učeničkim ilustracijama (Glasnović Gracin i Kuzle, 2021) jest u tome što je to istraživanje provedeno prije uvođenja CUR-a. Studija Glasnović Gracin i Krišto (2022) fokusirala se pak na analizu udžbeničkih zadataka u skladu s važećim novim kurikulom CUR.

Metoda

Ova se studija odnosi na dokumentacijsko istraživanje dvaju matematičkih planiranih kurikula u Hrvatskoj (MZOS, 2006; MZO, 2019) za razrednu nastavu (od1. do 4. razreda). Taj period je ključan za razvoj geometrijskoga mišljenja i za stvaranje osnova za daljnje usvajanje geometrijskih sadržaja (Franke i Reinhold, 2016; Mamanna i Villani, 1998). Nadalje, prijašnja istraživanja potencijalno primijenjenoga i postignutoga kurikula fokusirala su se na razrednu nastavu (Glasnović Gracin i Krišto, 2022; Glasnović Gracin i Kuzle, 2021). Na taj način analiza rezultata planiranoga kurikula razredne nastave može objasniti neke od prijašnjih nalaza.

Analiza je obuhvatila dijelove kurikula koji se odnose na geometrijske koncepte u razrednoj nastavi, uz korištenje kvalitativne sadržajne analize (Mayring, 2000). Slijedeći određene kategorije, obje su autorice neovisno analizirale svaki kurikul. Analiza je bazirana na maloj modifikaciji okvira prikazanoga u Tablici 1. Sljedeće potkategorije dodane su u instrument na temelju provedene analize: F1j (ravnina i prostor), F3d (vizualna percepcija), F3e (prostorna vizualizacija, odnosi i orijentacija) te F4j (računanje s mjernim brojevima). Ravnina i prostor odnose se na eksplicitno spominjanje ravnine i trodimenzionalnoga prostora kao geometrijskih ideja, a također i na termin „ravna ploha" kada se misli na ravninu. Vizualna percepcija odnosi se na opažanje vizualnih objekata i operiranje s njima. Računanje s mjernim brojevima obuhvaća aktivnosti računanja s mjernim jedinicama, primjerice, „Izračunaj 3 m + 8 m".

Obje su autorice dodijelile kodove fundamentalnih ideja (od F1 do F7) i njihove podkodove (npr., F1a, F1b) svakoj temi ili ishodu u oba kurikula. Analiza je obuhvatila sav tekst iz pojedine geometrijske teme u NPP-u (ključne riječi i obrazovna postignuća unutar određene teme), kao i iz pojedinoga ishoda iz CUR-a (njegovu razradu, sadržaj te preporuke). U sljedećem koraku autorice su usporedile dodijeljene kodove i podkodove. Sve nedosljednosti koje su se pojavile su prodiskutirane, što je dovelo do prilagodbe instrumenta te su se posljedično donijeli završni kodovi. Završno slaganje među ispitivačima je bilo 100 %. Primjer kodiranja teme „Pravi kut" u četvrtom razredu (MZOS, 2006, str. 243) dana je u Tablici 4.

Tablica 4

U ovom istraživanju smatra se da je prisutna koherentnost određene fundamentalne ideje i njezinih potkategorija ako se određeni koncept pojavljuje u uzastopnim razredima. Raznolikost obuhvaća širok raspon fundamentalnih ideja prisutnih unutar određenoga razreda. Na temelju dobivenih kodova i podkodova prikazana je deskriptivna statistika njihova pojavljivanja u svakom razredu. Ako je kod bio prisutan u određenom razredu, dodijeljen mu je znak „+", a u protivnom je ćelija ostala prazna. Tako su raznolikost i koherentnost fundamentalnih ideja jasno vidljive u svakom analiziranom kurikulu. Nadalje, istraživanje je nastojalo povezati dobivene podatke s rezultatima prethodnih studija: podatci dobiveni iz CUR-a uspoređeni su s nalazima iz Glasnović Gracin i Krišto (2022), dok su podatci dobiveni iz NPP-a uspoređeni s nalazima iz Glasnović Gracin i Kuzle (2021). Rezultati o određenoj fundamentalnoj ideji ili njezinoj potkategoriji planiranoga kurikula uspoređeni su u odgovarajućoj prethodnoj studiji. Ako je znak u planiranom kurikulu bio „+" i postotak u odgovarajućoj prethodnoj studiji bar 8 %, veza je naznačena sa „slaže se". Ako je ćelija u NPP-u ili CUR-u bila prazna, a udio u prethodnoj studiji manji od 8 %, veza je također naznačena kao „slaže se". U protivnom, veza je naznačena kao „nema slaganja".

