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Dynamic analysis and comparison of single-phase induction motor with a switching and double capacitors

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SUMMARY

In order to get maximum torque in the single-phase induction motor one of the methods is to use a switching capacitor in the auxiliary winding. This configuration with an electronic switch connected across the capacitor makes possible to obtain a maximum torque by using a single capacitor. Optimal capacitor values are used to obtain the maximum torque and to improve the performance of a single-phase induction motor at steady-state and transient operations. These variable capacitor values are obtained by controlling the duty cycle of the electronic switch. In this paper the dynamic model for the single-phase induction motor with the switching capacitor has been obtained by using MATLAB/Simulink software package program. Simulation results are presented to verify the high performance operation of the single-phase induction motor at transient and steady state.

***Key words**: single-phase induction motor, switching capacitor, dynamic modelling.*

1. INTRODUCTION

Single-phase induction motors are usually low power machines and are widely used in industry and home applicants. This motor cannot run directly from the mains because of its structure. The single-phase induction motor (SPIM) is operated with auxiliary windings having inductive or capacitive characteristic (Figure 1). However, SIMPs with auxiliary windings having capacitive characteristic are usually used. This type of the motor is manufactured either with single or double capacitors. While the capacitor in the SPIM with single capacitor is taken out together with the auxiliary winding at *75%* of the synchronous speed, in the SPIM with double capacitor one of the capacitor is taken out at *75%* of the synchronous speed and the other one remains continuously in the operation. Thus, the starting and running torque of the SPIM is improved [1].

In the SPIM one of the most important problems is to determine appropriate capacitor values. The SPIM with the switching capacitor eliminates this problem. The capacitor values can be changed by the electronic switch connected across the capacitor in the auxiliary winding [2-3]. Any capacitor value can be obtained by controlling on-off time of the electronic switch. Obtaining a maximum moment in the SPIM can be considered as an optimisation problem [4]. Thus, the optimal capacitor value which is a function of speed can be obtained.

In this work the dynamic operating model of the SPIM with the switching capacitor has been obtained using MATLAB/Simulink package program [5]. Previous works in this subject have not studied the dynamic performance of the SPIM with the switching capacitor. This paper intends to fill the gap in this subject.

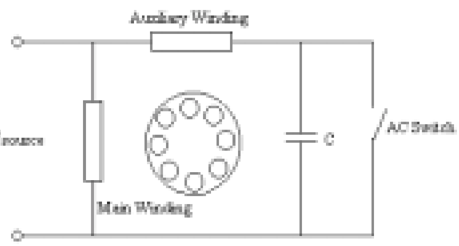

Fig. 1 Single-phase induction motor with switching capacitor

2. MATHEMATICAL MODEL

The mathematical model of the SPIM in *d-q* axis is given with next equations [6]:

$$V_{sq} = (R_{sq} + p \cdot L_{sq})i_{sq} + p \cdot L_{oq} \cdot i'_{rq} \tag{1}$$

$$V_{sd} = (R_{sd} + p \cdot L_{sd})i_{sd} + p \cdot L_{od} \cdot i'_{rd} \tag{2}$$

$$\begin{aligned} 0 = (p \cdot L_{oq})i_{sq} - (\frac{N_q}{N_d} \cdot \omega_r \cdot L_{od})i_{sd} + \\ + (R'_{rq} + p \cdot L'_{rq})i'_{rq} - (\frac{N_q}{N_d} \cdot \omega_r \cdot L'_{rd})i'_{rd} \end{aligned} \tag{3}$$

$$\begin{aligned} 0 = (p \cdot L_{od})i_{sd} - (\frac{N_d}{N_q} \cdot \omega_r \cdot L_{oq})i_{sq} + \\ + (R'_{rd} + p \cdot L'_{rd})i'_{rd} - (\frac{N_d}{N_q} \cdot \omega_r \cdot L'_{rq})i'_{rq} \end{aligned} \tag{4}$$

