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Some Historical and Economic Facts behind the Geometry of Circles and Squares

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Abstract: *The two popular geometrical shapes – circle and square – have long attracted the attention of scholars in mathematics, philosophy, religion, architecture, art and traditions of the human society. Following is a brief review of the use of these two images in historical perspective.*

Keywords: *ensō, frustum, feng shui, mandal, Pythagorean Theorem*

JEL Classification: B0, N0

The Geometry

In geometry we study the sizes, shapes, positions, angles, and dimensions of things. Since ancient times, practically in all civilizations, for over 5000 years, geometrical shapes – such as *triangles, circles, squares, pentagrams, hexagrams* and some other shapes, have widely been used. The geometrical designs have mostly been used in science, architecture, art, and religion to depict various aspects of human life – from days of ancient Egypt to date. Geometry as it was developed and used in Ancient Egypt was a necessary outgrowth of surveying to preserve the layout and ownership of farmland, which was flooded annually by the Nile. In ancient Egypt, geometry, was just a collection of rules and solutions aimed at specific circumstances, such as calculating the volume of a truncated pyramid. They also used trigonometry at that point, in development of a subset of geometry, for surveying and for measuring the dimensions of pyramids. The Babylonians too, may have known the general rules

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for measuring areas and volumes.[1] They measured the circumference of a circle as three times the diameter and the area as one-twelfth the square of the circumference, which would be correct if π is estimated as 3. The volume of a cylinder was taken as the product of the base and the height, however, the volume of the *frustum* of a cone or a square pyramid was incorrectly taken as the product of the height and half the sum of the bases.

Recently discovered **Babylonian clay tablet** appears to show that geometry was developed by humans some 1,400 years before than it is expected. The earliest recorded beginnings of geometry can be traced to the early peoples, such as in the ancient Indus Valley civilization, and ancient Babylonia from around 3000 BC. Early geometry was a collection of empirically discovered principles concerning lengths, angles, areas, and volumes, which were developed to meet some practical need in various crafts. The *Pythagorean Theorem* was also known to the Babylonians. Also, there is a recent discovery in which a tablet used π as 3 and 1/8. There have been more recent discoveries showing that ancient Babylonians may have discovered astronomical geometry nearly 1400 years before Europeans did.

Evidently, these ancient cultures laid down the foundations of the Greek geometry and influenced the Greeks, who would bring a deductive methodology to geometry, trying to find elegant rules underpinning the field. For the ancient Greeks, geometry was the crown jewel of their sciences, reaching a completeness and perfection of methodology that no other branch of their knowledge had attained. They expanded the range of geometry to many new kinds of figures, curves, surfaces, and solids; they changed its methodology from trial-and-error to logical deduction; they recognised that geometry studies the eternal forms, or abstractions[2].

Ancient India and China too had an advanced geometrical science. In India geometry was commonly used to construct altars (in 8^{th} and 7^{th} BC). In China definitive trace is in work (or at least oldest existent) on geometry was that of *Mo Jing*, then *Mohist* canon of the early philosopher *Mozi* (470-390 BC).

Geometry and ancient Economics

The geometric mean has often been used in business and economics for finding the average rates of change, average rates of growth, or other average ratios. Given n values (all of which are positive), the geometric mean is the n^{th} root of their product.

To understand the main streams of economic history, one needs to consider several key questions, e.g., what was produced? Primitive societies might think of it in terms of what they could hunt or plant. But this is also a question that entrepreneurs have been asking for thousands of years; how was it produced? , and, how did what was produced got distributed? Asking these questions help us to understand commerce, the allocation of resources, and social welfare practices.

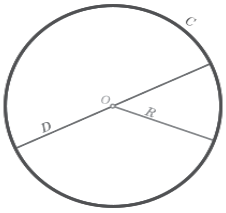
Mankind moved from hunter-gatherers to cultivators millennia ago. The land was cultivated in forms of squares, ovals and circles. Business contracts had existed for thousands of years, but the form these have changed over time and hundreds of years, giving rise to a variety of partnerships and corporative relationships. Greeks were the first to export classic products such as wine, olives, and pottery which helped to spread the Greek culture to the wider world.