Rezultati

Rezultati su podijeljeni u tri dijela, u skladu s istraživačkim pitanjima.

Fundamentalne ideje u planiranom (NPP) i postignutom kurikulu

Raznolikost u planiranom kurikulu NPP. Distribucija fundamentalnih ideja u planiranom kurikulu NPP (MZOS, 2006) za razrednu nastavu prikazana je u Tablici 5. Rezultati ukazuju da je fundamentalna ideja F1 (geometrijski oblici i njihova konstrukcija) prisutna u svakom razredu, F3 (koordinate, prostorni odnosi i mišljenje) prisutna je u prva tri razreda, a F4 (mjerenje) u 1., 3. i 4. razredu. Fundamentalna ideja F6 (oblici u okolini) eksplicitno se spominje samo u prvom razredu, iako NPP pod općim ciljevima i zadaćama spominje da učenici trebaju biti sposobni primijeniti matematička znanja u svakodnevici (MZOS, 2006, str. 238). Fundamentalne ideje F2 (operacije s oblicima), F5 (geometrijski uzorci) i F7 (geometrizacija) uopće nisu prisutne u NPP-u. Stoga rezultati ukazuju na nedostatak raznolikosti u geometriji u NPP-u, što je detaljnije ispitano kroz podkodove fundamentalnih ideja (tablice 6 - 8).

Tablica 5

Ako pogledamo dublje u F1, potkategorije F1a (točke), F1b (jednodimenzionalni objekti) i F1f (crtanje i alati za crtanje/konstruiranje) prisutne su u svima četirima razredima (Tablica 6). Potkategorije F1c (dvodimenzionalni objekti) i F1e (geometrijska svojstva) pojavljuju se u 1., 3. i 4. razredu, dok je ideja F1d (trodimenzionalni objekti) prisutna samo u 1. i 4. razredu. Gledajući po razredima, rezultati pokazuju raznolikost potkategorija od F1 u 1., 3. i 4 razredu. U drugom se razredu NPP-a geometriji posvećuju samo dvije teme (od njih 31), što može objasniti smanjenu raznolikost potkategorija od F1.

Tablica 6

Ako pogledamo dublje u F3, samo je jedna njezina potkategorija (F3b) prisutna od 1. do 3. razreda (Tablica 7). Radi se o pozicijskim odnosima poput unutar/izvan, pripada/ne pripada, paralelan, okomit. Ovaj rezultat ukazuje na manjak raznolikosti u potkategorijama od F3.

Tablica 7

Ako pogledamo dublje u ideju F4, raznolikost njezinih potkategorija prisutna je samo u četvrtom razredu, sa 6 od 10 prisutnih aspekata (Tablica 8).

