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Regression Approach in the Evaluation of White’s Effect Magnitude in Comparison to Lightness

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Abstract: In the achromatic White's grid, the grey patches between the grey lines are perceived as darker while the same elements inserted between white lines are perceived lighter than their true measured values. A number of authors attempted to calculate the magnitude and direction of this effect using mathematical models based on multiple spatial filters. This paper uses different mathematical model, based on regression analysis, which has shown itself as an excellent tool for prediction of direction and magnitude of White's effect. The psychophysical visual experiment was conducted on 38 subjects of both genders. The differences in lightness perception ΔL_{00} were calculated in CIE ΔE_2000 system. This paper determined the functional dependence of White's effect magnitude ΔL_{00} to the lightness L of rectangular elements in White's achromatic grid. The results gave the square polynomial of very high quality (R^2 = 0,974). Regression polynomials were also found. The gave numerical values ΔL_{00} of difference in perception of left and right elements in comparison to their physical values (left R^2 = 0,943, right R^2 = 0,938) in dependence to variation of lightness parameter *L*. Results of the research clearly show the mathematical pattern of White's effect based on the lightness of rectangular elements.

Keywords: assimilation; induction; lightness; regression analysis; White’s effect

1 INTRODUCTION

White's effect illustrates a variation in the lightness of grey elements when juxtaposed with black and white lines [1-4]. This can be characterized as the grey elements assimilating with the color of the solid lines. Grey-black lines are perceived darker in comparison to grey-white lines which are perceived as lighter (Fig. 1). Numerous scientific research attempted to interpret the appearance of the effect and its magnitude.

Figure 1 White's effect

Research conducted by Anderson [5, 6] showed that White's effect has the opposite effect from simultaneous contrast. The author defined new illusions containing layer dims, which are similar to White's effect in order to evaluate elements of organizational forces influencing the calculation of surface lightness.

Blakeslee and McCourt proved that mathematical tools such as a multiscale array of two-dimensional difference-of-Gaussian (DOG) filters represent a very successful tool for the prediction of a lot of psychophysical visual effects such as grating induction, induction, assimilation and Herman's grating [7]. However, Todorović [8, 9] determined that the DOG models can't be connected to a significant group of effects that include White’s effect.

Blakeslee and McCourt [10-13] attempted to interpret effects like White’s and others, for example simultaneous brightness contrast and grating induction using multiscale spatial filtering). They also determined that the effect also appears in the early stage of cortical filtering operations in the human visual system.

Robinson, Hammon and de Sa, developed and modified ODOG models. Their re-search produced locally normalized ODOG or LODOG model [14] LODOG model enables the interpretation of a range of effects like White's zig-zag effect [15] and radial White stimulus [16]. Their results increased the efficiency of lightness prediction models. Robin-son, Hammon & de Sa developed frequency-specific locally normalized models or FLODOG models [14]. Blakeslee, Cope and McCourt generated ODOG models by using software Mathematica based on Wolfram's research [17].

White's and other psychophysical visual effects are often researched with T-junctions. T-junctions are formed at the intersection of grey rectangular element, black and white parallel lines. Margaret S. Livingstone and Piers D. Howe researched White's effect samples without T-junctions and radial samples. The magnitude of the effect is similar in samples with or without T-junctions [18]. Authors used the results to explain White's effect on Gestalt theory. They also confirm the hypothesis by Gilchrist [19] that states that the illusion can be defined with Gestalt grouping laws and the anchoring theory of lightness perception.

Lin and Chen [20] found similarities in magnitudes of White’s chromatic effect on bull's eye radial and grated samples. They determined that there is a common mechanism influencing the appearance of chromatic assimilation and White’s effect. The research made by Altschuler and others [21] showed that the chromatic White’s effect is not dependent on the geometric structure of the stimuli. Their results show the possibility of defining chromatic White’s effect using the influence of background colour or lightness on the perception of test disc colour. They also determined the possibility of simultaneous colour contrast, afterimages, metameric intransitivity and chromatic White’s effect being caused by identical neurological mechanisms. According to

Clifford and Spehar [22], the appearance of chromatic White's effect can be explained by the contrast of rectangular elements in the grate with the colour of the same grate as the assimilation effect on neighbouring grates. Their research was made in DKL colour space, and the qualitative value of the magnitude was calculated on results gained with classical achromatic White's samples.

