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# AMPLIFICATION FACTOR OF A He-Ne MIXTURE IN A DETUNED LASER RESONATOR

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*Abstract:* Detuning effects of a plane-parallel resonator on the output power of a helium-neon laser have been considered. The dependence of the total quality factor of a plane-parallel resonator on detuning has been found. The dependence of the output power of a He-Ne laser ($\lambda = 1.15\ \mu$) on losses of the resonator has been established in the measurements performed.

The linear factor of amplification in a He-Ne mixture inside a resonator has been discussed in terms of geometrical optics theory. Experimental data have been used in calculating the linear amplification factor, and its dependence on the losses of the resonator due to detuning has been found.

The deviation of the amplification factor from linearity has been explained in terms of multimode effects of the He-Ne laser.

## 1. *Introduction*

The output power of a given type of laser depends on the input energy[1], on the parameters of the resonator[2, 3], and on the amplification of an active laser medium[4]. The resonator determines the type of radiation modes[5]. The radiation mode obtained in the resonator takes part in the stimulated process. Accordingly, the resonator parameters and the effective amplification of an active laser medium are mutually dependent.

The process of light amplification due to stimulated emission in a resonator is found to be nonlinear[6]. The intensity of the laser beam is proportional to the intensity of induced emitted light inside a laser resonator. On the other hand, an active medium has limited emission properties determined by the number of atoms capable to emit light through a stimulated process[7]. This causes saturation of amplification coefficients inside the resonator. The value of the saturated amplification coefficient depends on the density of induced radiation. This relationship is given by the relation[8]

$$K = \frac{K_0}{1 + a u}, \tag{1}$$

where $K_0$ is the unsaturated (linear) coefficient, and $\alpha$ the parameter of nonlinearity.

The integral output power of a laser beam, in the approximation of geometrical optics, is expressed by[9]

$$W = \frac{vsl}{a} [\,|K_0|\,l - \varrho l - \ln \frac{1}{r}] \frac{T \frac{1}{l} \ln \frac{1}{r}}{(\varrho l + \ln \frac{1}{r})\ (1 - r)} . \qquad (2)$$

Here $v$ is the velocity of light in an active laser medium, $s$ and $l$ are the diameter and the length of the active medium, $T$ and $r$ are the coefficients of transmission and reflection of the mirror laser resonator; $\varrho$ determines the losses in volume of stimulated radiation in the active medium.

In the case of a detuned resonator additional losses occur. In a plane-parallel resonator the predominant, additional losses are due to the tilt angle between the mirrors[5, 10, 11]. Tilting the mirrors decreases the efficiency of stimulated emission of an active laser medium, thus increasing its working threshold[12, 13].

Effects of tilting the mirrors in a plane-parallel laser resonator may be considered in terms of the theory of a classical passive Fabry-Perot interferometer. In such an interferometer, tilting the mirrors causes the interferential patterns to become weak[14] and the profile of the passband to become asymmetric[15]. Unlike a resonator with parallel mirrors, a classical passive Fabry-Perot interferometer, used as a laser resonator, brings about the variation in output power of the laser beam. The tilt angle of the mirrors in a plane parallel resonator gives rise to the variation of the quality factor.

The total quality factor $Q$ is determined by the relation[16]

$$\frac{1}{Q} = \frac{1}{Q_1} + \frac{1}{Q_2} + \frac{1}{Q_3} . \qquad (4)$$

Here $Q_1$, $Q_2$ and $Q_3$ are the partial quality factors determined by partial losses due to transmission of the mirrors, diffraction losses at the edge of the mirrors, and the angle between them.

## 2. *Experimental*

In order to study the effect of detuning on the effective amplification factor of a He-Ne mixture in a laser, a resonator 132 cm long and 1.8 cm in diameter was used. The active length $L$ of the emission medium was determined by the distance between the excitation electrodes, and it was 110 cm. The resonator was determined by plane-dielectric mirrors with a coefficient of reflection of 0.985, at a wavelength of 1.1 μ. The laser worked at a $2s_2$—$2p_4$ transition at an emission wavelength of 1.1523 μ. The discharge tube was filled with a mixture of helium and neon in the ratio 10:1, to a total pressure of 1.2 mm Hg. The mixture was excited by a radiofrequency field of 27 MHz, supplied by a generator of 150 W.

In order to avoid mechanical disturbances, the resonator should be suitably designed. Mirrors should be adjusted to mutually parallel spacings by means of a double system of micrometric screws. By turning one of the control screws it is possible to tilt carefully one of the mirrors.