Tablica 8

Povezivanje planiranoga i postignutoga kurikula. Usporedba tablica 2 i 5 omogućuje istraživanje veza između NPP-a (MZOS, 2006) i postignutoga kurikula (Glasnović Gracin i Kuzle, 2021). Obje tablice odražavaju prilično usku sliku geometrije s naglaskom na geometrijske oblike i njihovu konstrukciju (F1). Nedostatak fundamentalnih ideja F2, F5 i F7 u planiranom kurikulu (Tablica 5) može objasniti njihovu odsutnost u postignutom kurikulu (Tablica 2). Fundamentalna ideja F6 spominje se samo u prvom razredu i prisutna je u 8 % ilustriranih elemenata (8 % u drugom razredu, 9 % u trećem te 6 % u četvrtom razredu) (Glasnović Gracin i Kuzle, 2021). Iako F6 nije spomenuta pod temama, naglašena je u općim ciljevima i zadaćama nastave Matematike (MZOS, 2006, str. 238) i stoga se mogla očekivati u postignutom kurikulu. S druge strane, rezultati dobiveni za fundamentalne ideje F3 i F4 se ne podudaraju: dok su F3 i F4 prisutne u planiranom kurikulu u trima od četiriju razreda, jedva da su prisutne u učeničkim ilustracijama. Ovaj rezultat možda može objasniti nalaz da su geometrijski oblici (F1) lakši za ilustriranje nego prostorni odnosi i mjerenje (Kuzle i Glasnović Gracin, 2020). Rezultati o vezi između planiranoga i postignutog kurikula nalaze se u Tablici 9.

Tablica 9

S obzirom na potkategorije od F1, rezultati dobiveni iz crteža (Glasnović Gracin i Kuzle, 2021) pokazali su da su sudionici kao glavne predstavnike svoje slike o geometriji identificirali F1b (19 % elemenata na ilustracijama), F1c (38 %) i F1d (17 %). Među njima je samo F1b bio prisutan u svim razredima NPP-a (Tablica 6) i prikazuje slaganje među dvama kurikulima. Potkategorija F1c, iako najčešće korištena u crtežima, nije prisutna u 2. razredu NPP-a. Zanimljivo, trodimenzionalni objekti (F1d) bili su prisutni na 17 % svih ilustriranih elemenata (Glasnović Gracin i Kuzle, 2021), iako nisu mnogo zastupljeni u NPP-u (Tablica 6). Iako je potkategorija F1a (točke) bila prisutna u dječjim crtežima sa samo 2 %, ona se nalazi u svim razredima NPP-a. Slično tome, F1e i F1f prisutni su u NPP-u u trima razredima, ali su se nalazili na otprilike 4-5 % nacrtanih elemenata. Također, neke potkategorije koje nisu bile prisutne u NPP-u nisu bile prisutne ni u dječjim crtežima (npr. F1g, F1h, F1i).

Fundamentalne ideje u planiranom (CUR) i potencijalno primijenjenom kurikulu

Raznolikost u planiranom kurikulu CUR. Distribucija raznolikosti fundamentalnih ideja u planiranom kurikulu (MZO, 2019) za razrednu nastavu prikazana je u Tablici 10. Rezultati ukazuju da su fundamentalne ideje F1, F3 i F4 prisutne u svakom razredu. Međutim, fundamentalna ideja F2 nalazi se samo u prva dva razreda, a F7 samo u 3. i 4. razredu. Ideja F5 prisutna je samo u prvom razredu, dok je F6 pokrivena u 1., 2. i 4. razredu. Stoga je svih sedam fundamentalnih ideja geometrije pokriveno CUR-om. Ovi rezultati ukazuju na veću raznolikost fundamentalnih ideja u CUR-u u odnosu na NPP (tablice 5 i 10). Ipak, neke fundamentalne ideje, poput operacija s oblicima (F2) i geometrijskih uzoraka (F5), nisu prikazane kroz većinu razreda.

Tablica 10

Ako gledamo iz perspektive razreda, CUR pokriva šest fundamentalnih ideja u 1. razredu (sve osim F7), pet u 2. i 4. razredu te četiri u 3. razredu. Najveću raznolikost ideja opet pronalazimo u prvom razredu. Geometrizacija (F7) je prisutna samo u 3. i 4. razredu, što je očekivano s obzirom na njezinu apstraktnu prirodu (Kuzle i Glasnović Gracin, 2020). Ovi rezultati otkrivaju raznolikost geometrijskih ideja u CUR-u, posebice u usporedbi s NPP-om.

