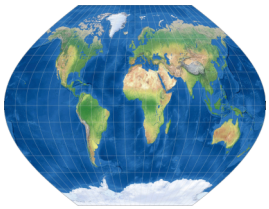
The Frančula Projections

Frančula I – Frančula V

Projection Projekcija	E_A E_A	Parenting Projection Roditeljska projekcija	Variant of Varijanta od	
Frančula I *Anlage 1*	0.3087	Sinuosidal/ Sinusna	Wagner III	
Frančula II *Anlage 2*	0.3115	Apian II	Wagner VI	
Frančula III *Anlage 3 & 4*	0.2797	American Polyconic američka polikonusna	—	
Frančula IV *Anlage 5 & 6*	0.2745	Equidistant azimuthal ekvidistantna azimutna	Wagner IX	
Frančula V * *Anlage 7*	0.3172	Equal-area azimuthal ekvivalentna azimutna	Wagner VII	

* equal-area/ * ekvivalentna
Lower values of E_A are better/ Manje vrijednosti od E_A su bolje

In the beginning of this year, I added the Györffy E projection to my website (https://blog.map-projections.net/the-francula-projections#cblox_144). Preparing that blogpost, I read Györffy's paper (Györffy 2018) where I stumbled across the mentioning of projections that "were created (...) by suitable renumbering of Aitoff and ordinary polyconic". Now, I have written about the *Umbeziffern* (renumbering, map-projections.net/wagner-umbeziffern.php) quite a lot, but I had never heard or read that this procedure had been applied to the American Polyconic. So I investigated and found out that in 1971, Nedjeljko Frančula introduced a bunch of projections obtained by *Umbeziffern* from various parent projections (Frančula 1971).

Frančuline projekcije

Početkom godine dodao sam projekciju Györffy E na moju web-stranicu (https://blog.map-projections. net /the-francula-projections#cblox_ 144). Pripremajući taj blog pročitao sam Györffyjev članak (Györffy 2018) pri čemu sam slučajno naišao na spominjanje projekcija koje su „stvorene (...) prikladnim prenumeriranjem Aitovljeve i obične polikonusne projekcije“. Iako sam dosta pisao o *Umbeziffern* (prenumeriranje, map-projections.net/wagner-umbeziffern.php), nikada nisam čuo ni pročitao da je taj postupak primijenjen na američku polikonusnu projekciju. Stoga sam istraživao i pronašao da je Nedjeljko Frančula 1971. godine predstavio veći broj projekcija dobivenih prenumeriranjem iz različitih parova projekcija (Frančula 1971). I evo ih ovdje. Četrnaest projekcija koje minimiziraju distorzije primjenom kriterija Airyja i Airy-Kavrayskoga.

Proslov

Prije nego što počnemo gledati projekcije, dozvolite mi da skinem nekoliko stvari sa srca ...

- Airyjev kriterij? Kriterij Airy-Kavrayskoga? Što je to?
  Pa, neću to objašnjavati. Kažimo samo da su to mjere za izražavanje količine distorzija kartografskih projekcija. Teorijski, karta s malom vrijednosti E_A (Airy) ili E_{AK} (Airy-Kavrayskiy) imat će malu distorziju kutova i površina. Postoje i druge mjere i raspravljalo se o tome koja je najbolja. Frančula je odlučio upotrijebiti kriterije Airyja i Airy-Kavrayskoga i razvio projekcije s minimalnim distorzijama prema *tim* mjerama. Moguće je da bi prošle lošije prema nekim drugim mjerama.
- Osim uobičajenih ilustracija s pomoću Tissotovih indikatrisa, mogu prikazati drukčiju vizualizaciju distorzija za samo dvije projekcije. To malo smeta jer Frančuline projekcije minimiziraju distorzije. Međutim, možete pogledati Frančulin izvorni tekst koji sadrži vizualizacije distorzija kutova i površina. Tekst je na njemačkom jeziku, ali to vas ne bi trebalo spriječiti da pregledavate slike.
  U tablicama dajem brojeve za *Anlage* (= privitak, oni počinju na stranici 72) u Frančulinu radu gdje možete pronaći vizualizacije. Kada postoje dva broja, prva slika prikazuje distorzije površina, a druga kutova. Nema slike za projekciju Frančula X. Projekcije V i XIV, su ekvivalentne, pa naravno imaju samo distorzije kutova. Ne znam zbog čega su slike distorzija kutova izostavljene za neke druge projekcije.
- Imena Frančula I do Frančula XIV nisu utvrđene oznake. Smislio sam ih kad sam pripremao ovaj blogpost. Redni brojevi odgovaraju redoslijedu kojim su navedeni u Frančulinu radu.
- Frančuline projekcije izvedene su s pomoću prenumeriranja (*Umbeziffern*) iz pet "roditeljskih" projekcija. Četiri od njih je već upotrijebio Wagner, pa ja odgovarajuće Frančuline nazivam “Wagnerovim varijantama”.
- Frančula je izjavio da su projekcije razvijene kako bi se koristile kao općenite referentne karte u atlasima, ali je istaknuo i da bi mogle poslužiti također i za tematske karte. Stoga izgledaju manje-više poput svih karata na koje smo navikli: karte svijeta s linijom pola, u uspravnom aspektu, bez prekida. Ne očekujte ništa neobično ili iznenađujuće!
- I na kraju, ali ne manje važno: Bez pomoći Petera Dennera ne bih vam mogao pokazati ove projekcije. Na temelju moje privremene provedbe prilagodljivog Wagnera IX, razvio je konačnu verziju koja ne samo da je općenito poboljšana, već je ponudio i mogućnost primjene *Umbeziffern* na druge roditeljske projekcije. Mnogo hvala Peter! Primjena d3-geo-projekcija bit će uskoro posebno prikazana.