Budimir, Mrvac and Matijević used nonlinear regression models to research regularities in defining White's effect in dependence on the percentage of grate coverage [23, 24]. Results represent a mathematical description of laws that define White's effect regarding grating coverage percent.

In order to determine regularities shown in White's effect, this paper researches different curves that show the dependence of magnitude to the lightness of the effect [25, 26]. Application of different numerical methods used in psychophysical research based on different interpolations was discussed.

Monte-Carlo simulation and parameter assessments with the method of maximal credentialing [27-29]. Methods of nonlinear regression analysis were chosen and gave excellent results [30, 31]. Results presented in this paper give analytical expressions for assessment of White effect magnitude for different lightness values of rectangular elements. The applied method differs from previously described and offers new possibilities for the study of many visual effects.

2 EXPERIMENTAL PART

2.1 Research Description

The experiment consists of two components: the instrumental (or measured) and the visual. The selected instrumental component involves the spectrophotometric measurement and the representation of CIE $L^*a^*b^*$ values. The visual component was employed to identify the pertinent fields utilizing the method of simultaneous binocular harmonization [32].

2.2 Research Description

Test sheet was designed in compliance to research methodology. Test sheet consisted of 9 achromatic variations (cards) of White's effect of identical 50% coverage, different lightness (L^*) of grey rectangular elements (Tab. 1) and reference sheet (Fig. 3). During print values of chromatic components, a^*, b^* varied, but in minute values which did not influence the research.

Table 1 Measured physical Lab values of the test sample

Sample number	L^*	a^*	b^*
1.	93,21	3,40	–7,93
2.	90,36	3,27	–8,32
3.	86,49	3,00	–6,68
4.	81,30	3,03	–7,41
5.	75,03	2,63	–6,88
6.	66,41	1,92	–6,30
7.	55,29	1,52	–6,23
8.	44,95	1,22	–5,90
9.	34,38	0,79	–5,65

Dimensions of test sheets in 183×110 mm (width×height) and the value of parallel white and black lines is identical $x = y = 6$ mm (Fig. 2).

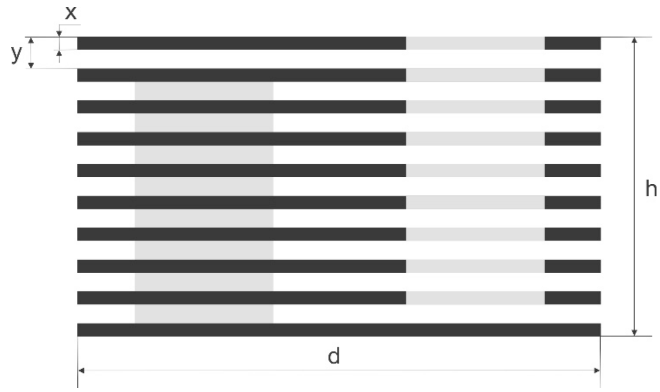

Figure 2 Test sheet dimensions

Test sheet size was created in compliance with standard observer conditions for graphic industry and professional photography (ISO 3664:2009). Conditions include 10° viewing angle and viewer distance of 60 cm according to the formula [33].

$$\tan\frac{VA}{2}=\frac{H}{2D} \tag{1}$$

VA stands for the viewing angle, H for test sample height and D for distance of the sample form the observer.