S obzirom na potkategorije od F1, pet ih je prisutno u CUR-u u svakom razredu, a to su F1a, F1b, F1c, F1e i F1f (Tablica 12). Potkategorije F1d i F1i prisutne su u svim razredima osim trećega. Ovi rezultati ukazuju na raznolikost potkategorija od F1 u CUR-u. Kutovi (F1h) su prisutni samo u 4. razredu. Gledajući po razredima, rezultati pokazuju raznolikost potkategorija od F1 u razrednoj nastavi, ali je ta raznolikost veća u 1., 2. i 4. razrednu u odnosu na treći.

Tablica 11

Ako se dublje pogleda u F2, prisutna je samo jedna od njezinih devet potkategorija, i to F2g (Tablica 12) samo u 1. i 2. razredu. Ovdje se kurikulske preporuke odnose na korištenje tangrama za sastavljanje i rastavljanje oblika u ravnini (MZO, 2019).

Tablica 12

Ako se dublje pogleda u F3, prisutne su tri od pet njezinih potkategorija, i to F3b, F3c i F3d (Tablica 13). Pritom se F3b proteže od 2. do 4. razreda.

Tablica 13

Ako se pogleda dublje u F4, prisutno je sedam od deset njezinih potkategorija, i to F4a–F4c, F4f–F4h te F4j (Tablica 14). Međutim, F4d, F4e i F4i uopće nisu pokrivene CUR-om[1]. Dok je F4g prisutan u svim razredima analiziranoga CUR-a, F4a, F4f i F4g se odnose na 2., 3. i 4. razred.

Tablica 14

Fundamentalna ideja F7 prisutna je u 3. i 4. razredu CUR-a (Tablica 10) i pokriva samo potkategoriju geometrijskih problema (Tablica 15).

Tablica 15

Povezivanje planiranoga i primijenjenoga kurikula. Usporedba tablica 3 i 10 otkriva neke razlike između CUR-a (MZO, 2019) i udžbeničkoga sadržaja (Glasnović Gracin i Krišto, 2022), vezano uz raznolikost fundamentalnih ideja. Ideja F1 je prisutna od 1. do 4. razreda u CUR-u (Tablica 10), a također je bila dominantna u udžbeničkim zadatcima: 79 % svih geometrijskih zadataka u analiziranim udžbenicima odnosilo se na F1 (Tablica 3). Ideja F4 prisutna je u svim analiziranim razredima CUR-a te u 16 % geometrijskih udžbeničkih zadataka. Međutim, F2, F3 i F7 uopće nisu bile prisutne u analiziranim udžbenicima, iako su pokrivene CUR-om u 3. i 4. razredu. Zanimljivo, F3 se nalazi u CUR-u u svima četirima razredima, ali uopće ne u udžbenicima. Ideje F5 i F6 bile su prisutne u samo 4 – 5 % zadataka iako je njihova prisutnost veća u CUR-u u odnosu na NPP. Ovi nalazi pokazuju da su mogućnosti za učenje geometrije u analiziranim udžbenicima prilično uske, što nije u skladu s kurikulskim zahtjevima. Stoga rezultati mogu upućivati da su, unatoč raznolikosti fundamentalnih ideja u CUR-u, analizirani udžbenici još uvijek bliže NPP-u u (ne)raznolikosti fundamentalnih ideja. Poveznica između planiranoga i potencijalno primijenjenoga kurikula nalazi se u Tablici 16.

Tablica 16

[1] Važno je napomenuti da je mjerenje tekućina (u litrama i decilitrima) uključeno u kurikul (MZO, 2019), ali taj sadržaj nije uključen u ovo istraživanje. To je stoga što mjerenje volumena tekućine u CUR-u stavlja snažan naglasak na fizikalne karakteristike, a ne na geometriju.