Nakon ovog uvodnog govora konačno možemo pogledati projekcije.

Projekcije Frančula I do Frančula V su optimirane i daju minimalne distorzije u skladu s kriterijem Airyja (E_A); manje vrijednosti pokazuju manju distorziju i bolje su. Projekcije I do IV imaju jednako razmaknute paralele (uzduž srednjeg meridijana) i nisu ni konformne ni ekvivalentne, br. V je ekvivalentna.

Iako pseudocilindrične projekcije I i II imaju male vrijednosti distorzije, izgledaju (po mom mišljenju) prilično neugodno zbog istezanja u smjeru sjever-jug. Mnogo su mi draže projekcije III i IV, a pokazuju i bolje vrijednosti E_A. Budući da su donekle slične na prvi pogled, evo poveznice na njihovu izravnu usporedbu: https://map-projections.net/compare.php?p1=francula-3&p2=francula-4&w=1&sm=1

Frančula V pokazuje najveću vrijednosti distorzije, ali ima često korisno svojstvo ekvivalentnosti. Usput, moj eksperiment kojeg sam nazvao Wagner BCW-E (https:// map-projections.net/img/jpg-w/wagner-var-67-85-60-0-198.jpg) je donekle sličan (ali stvarno nisam ništa znao o Frančulinim projekcijama kad sam to smislio).

And here they are. Fourteen projections, minimizing distortions by application of the Airy and the Airy-Kavrayskiy criterion.

A few words in advance

Before we start looking at the projections, let me get a few things off my chest...

- Airy criterion? Airy-Kavrayskiy criterion? What is that?
  Well, I am not going to explain that. Let us just say that these are metrics to express the amount of distortions in a map projection. In theory, a map with a low E_A (Airy) or low E_{AK} (Airy-Kavrayskiy) value will have low angular and areal distortions. There are other metrics and it has been argued upon which is best. Frančula decided to use the Airy and Airy-Kavrayskiy and developed projections with minimal distortions according *these* metrics. It is possible that they fare worse according to other metrics.
- Apart from the usual images showing Tissot's indicatrix, I can show another visualization of distortions only for two of the projections. That is a bit of a nuisance since Frančula's projections are about the minimization of distortions. However you can refer to Frančula's original text containing visualizations of angular and areal distortions. The text is in German but that should not keep you from examining images.
  In the tables, I will provide the numbers of the *Anlage* (= attachment, they start at page 72) in Frančula's paper where you can find the visualizations. When there are two numbers, the first image shows areal, the second angular distortions. There is no image for the Frančula X. Projections V and XIV, being equal-area, of course only have angular distortions. I do not know why the image for angular distortions also were omitted for some other projections.
- The names Frančula I to Frančula XIV are no established designations. So I just came up with them when I prepared this blogpost. The ordinal numbers correspond to the order in which they are listed in Frančula's paper.
- The Frančula projections are derived by *Umbeziffern* from five parenting projections. Four of them have already been used by Wagner, so I am referring to the corresponding Frančulas as "Wagner variants".
- Frančula stated that the projections were developed to be used as general reference maps in atlases, but pointed out that they also might serve well for thematic maps. Thus, they look more or less like all the maps we are used to: World maps with a pole line, in equatorial aspect, without interruptions. Don't expect anything unusual or surprising!
- And last but not least: Without the help from Peter Denner, I would not have been able to show you this projections. Based on my provisional implementation of the customizable Wagner IX he developed a final version that not only was generally improved but also offered the option to apply the *Umbeziffern* to other parent projections. Thanks a lot, Peter! The implementation for d3-geo-projections will be presented separately sometime soon.

After this opening speech we finally can look at the projections.

The projections Frančula I to Frančula V were optimized for minimal distortion according to the Airy criterion (E_A); lower values indicate lower distortion and are better. I to IV have equally spaced parallels (along the central meridian) and belong to the type of aphylactic (compromise) projections, No. V is equal-area.

While pseudocylindric projections I and II do have low distortion values, they look (in my opinion) quite unpleasing to the eye because of the stretch in north-south-direction. I much prefer the lenticular projections III and IV, they also show better E_A values. Since they are somewhat similar at first glance, here is the link to compare them directly: https://map-projections.net/compare.php?p1=francula-3&p2=francula-4&w=1&sm=1

Frančula V shows the highest distortions values but has the often-useful property of equivalence. By the way, my own experiment I called Wagner BCW-E (https://map-projections.net/img/jpg-w/wagner-var-67-85-60-0-198.jpg) is somewhat similar (but I promise that I knew nothing about Frančula's projections when I came up with it).

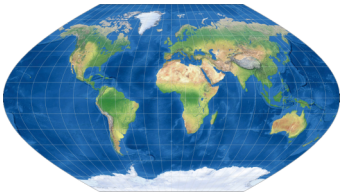
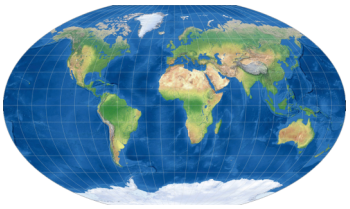
The projections VI to IX were developed using the Airy criterion, too. They are based on parent projection, we have already seen on Frančula I to III.