where: V_{sq} and V_{sd} are the *q* and *d* axis stator voltages, R_{sq} and R_{sd} are the *q* and *d* axis stator resistances, L_{sq} and L_{sd} are the *q* and *d* axis stator inductances, *p* is the differential operator (*d/dt*), i_{sq} and i_{sd} are the *q* and *d* axis stator currents, L_{oq} and L_{od} are the *q* and *d* axis mutual inductances, i'_{rq} and i'_{rd} are the *q* and *d* axis rotor currents, N_q and N_d are the *d* and *q* axis effective turns, R'_{rq} and R'_{rd} are the *q* and *d* axis rotor resistances, L'_{rq} and L'_{rd} are the *q* and *d* axis rotor inductances and ω_r is the rotor angular speed. In the *q* and *d* axis the stator and rotor inductances can be expressed as follow:

$$L_{sq} = L_{lsq} + L_{oq} \tag{5}$$

$$L_{sd} = L_{lsd} + L_{od} \tag{6}$$

$$L'_{rq} = L'_{lsq} + L'_{oq} \tag{7}$$

$$L'_{rd} = L'_{lsd} + L'_{od} \tag{8}$$

where: L_{lsq} and L_{lsd} are the *q* and *d* axis stator leakage inductances and L'_{lrq} and L'_{lrd} are the *q* and *d* axis rotor leakage inductances. The instantaneous electromagnetic torque is calculated as:

$$T_e = \frac{P}{2} \cdot \frac{N_d}{N_q} \cdot L_{oq}(i_{sq} \cdot i'_{rd} - i_{sd} \cdot i'_{rq}) \tag{9}$$

where *P* is the number of poles. The electromechanical equation of the SPIM is:

$$p \cdot \omega_r = \frac{1}{J_m}(T_e - T_l) \tag{10}$$

where J_m is the inertia constant of the motor and load and T_l is the external load. In these equations the *d* axis corresponds to the auxiliary winding and the *q* axis represents the main winding. Therefore, V_{sq} voltage will be equal to the source voltage V_{source}. The auxiliary winding voltage is expressed as:

$$V_{sd} = V_{source} - \frac{1}{C}\int i_{sd}\,dt \tag{11}$$

where V_{source} is the input voltage of the SPIM and *C* is the capacitor connected to the auxiliary winding in series.

3. DYNAMIC MODELLING

Simulink working under Matlab software package program has been used to get the dynamic model of the system. Equations (1) to (11) are used to obtain the SPIM model in *d-q* axis. Simulink is a simulation program, which uses toolboxes connected to each other. The motor model has been set up as single toolbox in Simulink. Hence, the motor parameters can be easily observed by following the toolboxes and therefore it will be simple to perform the simulation for different types of motors by modifying the related toll boxes. In the simulation there are basically three parameter groups. These are: input, output and calculation parameters. The calculation parameters contain constants required to perform the processes. The input parameters can be variable and are used in the processes in the model. The output parameters determine the results reached at the end of simulation. The inputs of the toolbox representing the SPIM are the source voltage and motor speed. The motor speed is a result of the simulation and it is also the input parameter for the motor toolbox by feedback. The output parameters of the SPIM are the *d-q* axis stator currents and motor torque. The calculation parameters of this block are the *d-q* axis stator and rotor resistors and inductances, mutual inductances, the number of poles and turn ratio, N_d/N_q. The switching capacitor connected to the auxiliary winding has been also modelled and the switching times have been calculated to get the optimal capacitor values [4]. The mechanical load fitted to the machine has been modelled and given in a separate toolbox as shown in Figure 2. While the input parameters for this toolbox are the electromagnetic torque of the SPIM and if available extra load, the output parameter is the motor speed. The inertia, viscous friction, aerodynamic friction and load torque are the calculation parameters of the toolbox for the mechanical load. This toolbox is used to calculate the motor speed using Eq. (10). Figure 2 shows the simulation model performed in Simulink [7].