Gledano po razredima, rezultati iz Glasnović Gracin i Krišto (2022) prikazuju da je ideja F5 prisutna samo u udžbeniku prvoga razreda (čak 19 % svih zadataka 1. razreda se odnosilo na F5), ali je bila posve izostavljena u ostalim razredima. Ovaj se nalaz slaže sa sadržajem Tablice 10, što znači da su geometrijski oblici prisutni samo u 1. razredu u oba dijela kurikula. Također, udio udžbeničkih zadataka koji se odnose na F6 smanjuje se počevši od prvoga razreda prema četvrtom (Glasnović Gracin i Krišto, 2022). Zapravo, u 4. razredu uopće nema udžbeničkih zadataka iz F6, iako je ta ideja prisutna u CUR-u (Tablica 10).

S obzirom na potkategorije od F1, rezultati pokazuju da su jedno- i dvodimenzionalni objekti (F1b i F1c) prisutni u svim razredima u CUR-u (Tablica 11) te da su prisutni u otprilike trećini udžbeničkih zadataka, u što se ubrajaju i kutovi (Glasnović Gracin i Krišto, 2022). Ovaj nalaz ukazuje na slaganje ovih dvaju dijela kurikula. Ipak, potkategorije F1a i F1d prisutne su u udžbenicima samo u 1. i 2. razredu, što nije u skladu sa zahtjevima CUR-a za 3. i 4. razred.

Koherentnost i raznolikost kroz vertikalnu perspektivu

Koherentnost. Vertikalna koherentnost fundamentalnih ideja u NPP-u vidljiva je za F1 i F3. Ideja F1 prisutna je u sva četiri razreda (Tablica 5). Koherentnost je posebno važna i u potkategorijama ideja. Tablica 6 prikazuje nedostatak vertikalne koherentnosti kod F1d jer je ona prisutna samo u 1. i 4. razredu. Također, F1e je prisutna u svim razredima osim u drugom. Daljnja analiza NPP-a (MZOS, 2006) otkriva da su geometrijske teme minimalno zastupljene u 2. razredu, što nikako ne doprinosi vertikalnoj koherentnosti geometrijskih sadržaja. Ideja F3 je zastupljena u NPP-u od 1. do 3. razreda (Tablica 7) i to samo s potkategorijom F3b. Ideja F4 prisutna je u 1., 3. i 4. razredu pa joj je vertikala prekinuta u 2. razredu (Tablica 5). Analiza potkategorija od F4 (Tablica 8) ukazuje na nedostatak koherentnosti vezano uz mjerenje. Primjerice, potkategorija F4g prisutna je samo u 1. i 4. razredu NPP-a. Fundamentalna ideja F6 eksplicitno je spomenuta samo u 1. razredu, iako se spominje u općem dijelu NPP-a (MZOS, 2006). Stoga se ona može odnositi na sve razrede te postoji vertikalna koherentnost za F6. Za fundamentalne ideje koje se uopće ne nalaze u NPP-u (npr. F2, F5, F7) ne možemo govoriti o koherentnosti.

Vertikalna koherentnost fundamentalnih ideja u CUR-u jasno je vidljiva za ideje F1, F3 i F4, koje su prisutne u svim analiziranim razredima (Tablica 10). Analiza potkategorija od F1 (Tablica 11) otkriva da su F1a, F1b, F1c, F1e i F1f prisutne u svim razredima razredne nastave. Prekid koherentnosti u 3. razredu vidljiv je kod F1d i F1i. Daljnja analiza CUR-a (MZO, 2019) pokazuje da je u 3. razredu naglasak stavljen na jedno- i dvodimenzionalne objekte i njihovo mjerenje (tj. duljina, opseg). Geometrijska tijela (F1d) nisu dio 3. razreda u CUR-u. U usporedbi s NPP-om, gdje je prekid u povezanosti učenja trodimenzionalnih oblika (F1d) još i veći (Tablica 6), ovdje se može vidjeti da je CUR učinio određene korake prema naprijed (Tablica 11), ali još uvijek nedostaje vertikalna koherentnost u F1d. Iako je ideja F3 prisutna u svim analiziranim razredima (Tablica 10), pogled u njezine potkategorije otkriva nedosljednosti u njezinoj unutarnjoj koherenciji (Tablica 13). Primjerice, potkategorija F3d prisutna je samo u 1. i 4. razredu, što ukazuje na nedostatak povezanosti u zahtjevima vizualne percepcije.