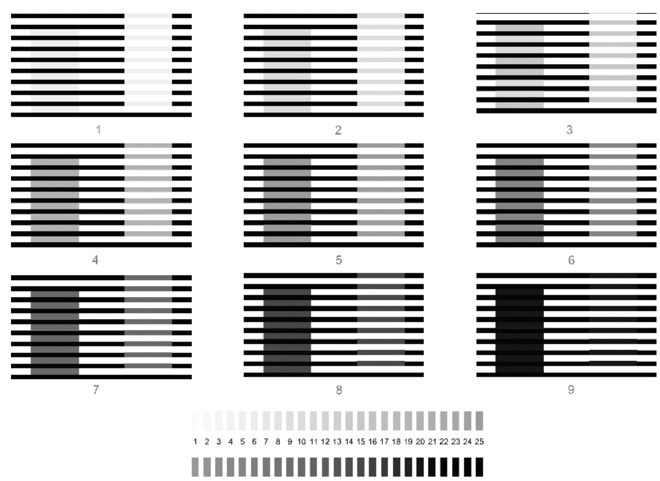

Figure 3 Created test sheets

Test sheets and reference sheets of achromatic grey were constructed in Adobe Photoshop 2020, utilizing Lab colour mode (as seen in Fig. 3). The sheet was devised to represent a variety of perceptible features in the Lab colour mode, spanning the entire perceptual range with incremental changes of 2%. Individual fields were accurately matched with the appropriate CIE $L^*a^*b^*$ values from spectrophotometric measurement.

The samples were printed using a calibrated Canon Pixma pro-100s printer. The rendering process occurred in Adobe Photoshop 2020, wherein the Lab colour mode was converted specifically for the Canon Pixma pro-100s. The conversion intent chosen was perceptive, and the conversion engine used was Adobe (ACE). Standardized Canon photo paper, with a matte finish and a weight of 170 g/m², served as the printing material. The paper underwent conditioning in a room for two days, under standardized environmental conditions with a temperature of 23 °C and a relative humidity of 55%. Finally, the samples were produced in a batch of 10 pieces.

2.3 Instrumental Analysis

In the visual segment of the experiment, 38 participants, encompassing both genders and averaging 20 years of age, were involved. Every participant had previously cleared the Ishihara test, employed to identify potential color vision impairments. Visual assessments were conducted in adherence to ISO 3664:2009 – utilizing a 10° viewing angle, maintaining a 60 cm distance between the observers and the test samples, set within natural grey surroundings, and under lighting conditions set to CIE D75 (7500K).

Visual assessment was carried out following the simultaneous binocular harmonization method. Both the test and reference sheets were positioned within the visual field concurrently. The participants were tasked with identifying the field on the test card that most closely matched the sample on the reference sheet (refer to Fig. 4).

Perceived lightness of left and right rectangular elements on the individual sheet was defined as the arithmetic centre of perceived lightness values of all subjects. Those values gave the lightness values of standard CIE observer [34].

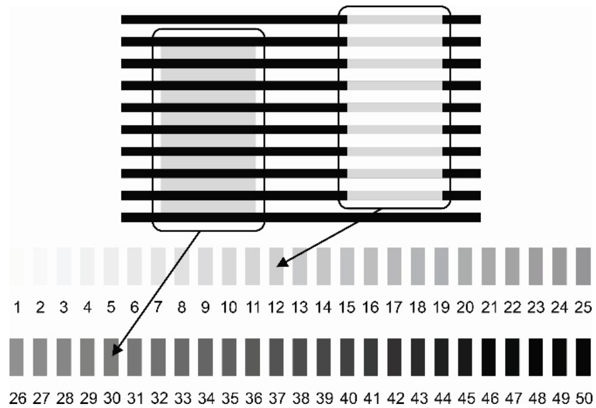

Figure 4 Visual matching principle with the reference sheet

Table 2 Descriptive statistic of perceived lightness *L*, *a* and *b* values of observed elements (left)