The first variants have a standard parallel (and thus, the area of lowest distortions) around 60° North/South. This not only explains their "stretched" appearance but also leads – despite their low overall distortion – to an unfavorable *distribution* of distortions. In order to fix this, Frančula set the standard parallel for the projections VI and VII to 40° North/South and adjusted the length of the pole line in order to reach minimum values of E_A in this configuration. The result indeed looks better to me, and probably fits better to the intended usage as general-references maps.

On the other hand, the variations of Francula III, i.e. Frančula VIII and IX, make me frown. The idea was the same, namely to enhance the distribution of distortions. But... Firstly, they look less attractive to me than No. III. Well, that is an objective, purely aesthetic judgement, so scratch that.

Secondly, I am not so sure whether the distribution of distortions is actually better. It is true, the areal distortions in regards of the nominal(!) scale of the map seem more balanced, but I think the distribution of angular distortions is even a bit worse. But more importantly I don't understand why the Frančula VIII is listed at all, when IX looks almost identical but has a lower E_A value. I just would have discarded the VIII. Well, I don't know the reason, and if it is mentioned in the paper, I managed to read over it so far. However:

Frančula VI – Frančula IX

Projekcija Projection	E_A E_A	Roditeljska projekcija Parenting Projection	Varijanta od Variant of	
Frančula VI *Anlage 9 & 10*	0.3825	Sinusna/ Sinuosidal	Wagner III	
Frančula VII *Anlage 12 & 13*	0.3830	Apian II	Wagner VI	
Frančula VIII *Anlage 15 & 16*	0.3303	američka polikonusna American Polyconic	—	
Frančula IX *Anlage 18 & 19*	0.3218	američka polikonusna American Polyconic	—	

Projekcije VI do IX su također razvijene prema Airyjevu kriteriju. Utemeljene su na roditeljskim projekcijama koje smo već vidjeli kod projekcija Frančula I do III.

Prve varijante imaju standardnu paralelu (i prema tome područje najmanjih distorzija) oko 60° sjeverno/južno. To ne samo da objašnjava njihov „rastegnuti" izgled već i dovodi – unatoč njihovoj manjoj ukupnoj distorziji – do nepovoljne *raspodjele* distorzije. Da bi to popravio, Frančula je postavio standardnu paralelu za projekcije VI i VII na 40° sjeverno/južno i prilagodio duljinu crte pola kako bi se postigle minimalne vrijednosti E_A u toj konfiguraciji. Rezultat mi zaista izgleda bolje i vjerojatno se bolje uklapa u namjeravanu upotrebu za karte za opću upotrebu.

S druge strane, s varijantama projekcije Frančula III, tj. Frančula VIII i IX, nisam oduševljen. Ideja je bila ista, naime da se poboljša distribucija distorzija. Ali ... Prvo, izgledaju mi manje privlačno od br. III. No, to je sasvim estetska prosudba, pa zaboravite to.

Kao drugo, nisam toliko siguran je li raspodjela distorzija zapravo bolja. Istina je, distorzija površina u pogledu nominalnog (!) mjerila karte izgleda uravnoteženija, ali mislim da je raspodjela distorzija kutova čak i malo gora. Ali što je još važnije, ne razumijem zašto je projekcija Frančula VIII uopće navedena, kad IX izgleda gotovo identično, a ima manju vrijednost E_A. Jednostavno bih odbacio VIII. Pa, ne znam razlog, a ako je spomenut u radu, do sada ga nisam uspio pročitati. Međutim:

Usporedite Frančula VIII i IX (https://map-projections.net/compare.php?p1=francula-8&p2=francula-9&w=1&sm=1) – morate dobro pogledati kako biste vidjeli razlike.

Da biste pregledali promjene primijenjene na prvu izvedbu:

Usporedite Frančula III i IX (https://map-projections.net/compare.php?p1=francula-3&p2=francula-9&w=1&sm=1)

Prije nego što prijeđem na sljedeći odjeljak, evo tablice svih Frančulinih projekcija koje smo do sada imali i nekoliko drugih poznatih projekcija, poredanih po njihovoj vrijednosti E_A, od najbolje do najgore. Sve vrijednosti su kopirane iz Frančulina rada.

Preostalih pet projekcija izvedeno je od istih roditelja kao i prije, ali su optimizirane za male vrijednosti distorzija prema kriteriju Airy-Kavrayskoga. Po mom mišljenju, pseudocilindrične (X i XI) izgledaju puno bolje od svojih parova dobivenih po Airyjevu kriteriju. Teže mi se odlučiti za lentikularne (lećaste): izgledaju drugačije, ali obje varijante (Airy i Airy-Kavrayskiy) prilično su ugodne.

A table of Frančula projections sorted by their E_A value best to worst.
Tablica Frančulinih projekcija poredanih po njihovoj vrijednosti E_A od najbolje do najgore.

Projection Projekcija	E_A E_A
Frančula IV	0.2745
Frančula III	0.2797
Frančula I	0.3087
Frančula II	0.3115
Frančula V *	0.3172
Frančula IX	0.3218
Winkel Tripel (Orig.)	0.3280
Frančula VIII	0.3303
Winkel Tripel	0.3674
Bartholomew	
Frančula VI	0.3825
Frančula VII	0.3830
Eckert V	0.4012
Eckert VI	0.4025
Eckert IV *	0.4068
Kavraiskiy VII	0.4126
Mollweide *	0.4419
Hammer *	0.4654
Aitoff	0.5041
Sinusoidal *	0.5581

* *equal-area*/ * *ekvivalentna*
Lower values of E_A are better/
Manje vrijednosti za E_A su bolje

Compare Frančula VIII and IX (https://map-projections.net/compare.php?p1=francula-8&p2=francula-9&w=1&sm=1) – you have got to look closely to see the differences.