In the other civilizations too agriculture was the main source of livelihood of the people. In Babylon, according to Herodotus, in comparison to other territories in the world, the land was very fertile. Farming was done by plough and oxen. Every ruler of Hammurabi dynasty built new canals for irrigation and renovated the old canals. To escape the land from floods, the dams were constructed. The surface of fields was upper than the canals and techniques were used for irrigation. The Babylonian people used to give more importance in growing dates, olive and grapes along with food grains. The land was mostly under the king, temple, feudal lords and rich merchants and collectively under the tribes. The land was given on lease for cultivation. The farmers had to pay 1/3 to 1/2 part of the total production as state tax. Hammurabi made strict rules about the sale of land. He encouraged the people to cultivate the new land. There was a provision of punishment for those who in any way harmed the agriculture. The taxes were exempted at the time of any natural calamity or famine. The state also used to give compensation for the encouragement of agriculture. Other than spinning, weaving clothes, making pottery and idols, weapons of metal, ornaments, wooden articles etc., were the industries prevailing there. Mainly the people of Babylonia were importing luxurious items, e.g., timber, led, glass, copper, gold, and silver. Trade was important in Sumerian society as Mesopotamia lacked essential materials such as stone, metals, and wood. Wool, *lapis lazuli*, gold, copper and iron were all very important resources in Mesopotamia.

In the Old Kingdom of Egypt, a period that stretches over roughly 500 years (2686–2181 BC), the economy was primarily agrarian and was heavily reliant on the Nile. The geometry as it was developed and used in Ancient Egypt, was a necessary outgrowth of surveying to preserve the layout and ownership of farmland. Agriculture thrived because Egypt has a favourable warm climate that provided with as many as three harvests each year. Ancient Egypt also had many natural resources, including flax, papyrus, stone, and gold.

The Circle

The word *circle* (Sans. मण्डल, *Maṇḍal or Chakra* चक्र*)*) derives from the Homeric Greek (κρίκος), meaning *hoop* or *ring*. The origins of the words *circus* and *circuity* are closely related. A circle is a simple closed curve that divides the plane into two regions: an interior and an exterior. In everyday use, the term *circle* may be used interchangeably to refer to either the boundary of the figure, or to the whole figure including its interior. In strict technical usage, the circle is only the boundary and the whole figure, is called a disc. A circle may also be defined as a special kind of *ellipse* in which the two *foci* are coincident, the *eccentricity* is 0, and the semi-major and semi minor axes are equal; or the two-dimensional shape enclosing the most area per unit perimeter squared, using calculus of variations.

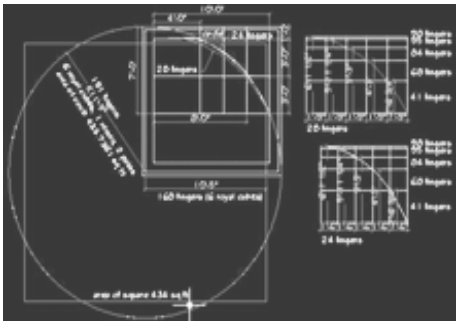


The circle has been known since before the beginning of recorded history. Natural circles would have been closely observed, such as the Moon, Sun, and a short plant stalk blowing in the wind on sand, which forms a circle shape in the sand. In mathematics, the study of the circle has helped inspire the development of geometry, astronomy and calculus.

Western biographers credit Archimedes of Syracuse (287-212 BC) with the analytical evaluation *n* of the actor *Pi* associated with circle, within a close range of $3^{1}/_{7}$ to $3^{10}/_{71}$. However, in the process of establishing this, many scholars recognized him as the first person to realize that the same factor is associated with both the perimeters which proves that the *area of the circle = ½ x perimeter x radius*.

A circle (*mandal*) in India, is a geometric configuration of symbols. In various spiritual traditions, mandal may was employed for focusing attention of practitioners and adepts, as a spiritual guidance tool, for establishing a sacred space and as an aid to meditation. In the eastern religions – Hinduism, Buddhism, Jainism and Shintoism, it is used as a map representing deities. A *mandal* generally represents the spiritual journey, starting from outside to the inner core, through layers.