Descriptive statistics (left rectangular elements)											
Nr.	Lightness	Lightness numerical	Variable	Expectation	Int. conf. –95%	Int. conf. 95%	Median	Min.	Max.	Variance	Std. dev.
1.	10	93.21	*L*	93,897	93,633	94,16	94,15	91,91	94,78	0,643	0,802
			a	3,581	3,547	3,614	3,6	3	3,64	0,01	0,102
			b	–7,889	–8,52	–7,259	–8,13	–8,53	3,6	3,682	1,919
2.	20	90.36	*L*	89,854	89,195	90,513	90,1	84,66	93,5	4,02	2,005
			a	3,456	3,425	3,486	3,44	3,26	3,64	0,009	0,092
			b	–8,501	–8,554	–8,447	–8,495	–8,8	–8,18	0,027	0,164
3.	30	86,49	*L*	84,286	83,131	85,441	84,535	76,17	91,38	12,338	3,513
			a	3,293	3,257	3,328	3,3	2,85	3,45	0,012	0,108
			b	–8,046	–8,141	–7,95	–7,9	–8,57	–7,45	0,084	0,289
4.	40	81,3	*L*	74,666	73,204	76,127	74,67	67,72	81,89	19,769	4,446
			a	2,8	2,685	2,915	2,85	2,1	3,32	0,123	0,35
			b	–7,744	–7,832	–7,657	–7,83	–8,14	–7,41	0,071	0,267
5.	50	75,03	*L*	63,947	61,641	66,252	64,15	39,87	74,67	49,182	7,013
			a	1,901	1,738	2,064	1,9	0,4	2,93	0,246	0,496
			b	–7,307	–7,465	–7,148	–7,315	–8,14	–5,92	0,232	0,482
6.	60	66,41	*L*	55,434	53,35	57,519	52,99	41,98	67,72	40,21	6,341
			a	1,37	1,259	1,48	1,17	0,99	2,1	0,113	0,336
			b	–6,745	–6,864	–6,626	–6,7	–7,53	–6,18	0,131	0,362
7.	70	55,29	*L*	46,139	44,208	48,071	47,43	34,88	60,17	34,524	5,876
			a	0,916	0,815	1,018	0,99	0,4	1,91	0,096	0,309
			b	–6,35	–6,434	–6,265	–6,275	–7,1	–5,92	0,066	0,257
8.	80	44,95	*L*	37,642	36,152	39,131	37,59	30,65	49,35	20,536	4,532
			a	0,692	0,592	0,792	0,46	0,4	1,13	0,093	0,304
			b	–5,91	–6,097	–5,724	–6,11	–6,57	–4,66	0,322	0,568
9.	90	34,38	*L*	30,324	28,396	32,253	30,65	20,65	41,98	34,411	5,866
			a	1,253	1,03	1,477	1,13	0,4	2,5	0,461	0,679
			b	–4,072	–4,695	–3,45	–4,66	–6,29	–0,04	3,588	1,894

3 RESULTS

Statistical analysis of experimental data was made with program Statistica 14. The statistical analysis contains descriptive statistic of all experiment results, correlative and regressive analysis. All data was gained from the psychophysical visual experiment of 38 subjects.

3.1 Descriptive Sample Statistic

Descriptive statistic contains medium values or arithmetic middle, reliability intervals, median, minimum, maximum, variance and a standard deviance of perceived *L*, *a* and *b* of left and right analysed samples from the data gained from the experiment.

Considering the relatively low variance values and standard deviation, descriptive statistic shows the high quality of analysed subject (Tab. 2 and 3). All reliability intervals have a diameter less than 2, which means that statistical deviations are not visually perceivable.