Owing to the geometry of the adjusting system the shift of the mirror, $\delta$, along the diameter of the laser beam is determined by the angle of rotation, $\alpha$, of the control screw. This is expressed by the relation

$$\delta = 8.6 \times 10^{-6} \alpha \qquad [\mathrm{mm}], \tag{5}$$

where $\alpha$ is given in arc degrees.

According to relation (5) it is possible to vary the tilting of the mirrors within an arc second. One of the screws is connected to the potentiometer and in this way the turning of the screw is controlled on the $x$ axis of an $xy$ recorder. The intensity of laser radiation is detected by means of a germanium photocell and recorded on the $y$ axis of the recorder.

## 3. *Results*

*3.1. Dependence of the quality factor on the angle between the mirrors.* By turning the control screw the tilt angle between the mirrors is varied and this brings about the variation of the partial quality factor $Q_3$. According to refs. 16 and 17 the total quality factor of a plane-parallel resonator in relation (4) can be expressed by the relation

$$Q_{\mathrm{tot}} = \frac{a\,Q(\Theta)}{b\,Q(\Theta) + c} \, . \tag{6}$$

Here $Q(\Theta)$ is the partial quality factor depending on the adjustment of the mirrors. $a$, $b$ and $c$ are constants, the values of which are $a = 1.34 \cdot 10^{10}$, $b = 40.6$, and $c = 1.34 \cdot 10^{10}$.

According to ref. 18, the partial quality factor is given by

$$Q(\Theta) = \frac{\omega}{v} \left( \frac{2lD}{\Theta} \right)^{1/2} , \tag{7}$$

where $\omega$ is the frequency of laser transition, $v$ the velovity of light in an active medium, $\Theta$ the angle between the mirrors, and $D$ the diameter of the resonator.

The total quality factor of the resonator under consideration, in dependence on the tilt angle between the mirrors, is given by the relation

$$Q_{\text{tot}} = \frac{1.58 \cdot 10^8}{4.74 + 1.34 \cdot 10^2 \, \Theta^{1/2}} \; . \tag{8}$$

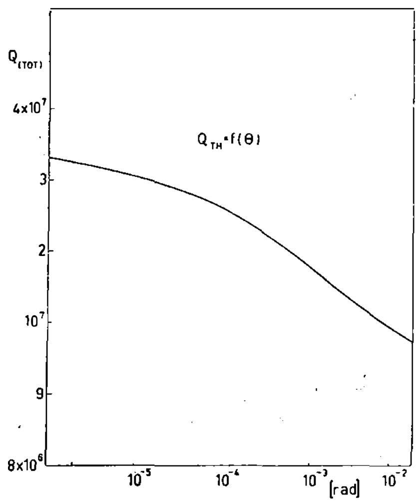


Fig. 1. Dependence of the total quality factor on the tilt angle between the mirrors.

The variation of the quality factor in dependence on the angle, expressed by relation (8), is shown in Fig. 1. As seen, the quality factor rapidly decreases with increasing angle between the mirrors. Since the quality factor is proportional to the ratio of the energy accumulated in the resonator to the energy lost per cycle[8)], an increase of the angle brings about losses in the resonator. It should be noticed that these losses are not included in the output power of the laser beam. Because of that the output laser power decreases with increasing angle between the mirrors.

To study the dependence of the output power on detuning, measurements were performed at different supply powers. The laser resonator was first tuned to the maximum power of the output beam. Then the control screw was turned to both sides from the maximum power position. The screw was turned until a position was reached where the output beam disappeared. The results of measurements are shown in Fig. 2. Each of the curves corresponds to a given supply power. Measurements were performed in the linear part of dependence on the input and output power. The tolerance range of adjustment was varied with the input power. At higher input powers the range of nonparallelism of the angle between the mirrors is larger. This is in agreement with the measurements performed in ref. 19. In the linear part of the dependence $P_{out} = f(P_{in})$ the increase in output energy of the laser beam is due to the increasing amount of population inversion[20] between the working laser levels. The threshold of the laser operation will be reached provided the critical value of population inversion