The model for the switching capacitor performed in Simulink is shown in Figure 3. A pulse generator provides the switching process in the model. There are two input parameters to this pulse generator. While one of the inputs represents the duty cycle of the pulse, the other one is used to determine the pulse starting time in the period. The output of the pulse generator is multiplied by the capacitor value. As a result the capacitor will be short circuited when the pulse generator output is zero and it will be present when the pulse generator output is high.

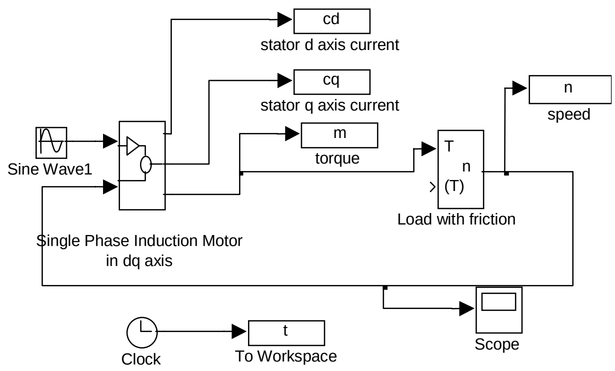

Fig. 2 Dynamic model of the SPIM drive system in Simulink

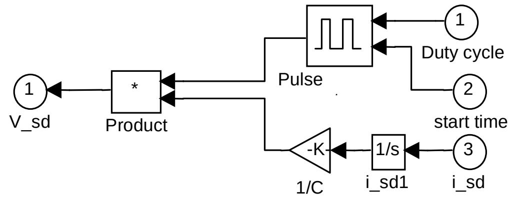

Fig. 3 Model for the switching capacitor in Simulink

4. DETERMINATION OF THE OPTIMAL CAPACITOR VALUES

According to the equivalent circuit of SPIM, the induction motor torque is expressed as:

$$\begin{aligned} T_e = &\frac{1}{\omega_e}\frac{2}{p}\Big\{\left[\left(R_{sq}+R_b+R_d\right)V_{sq-}+R_dV_{sq-}\right]^2+ \\ &+\left[\left(X_{lsq}+X_b+X_d\right)V_{sq-}+X_dV_{sq-}\right]^2\Big\}\cdot\left(R_a/\left(M_1^2+N_1^2\right)\right)- \\ &-\frac{1}{\omega_e}\frac{2}{p}\Big\{\left[\left(R_{sq}+R_a+R_d\right)V_{sq-}+R_dV_{sq+}\right]^2+ \\ &+\left[\left(X_{lsq}+X_a+X_d\right)V_{sq-}+X_dV_{sq+}\right]^2\Big\}\cdot\left(R_b/\left(M_1^2+N_1^2\right)\right) \end{aligned} \tag{12}$$

where R_a, X_a, R_b, X_b, R_d, X_d, M_1 and N_1 are dummy variables and can be expressed as:

$$R_a = \frac{\left[X_{os}^2 R'_{rq}/s\right]}{\left[\left(R'_{rq}/s\right)^2+\left(X_{os}+X_{lrq}\right)^2\right]} \tag{13}$$

$$X_a = \frac{\left[X_{os}\left(R'_{rq}/s\right)^2+\left(X_{os}+X_{lrq}\right)X_{os}X_{lrq}\right]}{\left[\left(R'_{rq}/s\right)^2+\left(X_{os}+X_{lrq}\right)^2\right]} \tag{14}$$

$$R_b = \frac{\left[X_{os}^2 R'_{rq}/(2-s)\right]}{\left[\left(R'_{rq}/(2-s)\right)^2+\left(X_{os}+X_{lrq}\right)^2\right]} \tag{15}$$

$$X_b = \frac{\left[X_{os}\left(R'_{rq}/(2-s)\right)^2+\left(X_{os}+X_{lrq}\right)X_{os}X_{lrq}\right]}{\left[\left(R'_{rq}/(2-s)\right)^2+\left(X_{os}+X_{lrq}\right)^2\right]} \tag{16}$$