Table 3 Descriptive statistic of perceived lightness *L*, *a* and *b* values of observed samples (right)

Descriptive statistics (right rectangular elements)											
Nr.	Lightness	Lightness numerical	Variable	Expectation	Int. conf. –95%	Int. conf. 95%	Median	Min.	Max.	Variance	Std. dev.
1.	10	93.21	*L*	93,587	93,309	93,864	93,92	90,65	94,78	0,715	0,845
			a	3,583	3,572	3,593	3,58	3,45	3,64	0,001	0,032
			b	–8,203	–8,259	–8,147	–8,13	–8,8	–8,05	0,029	0,17
2.	20	90.36	*L*	92,096	91,66	92,531	92,52	89,35	94,38	1,755	1,325
			a	3,539	3,519	3,56	3,56	3,43	3,64	0,004	0,062
			b	–8,437	–8,495	–8,378	–8,46	–8,8	–8,05	0,032	0,179
3.	30	86,49	*L*	88,851	87,964	89,738	89,725	80,83	91,91	7,282	2,699
			a	3,408	3,378	3,439	3,43	3,21	3,57	0,009	0,093
			b	–8,408	–8,489	–8,327	–8,445	–8,8	–7,79	0,06	0,245
4.	40	81,3	*L*	84,592	83,233	85,95	86,04	74,67	90,65	17,08	4,133
			a	3,25711	3,211	3,304	3,26	2,93	3,57	0,02	0,141
			b	–7,648	–8,48	–6,815	–8,18	–8,8	7,41	6,414	2,533
5.	50	75,03	*L*	79,173	77,846	80,5	79,74	69,35	88,42	16,296	4,037
			a	3,117	3,039	3,196	3,22	2,5	3,41	0,057	0,239
			b	–7,773	–7,86	–7,686	–7,835	–8,51	–7,41	0,07	0,264
6.	60	66,41	*L*	72,905	71,733	74,077	72,56	64,14	80,83	12,719	3,566
			a	2,609	2,482	2,737	2,51	1,58	3,31	0,151	0,388
			b	–7,662	–7,762	–7,562	–7,5	–8,14	–7,14	0,093	0,304
7.	70	55,29	*L*	64,717	62,306	67,129	66,05	41,98	74,67	53,823	7,336
			a	1,957	1,781	2,134	1,91	0,77	2,93	0,288	0,536
			b	–6,983	–7,758	–6,208	–7,37	–8,14	6,83	5,556	2,357
8.	80	44,95	*L*	56,827	54,913	58,74	55,94	45,64	69,35	33,876	5,82
			a	1,479	1,333	1,624	1,26	0,86	2,6	0,196	0,443
			b	–6,815	–6,979	–6,651	–6,7	–8,14	–6,26	0,248	0,498
9.	90	34,38	*L*	46,096	43,582	48,609	46,535	30,65	72,56	58,481	7,647
			a	0,892	0,759	1,025	0,86	0,4	2,5	0,164	0,405
			b	–6,283	–6,424	–6,143	–6,25	–7,83	–4,66	0,183	0,428

3.1.1 Descriptive Sample Statistic

The perceptual discrepancy induced by specific presentations of visual effects is represented as the lightness deviation ΔL_{00} between the reference sheet and the test sample [35, 36].

Difference values in perception are shown in Tab. 4.

Table 4 Difference values in lightness perception of analysed.

Lightness	Lightness numerical	ΔL_{00} (left)	ΔL_{00} (right)	ΔL_{00} (between)
10	93,21	0,42	0,23	0,19
20	90,36	–0,32	1,02	–1,4
30	86,49	–1,44	1,51	–2,95
40	81,3	–4,69	2,21	–6,89
50	75,03	–8,62	2,95	–11,56
60	66,41	–11,51	5,05	–14,52
70	55,29	–11,01	8,29	–17,48
80	44,95	–7,8	11,85	–18,78
90	34,38	–4,7	10,34	–13,54

4 DISCUSSIONS

4.1 Correlative Sample Analysis

Pearsons correlation coefficient ρ was used to perform correlative analysis of numeric lightness variable values and ΔL_{00} (left), ΔL_{00} (right) and ΔL_{00} (between). Marked correlations are significant, with significance levels $p < 0,05$ (Tab. 5).