Zaključno, rezultati ukazuju da su neke fundamentalne ideje i njihove potkategorije koherentne kroz razrednu nastavu, neke su povezanije u CUR-u nego u NPP-u, a neke će trebati bolje vertikalno povezati.

Raznolikost. Rezultati pokazuju da CUR nudi veću raznolikost fundamentalnih ideja u usporedbi s NPP-om (tablice 5 i 10). Zanimljiv je rezultat da se najslabija razredna raznolikost u CUR-u (četiri fundamentalne ideje) poklapa s najvećom iz NPP-a. Gledajući po razredima, NPP nudi najveću raznolikost u 1. razredu s četiri fundamentalne ideje, a najslabiju u 2. i 4. razredu sa samo dvije. U CUR-u je najveća raznolikost također u 1. razredu, sa šest fundamentalnih ideja geometrije, a najslabija je u 3. razredu s njih četiri.

Potkategorije od F1 također odražavaju veću raznolikost u CUR-u nego u NPP-u (tablice 6 i 11). Za razliku od CUR-a, NPP ne pokriva sve potkategorije od F1 u razrednoj nastavi. Najveća raznolikost unutar F1 je pronađena u 1. i 4. razredu sa sedam potkategorija, a najmanja u 2. razredu s njih tri. U CUR-u je najveća raznolikost također u 1. razredu (čak devet potkategorija), a najmanja u 3. razredu (s njih pet). Potkategorije od F3 ukazuju na nedostatak raznolikosti u obama planiranim kurikulima (tablice 7 i 13). Samo u 4. razredu CUR-a prisutne su tri (od pet) potkategorija. Vezano uz potkategorije od F4, NPP pokriva raznolikost samo u 4. razredu, dok CUR ovdje odražava raznolikost u svim razredima osim u prvome (tablice 8 i 14). Ovaj nalaz ukazuje da mjerenje nije u fokusu 1. razreda u CUR-u, ali se povećava u kasnijim razredima.

Zaključno, rezultati pokazuju veću raznolikost ideja u CUR-u nego u NPP-u, ali su također ukazali na neke probleme i izazove pronađene u CUR-u, posebice u 3. razredu.

Rasprava i zaključci

U fokusu ovoga rada je analiza dvaju planiranih kurikula i ispitivanje njihovih poveznica s prijašnjim studijama o postignutom i potencijalno primijenjenom kurikulu (Glasnović Gracin i Krišto, 2022; Glasnović Gracin i Kuzle, 2021) s obzirom na raznolikost i koherentnost geometrijskih sadržaja iz perspektive fundamentalnih ideja geometrije.

Analiza NPP-a pokazala je nedostatak raznolikosti geometrijskih sadržaja. Iako su ideje F1, F3, F4 i F6 pokrivene, samo je F1 prisutna kroz sve analizirane razrede. I NPP i dječji crteži (Glasnović Gracin i Kuzle, 2021) dali su prilično usku sliku o geometriji, s naglaskom na F1. Također, rezultati su otkrili nedostatak ideja F2, F5 i F7 kako u planiranom (NPP), tako i u postignutom kurikulu (Glasnović Gracin i Kuzle, 2021). Usporedba CUR-a i NPP-a pokazuje da CUR nudi veću raznolikost jer pokriva svih sedam fundamentalnih ideja geometrije, pri čemu su F1, F3 i F4 prisutne u svima četirima razredima. Po razredima, NPP pokazuje najveću raznolikost u 1. razredu s četiri različite ideje, a najmanju u 2. i 4. razredu, sa samo dvije. Slično, CUR ima najveću raznolikost u 1. razredu sa šest ideja, a najmanju u 3. razredu s njih četiri. Usporedba rezultata iz CUR-a i udžbeničkih zadataka (Glasnović Gracin i Krišto, 2022) pokazuje različitost kod fundamentalnih ideja F2, F3, F6 i F7. Dok je kod nekih fundamentalnih ideja vidljiva koherentnost kroz razrede (F1 i F3 u NPP-u te F1, F3 i F4 u CUR-u), drugim idejama i njihovim potkategorijama nedostajalo je koherentnosti, poput F3d u obama kurikulima.