Ancient Egypt



In ancient Egypt, the problem 49 from the Rhind Mathematical Papyrus *(RMP) finds the area of a rectangular plot of the land*. The Problem 6 of the Moscow Mathematical Papyrus (MMP) finds the lengths of the sides of a rectangular area given the ratio of the lengths of sides. According to a Rhind scroll, estimated area of the

(Note: Observe the Circle and Square)

circle is ≈ [(diameter) x 8/9]2. The Problem 48 of the RMP compares the area of a circle (approximated by an octagon) and its circumscribing square. The area of the octagonal figure is: $\pi = 3.14159....$*This* problem's result is used in problem 50[3].

As stated above, the ancient Egyptian geometry was developed and used in surveying to preserve the layout and ownership of farmland. Of course, they knew that they could approximate the area of a circle.

The S*hen-ring*, i.e., the circle - is represented as a double strand of rope, the ends of which are folded in such a way that a closed ring is formed with a knot on the underside. The *Shen-ring* appears as a hieroglyph, as a protective sign on stelae or in tombs, and as well as an amulet. It was used as a magical aid. During the 3rd and 4th dynasties it was worn around the neck for protection.

Because the circle had no end, the *Shen-ring* was regarded as the symbol of infinity, and when it encircled the sun, it was considered as a symbol of eternity of the universe. The notion of encirclement then led to an expansion in meaning – protection – added to that of infinity. The *Shen-ring* (*See*, Fig.1.a. and 1.b.) was particularly associated with *Horus* the falcon and *Nekhbet* (the vulture goddess was the protector of Upper Egypt and especially its rulers). Nekhbet, was frequently portrayed as spreading her wings over the pharaoh while the cartouche symbol is shown grasping in her claw (with other emblems) circle above the king to offer him eternal protection. Royal names were written in an elongated version of the *Shen-ring* – the *cartouche*. It was used as early as the IIIrd Dynasty where it can be seen in the reliefs from Djoser's step pyramid complex.

Mesopotamia

In Mesopotamian language, the word *zisurrû*, meant "the *magic circle* drawn with flour along with inscription *Zi-sur-ra-a*", was an ancient means of delineating, purifying and protecting from evil by enclosing of a ritual space in a circle of flour. It involved ritual drawings with a variety of powdered cereals to counter different threats and was accompanied by the gloss: *sag.ba sag.ba* (Akkadian: *māmīt*), the curse from a broken oath (cited in *The Exorcists Manual*), where it refers to a specific ritual on two tablets.

The word was used as a defensive measure and drawn on the ground around prophylactic figurines as part of a *Babylonian* ritual to thwart evil spirits, around a patient's bed to protect against ghosts or demons in much the same manner in which bowls thwart demons and curses, or as a component of another elaborate ritual. In the ritual tablet of the *Maqlu* incantation series, it instructs, "Thereafter, you encircle the bed with flour-paste and recite the incantation *sag.ba sag.ba* and the incantation *tummu bītu* (adjured is the house). The encrypt *én sag.ba sag.ba* also appears in the *Muššu's* ritual tablet, where the *circle* is rationalized in commentaries as representing certain protective deities, *lugal.gir.ra* and *Meslamtae'a*. In other rituals a circle might be painted in whitewash or dark wash on either side of a doorway for apotropaic purposes. The choice of flour was crucial to the purpose of the ritual for repelling ghosts, presumably to counter disease-carrying demons.

China, Japan and S. Korea

The *mandal* (*circle*) is also used as a graphic depiction of the spiritual universe and its myriad realms and deities. First in *Tibet* and *China* and then in Japan, the *mandal* rose to great prominence as a *living entity*, one that ensured the efficacy of esoteric rituals performed in its presence (*See Fig. 2. and 3.*). *Mandal* scrolls and paintings became popular in the 9th century onward with the growth of the *Shingon* (真言) and *Tendai* (天台) sects of Esoteric Buddhism – *Vajrayana*.