To inspect the changes applied to the first rendition:

Compare Frančula III and IX (https://map-projections.net/compare.php?p1=francula-3&p2=francula-9&w=1&sm=1)

Before me move on to the next section, here is a table of all the Frančula projections we have had so far and a few other well-known projections, sorted by their E_A value, best to worst. All values were copied from Frančula's thesis.

The final five projections are derived from the same parents as before but were optimized for low distortion values according to the Airy-Kavrayskiy criterion. In my opinion, the pseudocylindricals (X and XI) look much better than their Airy criterion counterparts. It is harder for me to decide on the lenticulars: They look different of course but both the Airy and the Airy-Kavrayskiy variants are quite pleasant.

Although derived from different parent projections, Frančula XII and XIII look somewhat similar.

Compare them directly to spot the differences (https://map-projections.net/compare.php?p1=francula-12&p2=francula-13&w=1&sm=1)

And again, the equal-area projection (XIV) has the highest distortion values but they still are quite good. Apart from its more pronounced North-South stretch, it is very similar to a variant Wagner himself introduced in 1941 which I have labelled Wagner VII.d:

Compare Wagner VII.d and Frančula XIV (https://map-projections.net/compare.php?p1=wagner-7d&p2=francula-14&w=1&sm=1)

And here is the table of the E_{AK} values of Frančula X to Frančula XIV, compared to the same well-known projections as before, plus Robinson and Eckert III. The E_{AK} for Robinson and Eckert III are not taken from Frančula's but from Györffy's paper (Györffy 2018). Note that Györffy calculates his E_K differently but the values can be converted to each other by dividing Györffy's E_K by the square root of 2 – or vice versa, by multiplying Frančula's E_{AK} with √2.

A few Words on Distortions

While I was writing this blogpost, it was pointed out to me that *"unlike the Airy-Kavrayskiy criterion, the Airy criterion gives different results for horizontal compression / vertical stretching than for horizontal stretching / vertical compression by the same factor"*. Which is why Frančula I to V, VIII and IX all seem vertically stretched (or horizontally compressed), especially the first two (VI and VII are exceptions here because Frančula deliberately deviated from the principle of achieving the lowest possible value of E_A).

Just mentioning that because it explains a lot. As I said in the beginning, currently I can show the usual visualization of distortions only for two projections, namely Frančula V and XIV. Since both are equivalent, there are only angular distortions. For comparison, I add the original Wagner VII. Lighter hues mean less distortion. Darker hues mean more distortion. The red lines mark the maximum angular distortion of 40° and 80° (Figures 1-3).

Well, it is always the same when you change parameters of a projection: Some areas win, others loose; and it is hard to say which variation is the best in total (basically, that is why people came up with such metrics as the Airy and Airy-Kavrayskiy criteria). So I will just say that regarding the distribution of distortions I think all three of them are appropriate for atlas cartography.

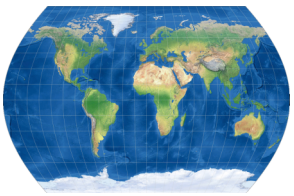
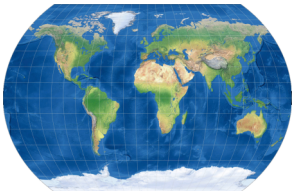
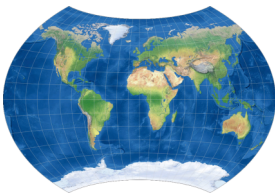
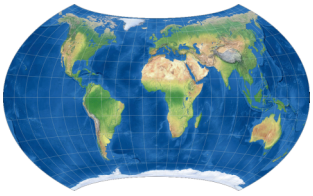
Résumé

Frančula introduced 14 projections with low distortion values. Do they fulfil the claim of being "advantageous projections for atlas cartography"?

Here's my personal opinion: Frančula I and II: Well, they might be useful if you really feel the need to cram a pseudocylindric projections into a tight space. I probably would not recommend them for general reference maps but *maybe* for one or the other thematic map. Although I have to say that in this case, I don't see any advantages over known projections (e.g. Wagner III with corresponding standard parallels).

Frančula III and IV, VIII and IX: There is still quite a vertical stretch, but aesthetically speaking, I like them nonetheless. I think they are suitable for atlas cartography *if* you want to save some space horizontally. However in the atlases I know there is no need to save space in this direction. Therefore, I would rather use them for thematic maps (usually displayed smaller) than for the main maps (which tend to be given a lot of space). In addition, they should also

Frančula X – Frančula XIV

Projekcija Projection	E_{AK} E_{AK}	Roditeljska projekcija Parenting Projection	Varijanta od Variant of	
Frančula X	0.2498	Sinusna/ Sinuosidal	Wagner III	
Frančula XI *Anlage 21*	0.2490	Apian II	Wagner VI	
Frančula XII *Anlage 22 & 23*	0.2388	američka polikonusna American Polyconic	—	
Frančula XIII *Anlage 25 & 26*	0.2359	ekvidistantna azimutna Equidistant azimuthal	Wagner IX	
Frančula XIV * *Anlage 28*	0.3177	ekvivalentna azimutna Equal-area azimuthal	Wagner VII	

* ekvivalentna/ **equal-area*
Manje vrijednosti za E_A su bolje/Lower values of E_A are better

Iako izvedene iz različitih roditeljskih projekcija, Frančula XII i XIII izgledaju donekle slično.