Rezultati ovoga istraživanja ukazuju na zabrinutost s obzirom na mogućnosti koje nastava geometrije treba donijeti, poput kritičkoga mišljenja i intuicije, razvoja vještina vizualizacije te rješavanja problema i logičkoga argumentiranja (Jones, 2002; Mammana i Villani, 1998). Ova studija skreće pozornost na zahtjeve u geometriji u planiranom kurikulu, posebice na raznolikost i koherentnost sadržaja te njihova povezivanja s postignutim i potencijalno primijenjenim kurikulom. Valverde i sur. (2002) naglasili su da trodijelni model kurikula predstavlja početnu točku za stvaranje čvrsto utemeljenih mogućnosti za učenje. Planirani kurikul ovdje ima ključnu ulogu jer on utječe i na primijenjeni i na postignuti kurikul, posebice u zemljama sa snažno centraliziranim obrazovnim sustavom poput Hrvatske. Što se tiče statusa geometrije kao „problematičnog djeteta" (Backe-Neuwald, 2000) nastave Matematike, upravo bi stavljanje naglaska na raznolikost i koherentnost geometrijskih sadržaja moglo biti zanimljivo autorima kurikula i istraživačima cijele međunarodne zajednice.

Nedostatak nekih fundamentalnih ideja u hrvatskom kurikulu u razrednoj nastavi

Razlika između mogućnosti koje treba donijeti nastava geometrije (Jones, 2002; Mammana i Villani, 1998) i rezultata koji pokazuju slabu raznolikost i koherentnost dovela je do mnogih opservacija o važnosti određenih fundamentalnih ideja u planiranom kurikulu. U nastavku teksta fokusiramo se na ideje F2, F3 i F5 jer su one posebno važne za razdoblje razredne nastave.

Operacije s oblicima. U kurikulu primarnoga obrazovanja od velike su važnosti sadržaji simetrije, sastavljanja, presavijanja i popločavanja. Mnogi autori (npr., Franke i Reinhold, 2016; Moor i Van den Brink, 1997; Winter, 1976) naglašavaju nezamjenjivu ulogu simetrije u primarnom obrazovanju jer ona pomaže razvijanju vizualnih i prostornih sposobnosti te potiče kreativnost i maštu. Na tragu toga, Jones (2002) ističe da ključne ideje geometrije involviraju invarijantnost, simetriju i transformaciju i da stoga trebaju biti uključene u sve dijelove geometrijskoga kurikula. Već učenici 1. razreda donose iskustva sa simetrijom iz svakodnevnoga života (Franke i Reinhold, 2016). Nadalje, koncept sastavljanja i rastavljanja ključan je za rano učenje geometrije jer čini osnovu za mjerenje u višim razredima, poput pronalaženja površine, oplošja ili volumena nepravilnih oblika (Van de Walle i sur., 2019). Ipak, unatoč važnosti presavijanja i rasklapanja zbog povećanja učeničke motivacije, poticanja geometrijskoga mišljenja, vizualnih sposobnosti, rješavanja problema, komunikacije i poboljšanja fine motorike (Schipper i sur., 2015), ove aktivnosti uopće nisu uključene ni u jedan analizirani matematički kurikul u Hrvatskoj. S druge strane, ti su aspekti uključeni u mnoge kurikule diljem svijeta (npr., u Njemačkoj, Senatsverwaltung für Bildung, Jugend und Wissenschaft Berlin, 2015). Za razliku od njemačkih učenika (Kuzle i Glasnović Gracin, 2020), hrvatski sudionici uopće nisu prikazali ovu fundamentalnu ideju u svojim crtežima. Stoga bi budući kurikuli za geometriju u Hrvatskoj trebali u svoj sadržaj i ishode uzeti u obzir i potkategorije od F2.