Table 5 Results of correlative analysis

Variables	Numerical lightness values, *L*	ΔL_{00} (left)	ΔL_{00} (right)	ΔL_{00} (between)
Numerical lightness Values, *L*	1	-	-	-
	p = ---			
ΔL_{00} (left)	0,5789	1	-	-
	$p = 0,102$	p = ---		
ΔL_{00} (right)	–0,9724	–0,5737	1	-
	$p = 0,000$	$p = 0,106$	p = ---	
ΔL_{00} (between)	0,8638	0,8878	–0,8816	1
	$p = ,003$	$p = 0,001$	P = 0,002	p = ---

Statistically significant correlation value of ($p < 0,05$) was determined among the variables L, ΔL_{00} (right) and ΔL_{00} (between) and among variables ΔL_{00} (left) and ΔL_{00} (between) and among variables ΔL_{00} (right) and ΔL_{00} (between) (Tab. 5). Other correlations are not statistically significant. High positive correlation values of $\rho = 0,8638$ was determined between L and ΔL_{00} (between). High negative correlation values of $\rho = –0,8816$ were determined between ΔL_{00} (between) and ΔL_{00} (right). ΔL_{00} (between) and ΔL_{00} (left) are also highly correlated with $\rho = 0,8878$. This shows that the Munker-White effect whose heading in visible in arable value $\rho = 0,8878$ acts according to physical values of grate L and perceived difference ΔL_{00} (left) and their opposite values ΔL_{00} (right).

4.2 Regression Analysis of Elements

For conducting polynomial regression analysis, CIE ΔE_{00} differences in the lightness of the samples under consideration were calculated, as depicted in Tab. 4. Numerical values of magnitude shift lightness appearance in White's grate are presented as difference ΔL_{00}. ΔL_{00} is defined as dependent variable for regressive analysis, and physical lightness values L as independent variable. Analysis of different regression models showed that the polynomial models as most suited for available data. Left analysed elements and the difference between left and right models were described with squared regression models. For the analysis of right elements linear regression model was used.

Results of the analysis are shown in Tabs. 6, 7 and 8. The R² value is nearing its maximum value of 1 for all three models. Their values are 0,978, 0,972 and 0,990, which means that they are representative. Namely, models can interpret 97,8%, 97,2% and 99,0% of squared errors. Regression levels are read from p-values which for those models are p = 0.00008, 0.00001 and 0.00001. All three p-values are below 0,01 and that marks them as high quality. Adjusted R^2 determination coefficients have values of 0,943, 0,938 and 0,974 which is another confirmation of model quality. This parameter is contingent on the degrees of freedom. The standard error (SE) quantifies the average deviation of the data points from the regression curve. Standard error value for all three models (1,702, 1,075, 1,144) also shows their quality. Standard error of specific coefficients (Tabs. 6, 7 and 8) are relatively low and show high quality. Test shown very low p-values empirical or noticed significance) of variable coefficient P and P^2 (Tabs. 6, 7 and 8) Stated p-values are below significance level (0,001) and this model also shows the quality of regression models.

Table 6 Results of regression analysis of variable ΔL_{00} or analysed elements in dependence to lightness (left)

$N = 9$	Regression analysis of left rectangular elements lightness; square model; dependence variable L (lightness); R = 0,978 R^2 = 0,957, Adjusted R^2 = 0,943, $F(2,6)$ = 67,078, p < 0.00008, Std. error assessment: 1,702					
	b^*	Std. error of b^*	b	Std. error of b	$t(6)$	p-value
Free coefficient			26,603	4,56	5,834	0,00112
L	–5,924	0,7016	–1,271	0,15	–8,444	0,00015
L^2	6,55	0,7016	0,011	0,001	9,337	8,60E–05

Table 7 Results of the regression analysis of ΔL_{00} or analysed samples in dependence to lightness (right)