Usporedite ih izravno kako biste uočili razlike (https://map-projections.net/compare.php?p1=francula-12&p2=francula-13&w=1&sm=1)

I opet, ekvivalentna projekcija (XIV) ima najveće vrijednosti distorzije, ali i dalje je prilično dobra. Osim izraženijeg rastezanja u smjeru sjever-jug, vrlo je slična varijanti koju je sam Wagner predstavio 1941. godine, a koju sam označio kao Wagner VII.d:

Usporedite Wagner VII.d i Frančula XIV (https://map-projections.net/compare.php?p1=wagner-7d&p2=francula-14&w=1&sm=1)

Tu je i tablica s vrijednostima E_{AK} za projekcije Frančula X do Frančula XIV, uspoređene s istim dobro poznatim projekcijama kao prije, uključujući Robinsonovu i Eckert III. E_{AK} za Robinsonovu i Eckert III nisu preuzete od Frančule nego iz Györffyjeva članka (Györffy 2018). Uočimo da je Györffy računao E_K na drugi način, ali vrijednosti se mogu transformirati jedna na drugu dijeljenjem Györffyjeva E_K s drugim korijenom iz 2 – ili obratno, množenjem Frančulina E_{AK} s $\sqrt{2}$.

Nekoliko riječi o distorzijama

Dok sam pisao ovaj blogpost, istaknuto mi je da *"za razliku od kriterija Airy-Kavrayskoga, Airyjev kriterij daje različite rezultate za vodoravno stiskanje / vertikalno rastezanje nego za vodoravno rastezanje / vertikalno stiskanje istim faktorom"*. Zbog toga se sve Frančuline projekcije od I do V, VIII i IX doimaju vertikalno rastegnutima (ili vodoravno stisnutima), pogotovo prve dvije (VI i VII su iznimke jer je Frančula namjerno odstupio od načela postizanja najniže moguće vrijednosti E_A).

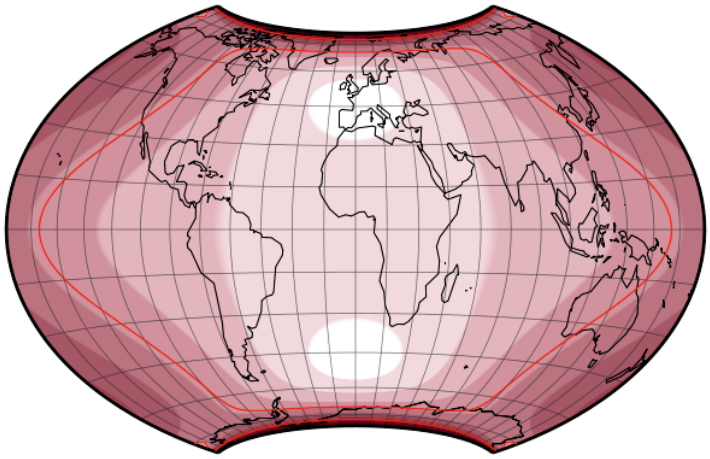


Fig. 1 Distribution of distortion of angles in the Frančula V projection.
Slika 1. Raspored distorzija kutova projekcije Frančula V.

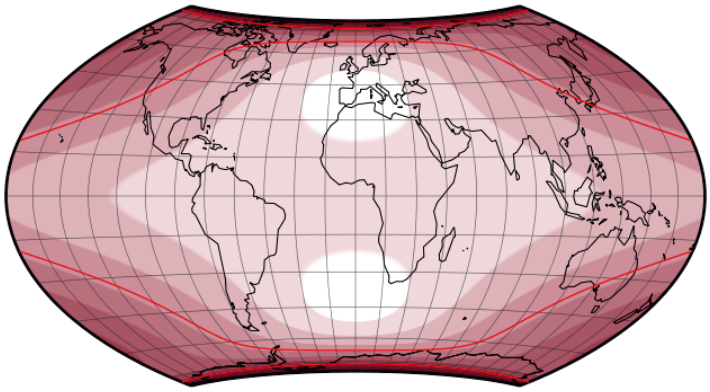


Fig. 2 Distribution of distortion of angles in the Wagner VII projection.
Slika 2. Raspored distorzija kutova projekcije Wagner VII.

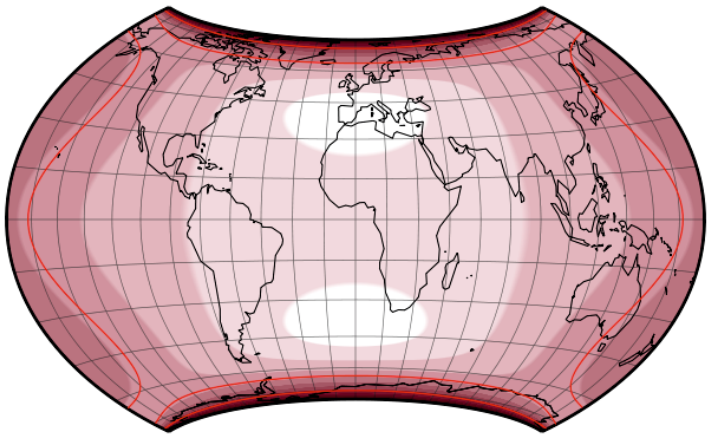


Fig. 3 Distribution of distortion of angles in the Frančula XIV projection.
Slika 3. Raspored distorzija kutova projekcije Frančula XIV.

be suitable for general reference wall maps where aesthetic considerations might be more important than in an atlas.

Frančula VI and VII, X to XIII: Now we are talking. I think they are very usable for general reference maps, both in atlases and on wall maps. I prefer the series that was optimized according to the Airy-Kavrayskiy criterion and among them, the lenticular projections.