Koordinate, prostorni odnosi i mišljenje. Pregled literature donosi snažne argumente zašto bi vizualna percepcija i prostorni zor trebali biti uključeni u kurikule primarnoga obrazovanja. Primjerice, Franke i Reinhold (2016) naglašavaju ove aktivnosti kao jedne od najvažnijih ciljeva geometrijskoga kurikula u primarnom obrazovanju, a također ističu važnost vizualne percepcije i koordinata kao preduvjeta za prostornu vizualizaciju. Ove teme podržavaju razvoj percepcije u geometriji i čine bazu za matematičko mišljenje (Schipper i sur., 2015). Konstrukt prostorne vizualizacije je sam po sebi složen jer obuhvaća različite mentalne aktivnosti, poput mentalne rotacije, prostornih odnosa i orijentacije u prostoru (Franke i Reinhold, 2016), stoga bi sve ove komponente trebale biti prisutne u kurikulu.

Geometrijski uzorci. Geometrijski uzorci smatraju se važnim dijelom ranoga geometrijskog kurikula jer ova fundamentalna ideja podržava sadržajne kompetencije u geometriji (prepoznavanje identičnih oblika i njihovih pozicija, implementacija simetrije, iskustvo s kutovima), kompetencije procesa (rješavanje problema i razvoj jezika kroz opisivanje pravilnosti i principa), opće kompetencije (fina motorika i sposobnosti percepcije), umjetničke kompetencije (mašta i kreativnost, estetski osjećaj za boje i oblike), socijalne kompetencije (uživati u nastavi Matematike kroz grupni rad i komunikaciju) te prostorne odnose (Franke i Reinhold, 2016; Schipper i sur., 2015). Osim toga, Winter (1976) je istaknuo algebarski aspekt geometrijskih uzoraka jer oni čine važnu vezu između geometrije i algebre. Stoga bi budući autori (hrvatskoga) kurikula trebali uključiti geometrijske uzorke kroz cijelo primarno obrazovanje.

Povezivanje istraživanja, ograničenja studije i buduće istraživanje

Ova je studija fokusirana na povezivanje analize dvaju hrvatskih kurikula s dvjema prethodnim studijama (Glasnović Gracin i Krišto, 2022; Glasnović Gracin i Kuzle, 2021) koje su bile usmjerene na potencijalno primijenjeni i postignuti kurikul. Njihova „karika koja nedostaje" bio je planirani kurikul koji je mogao utjecati na ostale dijelove kurikula. U ovoj se studiji analizirala prisutnost određenih fundamentalnih ideja i njihovih potkategorija u novom i starom kurikulu. Usporedba NPP-a i CUR-a uz korištenje apsolutnih i relativnih frekvencija nije bila moguća jer se ta dva dokumenta bitno razlikuju u svojoj strukturi i složenosti. Također, rezultati dobiveni u Glasnović Gracin i Krišto (2022) nisu obuhvatili sve potkategorije fundamentalnih ideja, tako da usporedba nije potpuna.

Instrument za analizu koji je korišten i modificiran u ovom istraživanju otvara mogućnost za analizu raznih kurikula diljem svijeta i daje mogućnost istraživačima koji se bave ovim područjem. Stoga buduća istraživanja mogu uključiti primjenu ovoga instrumenta na kurikule drugih zemalja, uz uključivanje problematike njihove raznolikosti i koherentnosti. Studije koje obuhvaćaju sve dijelove kurikula mogu pridonijeti dubljem razumijevanju geometrijskoga kurikula i poboljšanju pozicije školske geometrije unutar nastave Matematike.