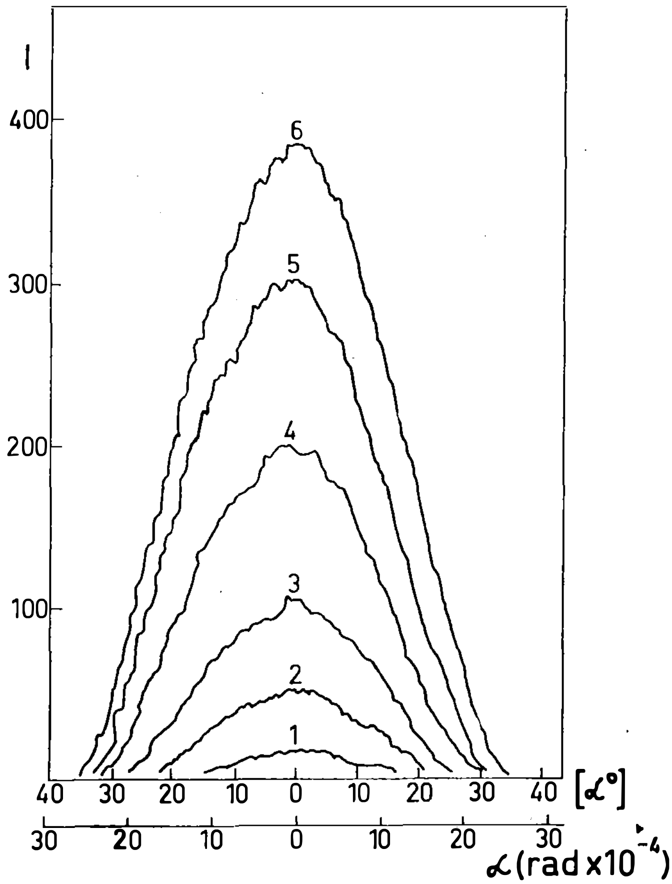


Fig. 2. Dependence of the output power of a plane-parallel resonator on the tilt angle of the mirrors. The upper scale on the abscissa indicates the values of the angle of rotation of the control screw. The lower scale indicates the values of the tilt angle between the mirrors.

is obtained. This value depends on losses in the resonator[21]. At higher input powers an active medium is able to compensate for losses in a larger range than at smaller input powers. Thus at higher input powers tolerances in adjustment are smaller.

*3.2. Effects of additional losses on the factor of amplification.* The effects of losses on the output power, which are the result of detuning of the resonator, are considered using relation (2). For additional losses in the resonator, relation (2) is transformed into[22]

$$W(x) = \frac{vs\,l}{a} \, \frac{(|K_o|\,l - \varrho l - \ln \frac{1}{r} - x)}{\varrho l + \ln \frac{x}{r} + x} \, \frac{T \frac{1}{l} \ln \frac{1}{r}}{1 - r} , \tag{9}$$

where $x$ denotes additional losses of the resonator.

To apply relation (9), we should introduce the notations described below.

From relations (9) and (2) it is easy to find the linear factor of amplification $K_o$

$$K_o = \frac{b(b+x)\ [f(x) - 1]}{1\ [(b+x)\ f(x) - f(x)^{-1}]} , \tag{10}$$

where the following notations are used:

$$f(x) = \frac{W(x)}{W_o} ,$$

$$a = |K_o|\,l - \varrho l - \ln \frac{1}{r} ,$$

$$b = \varrho l - \ln \frac{1}{r} ,$$

and

$$a + b = |K_o|\,l . \tag{11}$$

Accordingly, in order to express $K_o$, it is necessary to know the functional relationship $x = x(\Theta)$. From the knowledge of the tilt angle between the mirrors total losses in the resonator can be expressed by successive calculations of losses in power in a single transition from mirror to mirror. According to refs. 11 and 22 the losses of the plane-parallel resonator were calculated in dependence on the angle between the mirrors. This dependence is shown in Fig. 3.

As seen, for small values of $x$ the losses slowly increase with increasing angle between the mirrors. At values above $10^{-3}$ the angle increases exponentially. From Fig. 3 and relation (10) the dependence of the amplification factor $K_o$ of the He-Ne mixture on the losses of the resonator (Fig. 4) has been found.

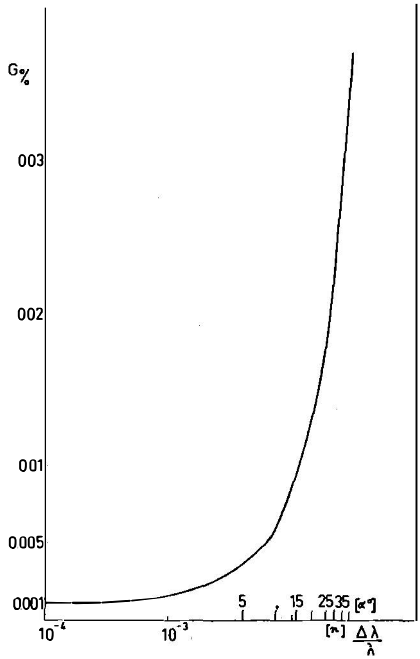


Fig. 3. Dependence of losses of a plane-parallel resonator on the tilt angle between the mirrors. The upper scale on the abscissa indicates the angle of rotation of the control screw. The lower scale indicates the tilt angle of the mirrors as distance of the edge of the mirrors from the parallel position. This distance is defined as fraction of the wavelength of the output laser beam, $\lambda = 1.15\ \mu$.