The equal-area projections Frančula V and XIV: Both are suitable for the cause. Each has its own pros and cons, all in all I prefer No. XIV.

Résumé of the résumé: Except for Frančula I and II, I think all projections are suitable for atlas cartography. Some more than others, in terms of the usual layout of atlases I know. But even those "others" might prove valuable in certain situations. In any case, they are interesting alternatives to the "usual suspects" like Robinson, Winkel Tripel etc.

Györffy's Comment Regarding the Shape

In the beginning I said that my attention was drawn to the Frančula projections when I read Györffy's paper (Györffy 2018). What he actually said about them was this:

(...) some projections were created representing the poles as concave curves (bent towards the equator) by suitable renumbering of Aitoff and ordinary polyconic, with even lower EK values (...) They are ignored in the praxis of cartography because of their appearance.

(...)

The shape of the mapped Earth in the renumbered Aitoff and ordinary polyconic (...) suggest that a minimized distortion projection, representing the pole as a line, generates an outline with concave pole lines (...) The unfamiliar shape of the mapped Earth provokes the neglect of these projections.

Apparently it is true that the Frančula projections so far have been "ignored in the praxis of cartography" (at least I don't think that I have ever seen them anywhere), but I do not agree with the reason he is giving for this. Their appearance merely is *one of the possible* reasons – but in my opinion there is no evidence that it is the actual reason. I think it is just as likely that the Frančula projections rarely have been used because only very few people actually know about them.

To spominjem samo zato jer puno toga objašnjava. Kao što sam rekao na početku, trenutačno mogu pokazati uobičajenu vizualizaciju distorzija samo za dvije projekcije, za projekcije Frančula V i XIV. Budući da su obje ekvivalentne, postoje samo distorzije kutova. Za usporedbu, dajem izvornu projekciju Wagner VII. Svjetlija boja znači manju distorziju. Tamnija boja znači veću distorziju. Crvene crte označavaju maksimalnu distorziju kutova od 40° i 80° (slike 1-3).

Pa, uvijek je tako kad mijenjate parametre projekcije: neka područja pobjeđuju, druga gube; i teško je reći koja je varijanta ukupno najbolja (u osnovi, zato su ljudi smislili takve mjere kao što su kriteriji Airyja i Airy-Kavrayskoga). Tako ću samo reći da mislim da su sve tri prikladne za atlasnu kartografiju s obzirom na raspodjelu distorzija.

Résumé

Frančula je uveo 14 projekcija s malim vrijednostima distorzija. Ispunjavaju li tvrdnju da su "povoljne projekcije za atlasnu kartografiju"?

Evo mog osobnog mišljenja: Frančula I i II: Pa, mogle bi biti korisne ako zaista osjećate potrebu da pseudocilindrične projekcije strpate u uski prostor. Vjerojatno ih ne bih preporučio za opće karte, ali *možda* za neku tematsku kartu. Iako moram reći da u ovom slučaju ne vidim nikakve prednosti nad poznatim projekcijama (npr. Wagner III s odgovarajućim standardnim paralelama).

Frančula III i IV, VIII i IX: Još uvijek postoji prilično vertikalno rastezanje, ali estetski gledano, ipak mi se sviđaju. Mislim da su prikladne za atlasnu kartografiju *ako* želite uštedjeti malo prostora vodoravno. Međutim, znam da u atlasima nema potrebe za uštedom prostora u tom smjeru. Stoga bih ih radije koristio za tematske karte (obično se prikazuju manje) nego za glavne karte (kojima se obično daje puno prostora). Uz to, one bi također trebale biti prikladne za općenite zidne karte gdje bi estetska razmatranja mogla biti važnija nego u atlasu.

Frančula VI i VII, X do XIII: Sad razgovaramo. Mislim da su te projekcije vrlo korisne za opće karte, kako u atlasima, tako i za zidne karte. Više volim seriju koja je optimirana prema kriteriju Airy-Kavrayskoga, a među njima lentikularne projekcije.

Ekvivalentne projekcije Frančula V i XIV: Obje su pogodne. Svaka ima svoje prednosti i nedostatke, sve u svemu više volim br. XIV.

Résumé résuméa: Osim projekcija Frančula I i II, mislim da su sve projekcije prikladne za atlasnu kartografiju. Neke više od drugih, u smislu uobičajenog izgleda atlasa kakve poznajem. Ali čak i one "druge" mogu se pokazati vrijednima u određenim situacijama. U svakom su slučaju zanimljive alternative "uobičajenim sumnjivcima" poput Robinsonove, Winkel Tripel itd.

Györffyjev komentar u pogledu oblika

Na početku sam rekao da su mi pažnju privukle Frančuline projekcije kada sam čitao Györffyjev članak (Györffy 2018). Ono što je on zapravo rekao o njima je ovo:

(...) kreirane su neke projekcije koje prikazuju polove kao konkavne krivulje (savijene prema ekvatoru) prikladnom prenumeracijom Aitovljeve i obične polikonusne projekcije, s još manjim vrijednostima E_K (...). One su u kartografskoj praksi ignorirane zbog svog izgleda.

Oblik preslikane Zemlje pri prenumeriranoj Aitovljevoj i običnoj polikonusnoj projekciji (...) sugeriraju da projekcija minimalne distorzije, koja prikazuje pol kao liniju, generira obris s udubljenim linijama pola (...). Neobični oblik preslikane Zemlje izaziva zanemarivanje tih projekcija.