$N = 9$	Regression analysis of right rectangular elements lightness; linear model; dependence variable L (lightness); R = 0,972, R^2 = 0,946, Adjusted R^2 = 0 ,938, $F(1,7)$ = 121,66, p < 0.00001, Std. error assessment.: 1,075					
	b^*	Std. error of b^*	b	Std. error of b	$t(7)$	p-value
Free coefficient			18,798	1,316	14,281	0,000002
L	–0,972	0,088	–0,2	0,018	–11,03	0,000011

Table 8 Results of regression analysis of variable ΔL_{00} or analysed elements in dependence to lightness (between)

$N = 9$	Regression analysis of elements lightness (between); square model; dependence variable L (lightness); R = 0,990, R^2 = 0,981, Adjusted R^2 = 0, 974, $F(2,6)$=151,61, p < 0,00001, Std. error assessment: 1,144					
	b^*	Std. error of b^*	b	Std. error of b	$t(6)$	p-value
Free coefficient			9,538	4,864	1,961	0,0976
L	–3,128	0,472	–1,063	0,161	–6,623	0,000571
L^2	4,021	0,472	0,01	0,001	8,514	0,000144

As a result, of regression analysis regression polynomials give functional dependence of the intensity of the effect shift the incidence of the lightness of the lightness of analysed elements.

Squared regression polynomial (Fig. 5) was used to describe perception deviation ΔL_{00} for left elements.

$$\Delta L_{00}(L) = 26,610 - 1,276L + 0,011L^2 \tag{2}$$

Perception deviations of right elements from their physical values are calculated with linear regression polynomial (Fig. 6):

$$\Delta L_{00}(L) = 18,798 - 0,200P \tag{3}$$

Mutual deviations of perceived lightness between left and right rectangular elements (White's effect) are calculated with squared regression expression (Fig. 7):

$$\Delta L_{00}(L) = 9,538 - 1,063L + 0,010L^2 \tag{4}$$

Graphical display of regression curves and their 95% reliability intervals is shown in Figs. 5, 6 and 7.

Analysis of extreme regression polynomial for left analysed elements was calculated as:

$$\min\left\{26,610 - 1,276L + 0,011L^2\right\} = 11,2258 \text{ in } L = 59,2983 \tag{5}$$

The deviation is largest at lightness value L = 59.3. At that value the degree of deviation is 11.23. That value shows high deviation at those levels of lightness.

After At interval $L < 59{,}3$ deviation intensity grows on absolute value while it shrinks at interval $L > 59{,}3$.

Zero points of this polynomial are $L_1 = 27{,}0$ and $L_2 = 91{,}6$. That means that there is no deviation in perception at those values.

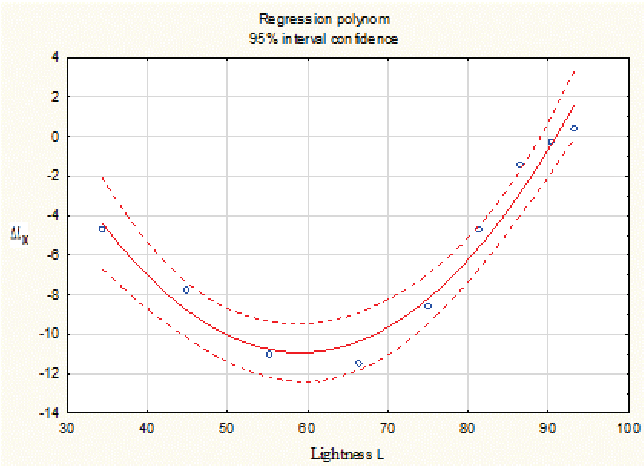

Figure 5 Polynomial regression of left elements (lightness influence)