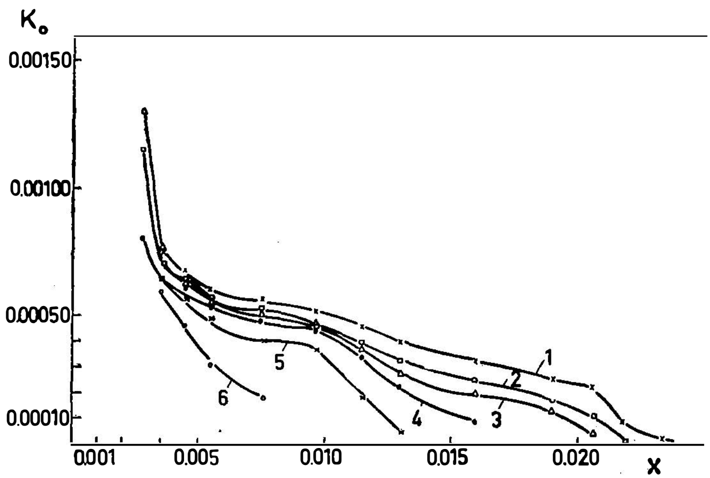


Fig. 4. Dependence of the amplification factor of a He-Ne mixture on losses of a plane parallel resonator.

The graph in Fig. 5 was deduced from the results for the dependence of the output power on the tilt angle of the mirrors (Fig. 2) and for the dependence of the resonator losses on the tilt angle of the mirrors (Fig. 3), showing the function $W(c)/W_0$ in dependence on $x$. Comparison of graphs 4 and 5 shows that the dependence of the output power on $x$ is similar to that of the amplification factor. For values of $x$ above $5 \times 10^{-3}$ the coefficient of amplification is a linear function of $x$ at higher output powers.

It may be concluded that for small losses the coefficient of amplification is saturated. Graph 4 shows that differences in values of linear coefficients of amplification become more pronounced with increasing losses at different input powers. This is due to the fact that $K_0$ is the effective coefficient of amplification averaged over all modes. For small losses it may be assumed

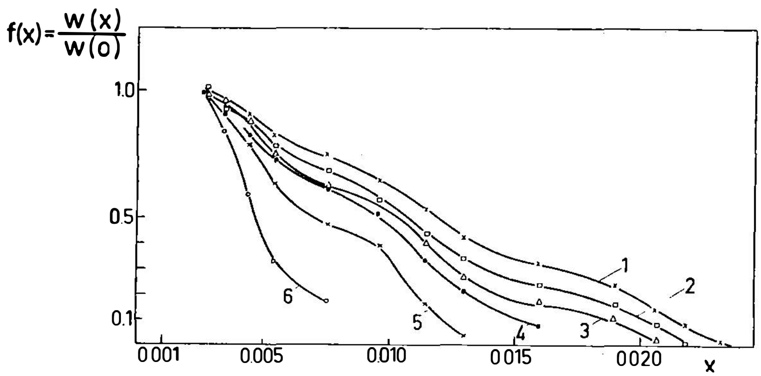


Fig. 5. Dependence $W(x)/W_0$ on losses of the resonator.

that the laser works at a constant number of modes regardless of the input power, since the coefficient of amplification is saturated. Because of that the values of $K_0$ at lower values of $x$ form well-arranged groups. At larger tilt angles of the mirrors and for higher losses the curves obtained at lower input power (graph 1) are below the threshold of multimode operation. In fact, at the same tilt angle between the mirrors the laser works with a smaller number of modes at lower input power. Consequently, for larger losses differences in values of amplification factors obtained at different powers are more remarkable.

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# POJAČANJE He-Ne SMJESE UNUTAR NEUGOĐENOG LASERSKOG REZONATORA


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## Sadržaj

U radu se istražuje utjecaj neugođenosti plan-paralelnog rezonatora na snagu izlaznog snopa helij-neonskog lasera. Nađena je zavisnost totalnog faktora dobrote plan-paralelnog rezonatora o neugođenosti rezonatora. Mjerenjem je utvrđena zavisnost izlazne snage He-Ne lasera ($\lambda = 1.15\ \mu$) o gubicima rezonatora.

Na temelju teorije bazirane na geometrijskoj optici diskutiran je linearni faktor pojačanja smjese He-Ne unutar rezonatora. Pomoću eksperimentalnih podataka računat je linearni faktor pojačanja te je nađena njegova ovisnost o gubicima rezonatora zbog neugođenosti. Na temelju multimodne slike djelovanja He-Ne lasera pokušalo se objasniti odstupanje faktora pojačanja od linearnosti.