$$R_d = 0.5\left[R_{sd}\left(N_q/N_d\right)^2 - R_{sq}\right] \tag{17}$$

$$X_d = 0.5X_{lsd}\left(N_q/N_d\right)^2 - 0.5X_{lsq} - X_C \tag{18}$$

$$\begin{aligned} M_1 = &\left(R_{sq}+R_a+R_d\right)\left(R_{sq}+R_b+R_d\right)- \\ &-\left(X_{lsq}+X_a+X_d\right)\left(X_{lsq}+X_b+X_d\right)- \\ &-\left(R_d^2+X_d^2\right) \end{aligned} \tag{19}$$

$$\begin{aligned} N_1 = &\left(R_{sq}+R_a+R_d\right)\left(X_{lsq}+X_b+X_d\right)+ \\ &+\left(R_{sq}+R_b+R_d\right)\left(X_{lsq}+X_a+X_d\right)- \\ &-2R_dX_d \end{aligned} \tag{20}$$

The maximum torque at various speeds is achieved by letting $dT_e/dX_c=0$ and from Eq. (13) the condition of the maximum torque can be derived as following:

$$\begin{aligned} &V_{sq}\left(M_1^2+N_1^2\right)\left(R_aS_1-R_bQ_1\right)+ \\ &+\left\{M_1\left(-2X_{lsq}-X_a-X_b\right)+N_1\left(2R_{sq}+R_a+R_b\right)\right\}\cdot \\ &\cdot\left\{R_b\left(P_1^2+Q_1^2\right)-R_a\left(T_1^2+S_1^2\right)\right\}=0 \end{aligned} \tag{21}$$

where P_1, Q_1, S_1 and T_1 are dummy variables and expressed as:

$$P_1 = \left(R_{sq}+R_a+R_d\right)V_{sq-}+R_dV_{sq+} \tag{22}$$

$$Q_1 = \left(X_{lsq}+X_a+X_d\right)V_{sq-}+X_dV_{sq+} \tag{23}$$

$$S_1 = \left(X_{lsq}+X_b+X_d\right)V_{sq+}+X_dV_{sq-} \tag{24}$$

$$T_1 = \left(R_{sq}+R_b+R_d\right)V_{sq+}+R_dV_{sq-} \tag{25}$$

As it can be seen from Eq. (21) the reactance of the capacitor varies with the speed and motor parameters. Therefore, Eq. (21) can be used to obtain maximum torque. Based on the simulated motor parameters, an optimal capacitor values versus motor speed is shown in Figure 4. This curve is used to achieve maximum torque. The effective reactance of the capacitor can be changed by controlling the duty cycle of the electronic switch. The relationship between the real and effective reactances of the capacitor is given by:

$$X_{C\,effective} = X_{C\,real}\frac{t_{off}}{T} \tag{26}$$

where t_{off} is off time of the switch and T is the switching period.