Očito je istina da su dosad Frančuline projekcije bile "ignorirane u kartografskoj praksi" (barem mislim da ih nikada nisam vidio), ali ne slažem se s Györffyjevim razlogom za to. Njihova pojava samo je *jedan od mogućih* razloga – ali po mom mišljenju nema dokaza da je to stvarni razlog. Mislim da je jednako vjerojatno da su se Frančuline projekcije rijetko koristile jer zapravo vrlo malo ljudi zna za njih. Međutim, ako vam se u osnovi sviđa npr. Frančula XIII, ali mrzite udubljene linije polova ... samo odrežite polarna područja (slike 4-5):

Mogli biste tvrditi da je takvo rezanje loša praksa, s čime se ja obično slažem (čak i zanemarujući činjenicu da bez obzira na to volim udubljene crte polova). Međutim, vidio sam lošija rezanja u atlasima i na zidnim kartama. Pa čak i kada mi se praksa ne sviđa, moram priznati da ne biste odsijekli ništa što je zaista važno imati na općenitoj karti svijeta. Ispod su dijelovi koji su uklonjeni – pretpostavljam da je to prihvatljiv gubitak.

Tablica s vrijednostima E_{AK} za projekcije Frančula X do Frančula XIV, uspoređene s poznatim projekcijama, uključujući Robinsonovu i Eckert III.
The table of the E_{AK} values of Frančula X to Frančula XIV, compared to the well-known projections, plus Robinson and Eckert III.

Projekcija Projection	E_{AK} E_{AK}
Frančula XIII	0.2359
Frančula XII	0.2388
Frančula XI	0.2490
Frančula X	0.2498
Winkel Tripel (Orig.)	0.2597
Winkel Tripel	0.2603
Bartholomew Kavraiskiy VII	0.2614
Robinson	0.2778
Eckert III	0.2853
Eckert V	0.2972
Frančula XIV *	0.3177
Eckert IV *	0.3326
Eckert VI	0.3496
Aitoff	0.3690
Mollweide *	0.3774
Hammer *	0.4096
Sinusoidal *	0.4701

* *ekvivalentna/* * *equal-area*
Manje vrijednosti za E_{AK} *su bolje*
Lower values of E_{AK} *are better*

Fig. 4 Trancated map in the Frančula XIII projection.
Slika 4. Okrnjena karta u projekciji Frančula XIII.

Fig. 5 Truncated part of the map in the Frančula XIII projection.
Slika 5. Uklonjeni dio karte u projekciji Frančula XIII.

However, if you basically like e.g. the Frančula XIII but hate the concave poles lines ... just cut off the polar areas (Figures 4-5).

You might argue that such a truncation is a bad practice, and I tend to agree (even disregarding the fact that I like concave pole lines anyway). However I have seen worse truncations in atlases and on wall maps. And even disliking the practice, I have to admit that you don't cut off anything that is really important to have on a general reference world map. Below are the parts that were removed – I guess that is an acceptable loss.

And an Experiment

Just out of curiosity, I used the *Umbeziffern* to create a projection that has some similarity with the Frančula XIII – more or less the same length and curvature of the pole line and the bounding meridians – but uses the equal-area azimuthal projection as parent, with "controlled areal inflation" (as Wagner called it), which makes it a variant of Wagner VII/VIII instead of Wagner IX. I chose a configuration that results in a smaller amount of areal inflation than the Frančula XIII has.

I liked the result, but at first I was not sure if I should add it to the website... but then, I realized something: Using Richard Capek's distortion characterization *Q* (another one of these metrics to measure overall distortion in a map projection), my experiment ended up with *Q* value of 88.1 – which is excellent, more accurately: Better than *any* of the 100 (uninterrupted) projections that were listed in Capek's paper!

Granted, the *Q* for my projection was not calculated from the projections' formula but determined by a certain way of cartometry (about which I am going to write a long and tiresome blogpost sometime soon); and the *Q* has certain deficits (I will elaborate on this in the afforementioned blogpost); and of course there may be projections not surveyed by Capek which have an even better *Q* value...

... But, hey, it was a nice surprise anyway!

It is inspired by the Frančula XIII so there should be some relation to that name. It is surely no *approximation* because the distribution of meridians and parallels differs quite a bit. I will just call it the F13 Copycat (https://map-projections.net/img/jpg-w/f13-copycat-60-77-60-45-170.jpg, Figure 6).

This time, I use a different visualization of distortions, namely isolines of areal and maximum angular distortion, which makes it easier to compare to the images of the Frančula XIII given on page 86, *Anlage* 25 and 26, in Frančula's thesis (Figures 7-8).

You will notice that the isoline indicate an areal inflation by 3.0 runs along 80° North/South, very much like it does on the Frančula XIII. So why am I saying that the Copycat has "a smaller amount of areal inflation"?

Note that at the center of the map, the areal inflation is given as 0.77 (so it is an *de*flation) on the Frančula XIII while it is 1.0 (= equal-area along the equator) on the copycat, so the latter actually has less areal inflation. Or, more precisely, a smaller variance of areal distortions. Regarding the angular distortions, Frančula's original clearly shows a better distribution, but this was to be expected.