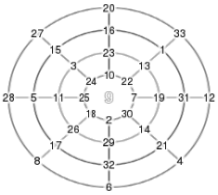
In ancient **China**, the circle was an important cultural symbol. It represents harmony, the balance of the *yin* and *yang* and the duality of nature, which is an essential part of *Daoist* and Chinese medical philosophies. For centuries, the circle has influenced scores of Chinese philosophers, scholars and literary figures.

Yin and *yang* representing elements of a circle in China is a philosophical concept. It describes opposite but interconnected forces. In Chinese cosmology, the universe creates itself out of a primary chaos of material energy, organized into the cycles of *yin* and *yang* and formed into objects and lives. *Yin* is the receptive and *yang* the active principle, seen in all forms of change and difference such as the annual cycle

(winter and summer), the landscape (north-facing shade and south-facing brightness), sexual coupling (female and male), the formation of both men and women as characters and socio-political history (disorder and order).

A perfect circle 圆, [yinpin] or [pinyin]) is almost impossible to produce by unaided human hands, and that is why the circle were invented by a Chinese mathematician, Yang Hui, (AD 1238–1298). It is the arrangement of natural numbers on a circle where the sum of the numbers on each circle and the sum of numbers on diameters are identical i.e. 138.

A hand drawn Japanese circle ensō



Yang Hui's circle

Further, *Taiichi* (太极) is a Chinese cosmological term for the *Supreme Ultimate* state of undifferentiated absolute and infinite potential, the oneness before duality, from which *yin* and *yang* originate.

In many *Feng shui* workshops one talks about the energetic grids (*bagua*) that are placed over structures to help us divine and decode what is going on in one's homes as a mirror reflection of what is going on inside one's mind and lives. Each *bagua* is represented by one of the five elements of wood, fire, earth, water, and metal, and each of the area has layers of symbolism. The circle is associated with earth element. While the *Earth* is related to female energy, the *Metal* is related to square, angular shapes and male energy, because the ancients observed the heavens as a sphere above us – therefore a circular shape. The earth however was a horizontal plane, therefore a square.

The energy of *Earth* is low and horizontal, as it follows the surface of the earth. Along with much of the ancient world, in ancient Chinese culture, associated with the circle is unity, perfection and completeness. A circle was considered mysterious and therefore, it was the shape of the unknowable, the heaven.

The *Yin*/*Yang* symbol is fairly popular in East Asia (China, Japan, and S. Korea). It is a circular form. Although, many entrances to public parks, shrines and some public buildings symbolize the circle, but the most impressive depiction of it is in the S. Korean national flag. The flag was created in 1882. It symbolizes the dual forces of nature *yin* and *yang*. The red and blue circle in the middle of the flag is called *taegeuk*, which means *supreme ultimate*.

The circle is divided into two parts, each of which resembles a comma. The upper, red part represents the forces of yang, and the lower, blue part represents the forces of yin. Together, the two forms signify the perpetually changing opposite yet complementary forces or principles embodied in all aspects of life. The thick round part of each comma represents the beginning of all things and the tail section represents the end. In the flag, the four sets of trigrams further convey the idea of the dualism of the cosmos. Heaven, the manifestation of the pure yang principle, is represented by the three unbroken lines; a set of three broken lines placed opposite represent the earth. The stages between the two extremes of yang and yin are represented by the two lines with a broken line between them signifying fire, and the two broken lines with an unbroken line in the middle, water. Together, these four trigrams also symbolize the seasons and the cardinal directions.

In **Taoist** philosophy the concepts of *circle* vs. *square* is usually used to refer to how a Taoist should deal with worldly interpersonal matter. Taoists are advised to display *circle* characteristics in the outer appeal but *square* characteristics in the inner core. A Taoist external appeal is preferably *circle* under most circumstances, but exceptions should be exercised as the need arises. However, the inner core must always be *square*, which means being straight and uncompromising in being virtuous. A Taoist should cultivate himself in terms of virtue, equanimity and wisdom. He must minimize his desire.