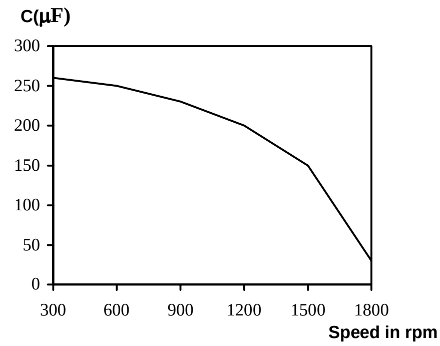

Fig. 4 Speed versus the optimal capacitor values calculated for the SPIM

5. SIMULATION RESULTS

The dynamic performances of the SPIM with the switching, double and single capacitor on no-load are shown in Figures 5 to 10. Figure 5 shows the speed characteristics for the SPIM with switching capacitor and for the SPIM with single and double capacitors in acceleration mode. As it can be seen from Figure 5 the SPIM with switching capacitor has better transient characteristic than those of the other operating conditions of the SPIM. In addition, although all the machines have the same load condition, the SPIM with switching capacitor has a higher rotation. This result shows that the SPIM with switching capacitor operates with the maximum torque at transient and steady state. Figure 6 shows a comparison for the torque between the SPIM with switching capacitor and SPIM using double capacitor at the same operating conditions. Again, the SPIM with switching capacitor reaches higher torque values than those of the SPIM with double capacitor. If the SPIMs with switching capacitor and double capacitor are examined from the point of the main winding currents (Figures 7 and 8) it will be seen that the SPIM with switching capacitor reaches faster to steady-state operation. In addition, the starting current in Figure 7 falls down in less time. However, the starting capacitor of the SPIM with double capacitor has longer effect as shown in Figure 8. Figures 9 and 10 demonstrate the auxiliary winding currents of the SPIM with switching and double capacitors in transient, respectively. In Figure 10, the starting current of the SPIM with double capacitor is high and falls suddenly down at *0.2 s* where the running capacitor (*40 μF*) remains on and starting capacitor (*300 μF*) is switched off. In the SPIM with switching capacitor, while the value of the switching capacitor is reduced with the increase of the speed according to the curve in Figure 4, the current also gets smaller as it can be seen in Figure 10. However, sudden change in the current does not occur as it does in the SPIM with double capacitor.

The dynamic performance of the SPIM in acceleration on load condition is shown in Figure 11. The acceleration time of the SPIM with single capacitor (*40 μF*) is much longer than those of the SPIM with switching and double capacitor. The SPIM with switching capacitor has better transient and steady state performances than the others because of the optimum capacitor. It has been noticed that the SPIM with single capacitor (*40 μF*) does not start running at more than *0.9 Nm* load conditions.

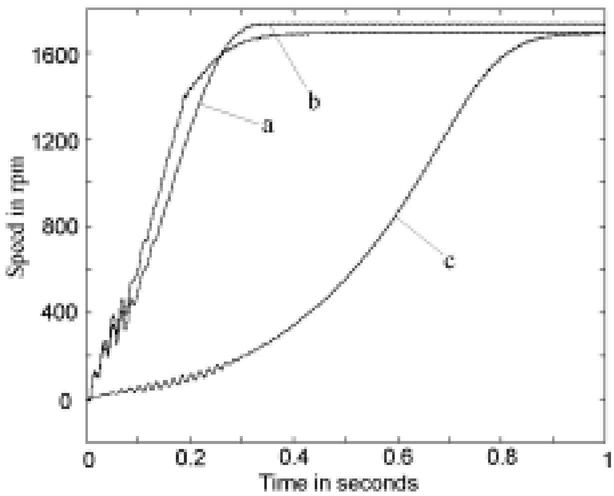

Fig. 5 Simulation results for SPIM on no-load: a) using optimum capacitor; b) using installed starting (300 μF) and running capacitor (40 μF); c) using single capacitor (40 μF)

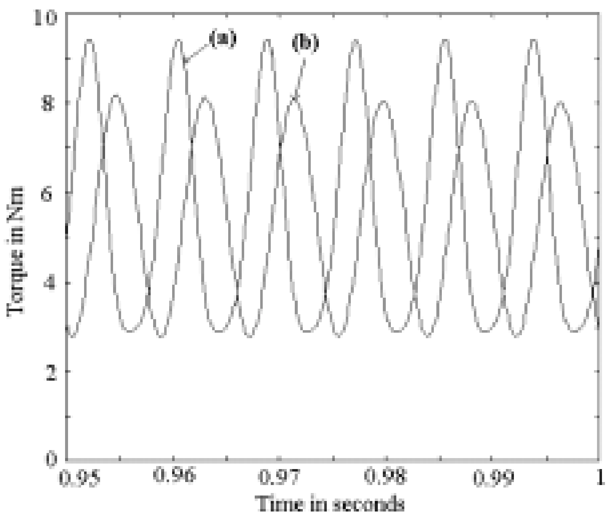

Fig. 6 Torque comparison of SPIM with (a) Switching capacitor and (b) Double capacitor