Using the Böhm notation (map-projections.net/wagner-umbeziffern.php), the F13 Copycat is called vii@60-77-60-45-170. To render the projection in Geocart (https://www.mapthematics.com/) use the *generalized Wagner* with the parameters:
a = 2.148298
b = 1.573349
m = 0.953264
m_2 = 0.725519
n = 0.427778
or use the d3-geo-projection scripts (https://github.com/d3/d3-geo-projection) with:
d3.geoWagner()
.poleline(60)
.parallels(77)
.inflation(45)
.ratio(170)

References / Literatura

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Györffy, J. (2018): Minimum distortion pointed-polar projections for world maps by applying graticule transformation, International Journal of Cartography 4(1):1-17, doi.org/10.1080/23729333.2018.1455263

Wagner, K. (1941): Neue ökumenische Netzentwürfe für die kartographische Praxis. In: E. Lehmann, ed., Jahrbuch der Kartographie, Leipzig: Bibliographisches Institut, 176–202

Tobias Jung

I jedan eksperiment

Samo iz znatiželje, upotrijebio sam *Umbeziffern* za izradu projekcije koja ima neku sličnost s Frančulinom XIII – više-manje iste duljine i zakrivljenosti linije pola i graničnih meridijana – ali koristi ekvivalentnu azimutnu projekciju kao roditelja, uz "kontroliranu inflaciju površina" (kako ju je Wagner nazvao), što je čini varijantom Wagnera VII/VIII umjesto Wagnera IX. Odabrao sam konfiguraciju koja rezultira manjom količinom površinske inflacije od one koju ima Frančula XIII.

Rezultat mi se svidio, ali u početku nisam bio siguran bih li ga trebao dodati na web stranicu ... ali onda sam shvatio nešto: Koristeći karakterizaciju *Q* distorzije prema Richardu Capeku (još jednu od mjera za mjerenje ukupne distorzije projekcije karte), moj je eksperiment završio s vrijednošću *Q* = 88,1 – što je izvrsno, točnije: bolje od bilo koje od 100 (neprekinutih) projekcija koje su navedene u Capekovom radu!

Doduše, *Q* za moju projekciju nije izračunan iz jednadžbi projekcija, već je određen s pomoću kartometrije (o čemu ću uskoro napisati dugačak i zamoran blog); a *Q* ima određene nedostatke (to ću detaljno objasniti u spomenutom blogu); i naravno mogu postojati projekcije koje Capek nije pregledao, a imaju još bolju vrijednost *Q* ...

... Ali, hej, ionako je to bilo lijepo iznenađenje!

Inspirirano je Frančulom XIII, pa bi trebala postojati neka veza s tim imenom. To sigurno nije *aproksimacija*, jer se raspodjela meridijana i paralela prilično razlikuje. Nazvat ću je F13 Copycat (https://map-projections.net/img/jpg-w/f13-copycat-60-77-60-45-170.jpg, slika 6).

Ovaj put koristim drugačiju vizualizaciju distorzija, naime izokole maksimalnih distorzija kutova i površina, što olakšava usporedbu sa slikama Frančule XIII dane na stranici 86, *Anlage* 25 i 26, u Frančulinu radu (slike 7-8)

Primijetit ćete da izoklina koja ukazuje na površinsku inflaciju od 3,0 ide uzduž paralele 80° sjeverno/južno, slično kao i kod Frančule XIII. Pa zašto onda kažem da Copycat ima "manju količinu površinske inflacije"?

Imajte na umu da je u središtu karte površinska inflacija 0,77 (dakle, to je *de*flacija) kod Frančule XIII, dok je na Copycatu 1,0 (= ekvivalentna uzduž ekvatora), tako da potonja zapravo ima manju inflaciju površina. Ili, preciznije, manje variranje distorzije površina.

Što se tiče distorzije kutova, Frančulin original očito pokazuje bolju raspodjelu, ali to je bilo za očekivati.

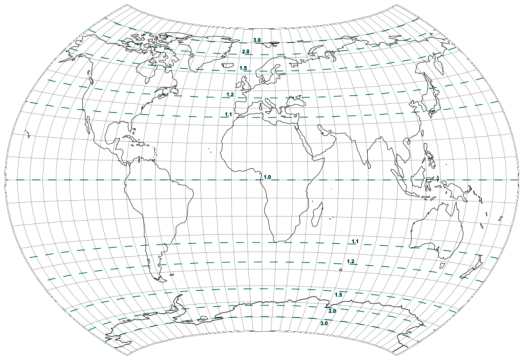
Prema Böhmovoj notaciji (map-projections.net/wagner-umbeziffern.php), F13 Copycat se zove vii@60-77-60-45-170. Da biste nacrtali projekciju s pomoću Geocarta (https://www.mapthematics.com/) upotrijebite *generalized Wagner* s ovim parametrima:

a = 2.148298
b = 1.573349
m = 0.953264
m_2 = 0.725519
n = 0.427778

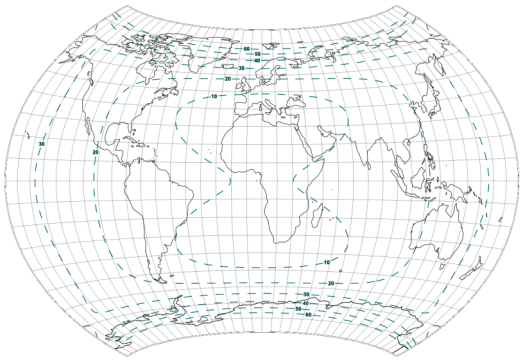
ili upotrijebite d3-geo-projection scripts (https://github.com/d3/d3-geo-projection) with:

d3.geoWagner()
.poleline(60)
.parallels(77)
.inflation(45)
.ratio(170)

Slika 6. F13 Copycat.
Fig. 6 F13 Copycat.



Slika 7. Razdioba distorzija površina u projekciji F13 Copycat.
Fig. 7 Distribution of areal distortions in the F13 Copycat projection.



Slika 8. Razdioba distorzija kutova u projekciji F13 Copycat.
Fig. 8 Distribution of angular distortions in the F13 Copycat projection.

Tobias Jung ■