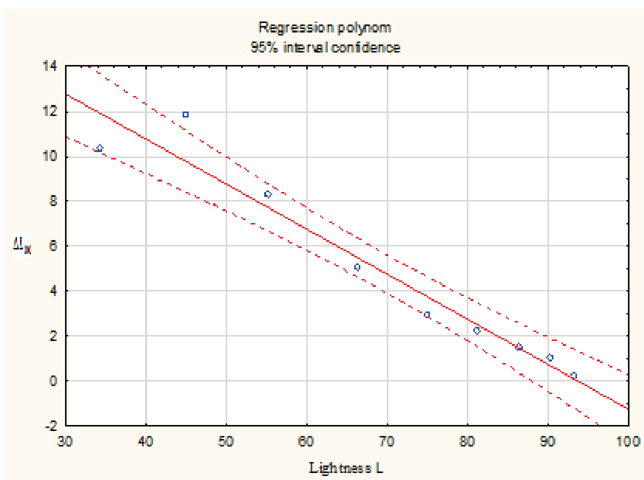

Figure 6 Polynomial regression of right elements (lightness influence)

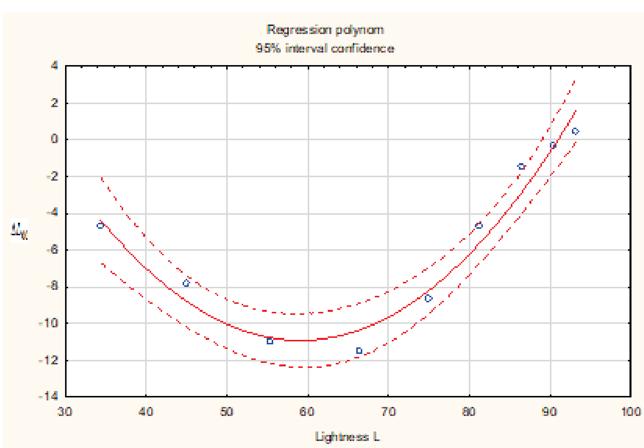

Figure 7 Polynomial regression of White's effect

The linear function describing lightness difference of right perceived elements falls on entire domain. The coefficient of this heading is –0,2. This means that an enhancement of lightness of rectangular elements for 1 unit can be expected to cause a perception shift of –0,2. Zero point of this heading is 93,81. Right rectangular elements are perceived in accordance with their physical values for that value.

Analysis of extreme polynomial difference in perception of left and right rectangular elements give the result:

$$\min\left\{9{,}538-1{,}063L+0{,}010L^2\right\}=-17{,}4702 \text{ in } L=50{,}8136 \quad (6)$$

According to that, White's effect has the highest intensity at lightness $L = 50{,}814$. In this case the intensity of White's effect, calculated through difference in perception of left and right elements is 17,47.

Intensity deviation enlarges at absolute values at interval $L < 50{,}814$, while it shrinks at interval $L > 50{,}814$.

Polynomial zero-points are:

$$L_1 = 9{,}95 \text{ and } L_2 = 91{,}65 \quad (7)$$

This White's effect is not present at lightness of rectangular elements of $L_2 = 91{,}65$.

5 CONCLUSIONS

The paper presents regression models of very high quality obtained from experiment data. Results show in what way the lightness of rectangular elements influence the magnitude of appearing psychophysical visual effect on lightness shift in White's grating. Obtained polynomials connect the shift of lightness appearance ΔL_{00} and physical lightness L of analysed elements. ΔL_{00} is defined as variable dependable of independent variable L. It is ascertainable that polynomials facilitate precise forecasts of the direction and magnitude of White's effect. The findings illustrate the principle governing the visual perception of White's effect relative to lightness. The mentioned principle reveals a notable intensity of White's effect that amplifies as the lightness of the rectangular elements nears the value $L = 50.814$. At this level of lightness, it is anticipated that a standard observer would perceive a lightness shift valued at $\Delta L = -17.4702$, a value considered to be relatively high. It was also determined that the effect itself has a much lower, but still rather high intensity in very low and very high values of lightness of rectangular elements. This is an exact, mathematical description of the White's effect, what offers many possibilities of estimating rather precisely the results of this effect for all lightness variations of the achromatic grey color of rectangular elements. It is to be expected that the effect would have similar results also in the case of coloured patterns.

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