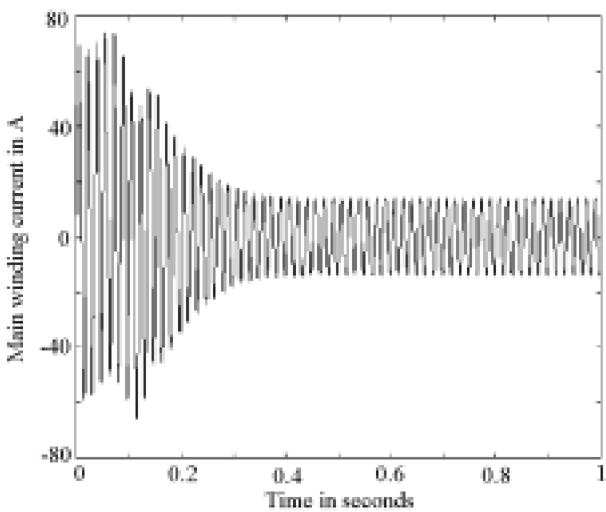

Fig. 7 Main winding current of SPIM with switching capacitor

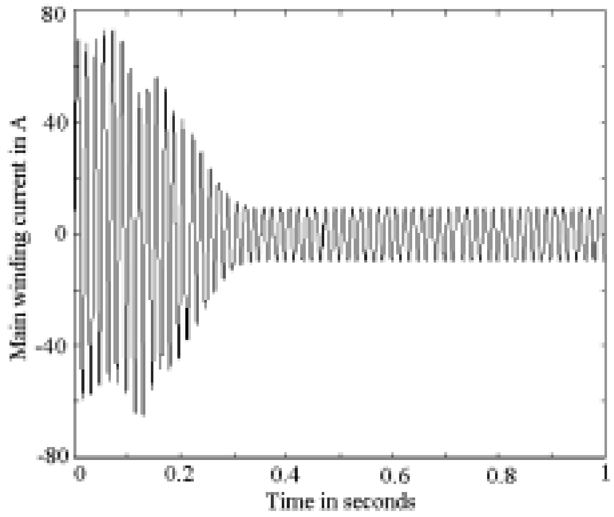

Fig. 8 Main winding current of SPIM with double capacitor

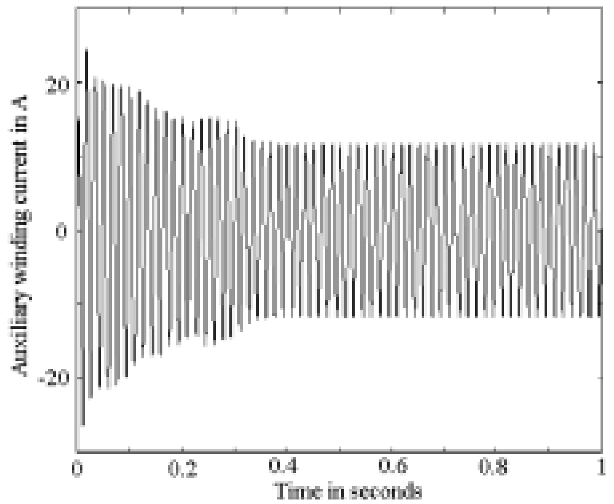

Fig. 9 Auxiliary winding current of switching capacitor SPIM

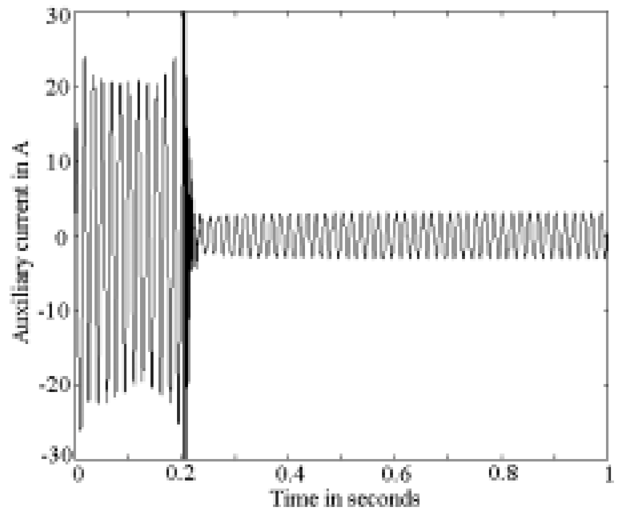

Fig. 10 Auxiliary winding current of double capacitor of SPIM

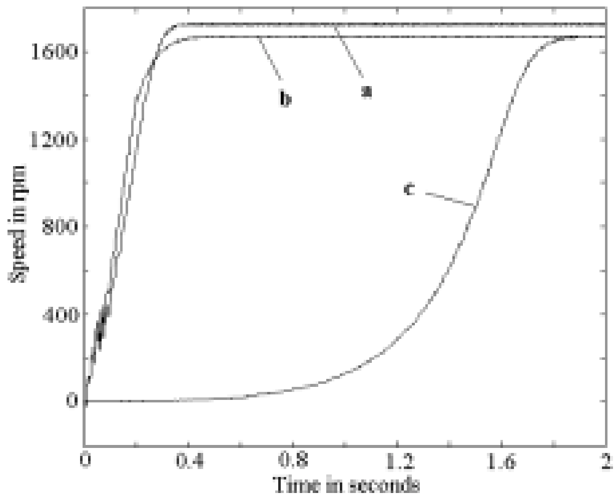

Fig. 11 Simulation results of speed of SPIM with 0.9 Nm loaded: a) using optimum capacitor; b) using installed starting (300 μF) and running (40 μF) capacitor; c) using single capacitor (40 μF)

6. CONCLUSIONS

In this paper, dynamic simulation of the SPIM with switching capacitor has been performed and the advantages of this drive system have been discussed. It has been shown that it is possible to obtain maximum torque by using optimum capacitor values in the SPIM with switching capacitor. This provides an advantage for both under starting condition with load and bigger load torque in steady state. In addition, this system does not require a centrifugal switch and uses one capacitor instead of two. Practical implementation of this work is under construction and in the implementation stage; the electronic switch will be controlled by a DSP to obtain optimum capacitor values resulting in maximum torque.

7. APPENDIX

Parameters of the SPIM:

$R_{sq}=0.542\ \Omega$	$L_{sq}=0.05027\ H$	$L_{od}=0.1348\ H$
$R_{sd}=2.4\ \Omega$	$L_{sd}=0.14185\ H$	$L_{oq}=0.0483\ H$
$R'_{rq}=0.533\ \Omega$	$L'_{rq}=0.05025\ H$	$J_m=0.0146\ Kgm^2$
$R'_{rd}=1.486\ \Omega$	$L'_{rd}=0.14016\ H$	$N_d/N_q=1.67$
$P=4$	$Load=0.02\ Nm$	

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DINAMIČKA ANALIZA I USPOREĐIVANJE JEDNOFAZNOG INDUKCIJSKOG MOTORA S PREKLOPNIM I DVOSTRUKIM KONDENZATORIMA

SAŽETAK

Da bi se dobio najveći moment zaokreta u jednofaznom indukcijskom motoru koristi se metoda uporabe preklopnog kondenzatora u pomoćnom namotu. Ovaj oblik, s elektroničkim prekidačem spojenim preko kondenzatora, omogućava dobivanje najvećeg zakretnog momenta uporabom jednog kondenzatora. Optimalne vrijednosti kondenzatora koriste se u postizanju maksimalnog momenta zaokreta i poboljšanju rada jednofaznog indukcijskog motora u ustaljenom stanju i prijelaznim radnjama. Te vrijednosti varijable kondenzatora dobiju se kontroliranjem pogonskog ciklusa elektroničkog prekidača. U ovom radu prikazuje se model za jednofazni indukcijski motor s preklopnim kondenzatorima, koristeći MATLAB / Simulink software programski paket. Predstavljeni su rezultati simulacije kako bi se potvrdile velike radne karakteristike jednofaznog indukcijskog motora u prijelaznom i ustaljenom stanju.

Ključne riječi: *jednofazni indukcijski motor, preklopni kondenzator, dinamičko modeliranje.*