India

The design principle of **Hindu** temples extensively uses circles in its horizontal layout and vertical spire design. This is almost universal in ancient Hindu temples in India, and South East Asia.

Turning square and circumscribing circle geometry is very commonly visible in Hindu Temples. This is both a structural design derivation as well as representative of philosophical beliefs of Hindus. These are described in ancient Sanskrit texts called *Vastu Shastras* (science of dwelling).

At the centre of these designs, underneath the spire is the main deity of the temple as well as the *Purusha* (a universal principle with no form, only knowledge and meaning, reflecting *Truth* and *Reality* for Hindus). The spires rising towards the sky, do represent the world of God *Brahma*. An Indian philosopher and mathematician named Baudhayana[4], had known very well the geometry of a circle much earlier than other people before him. He worked on the circles a few centuries earlier then Archimedes.

In Hinduism, a basic *mandal*, also called a *yantra*[5], takes the form of a *square* with four gates containing a *circle* with a central point. Mandals often have radial

balance. A *yantra* is similar to a mandal, usually smaller and use a more limited colour palette. It may be a two- or three-dimensional geometric composition used in meditative rituals, and may incorporate a *mantra* (Sans. मन्त्रम्, meaning sacred message) into its design. It is considered to represent the abode of the deity (*See Fig. 4.*).

Note that in the illustrated mandal, painted in 17th century, Tibetan *Five Deity Mandal*, in the centre is the *Red Enemy of Death*, embracing his consort, in the corners are the red, green, white and yellow *yamaris* (*See Fig. 3.*). Sand painting showing Buddha mandal, is what is made as part of the death rituals among *Buddhist Newars* of Nepal.

Ancient Greece

It is a known historical fact that geometry is an ancient Greek knowledge that arose as the field of dealing with spatial relationships. Euclid is considered to be the *father of geometry.* He is chiefly known for his *Elements,* a treatise that established the foundations of geometry that largely dominated the field until the early 19th century. Some innovations were made by Eudoxus, Hippocrates, Thales and Theaetetus. Because of their symmetry, circles were seen as representations of the *divine and natural balance* in ancient Greece (*in Fig. 4., one can observe the wheels on the Greek pottery*).

A focus on circles is evident among structures built throughout Greek history. The challenge was to construct a square with exactly the same area as a given circle, using only a set of compasses and a straight edge. People have been trying for Ages to solve it, but it was only in 1882, that it proved to be impossible.

The Romans

Ancient Roman architecture featured a number of incredible innovations that allowed them to grow and maintain a colossal Empire that stood for centuries. Important religious and political structures were, of course, incredibly important, but the centre of attention for most of the public were the structures created for entertainment. The Romans were fascinated by the circular forms.

This is evident from their massive construction of circular arenas (*See Fig. 5.*) throughout the Empire — colosseums. These were large, amphitheatres, circular, open-air venues stood at the heart of Ancient Roman events from gladiatorial contents to such public spectacles as executions.

Throughout the Roman Empire, in Catholic Churches, in palatial buildings, circles and squares have been widely used in wall-paintings and architectural designs.

With over 200 amphitheatres known to public record, one stands out as by the most famous and is still a popular attraction to this day: The Colosseum of Rome. Its construction began during the reign of Vespasian in 72 AD, only to be completed 8 years later during the rule of Emperor Titus.

In Religions

In **Hindu religions**, a *mandal* (Sans. *chakra* चक्र् i.e. *circle*) is an artistic representation. The image first appears in India in *Rig Veda* (cca. 1500-500 BC). The image has been used by cultures around the world in many different periods up to the present. The details of the meaning of a given *mandal* depend on the individual of higher thought and deeper meaning given as a geometric symbol used in spiritual, emotional, or psychological work to focus one's attention[6]. Creating or observing the image, but mandals in every culture serve, more or less, the same purpose of centring an individual or community on a given narrative in order to encourage introspection and, ultimately, an awareness of one's place and purpose in the world; this awareness then allows for peace of mind. It was, and is, used as a meditative tool and spiritual exercise in the belief systems of Jainism, Buddhism, and Shintoism, it also appears in Persian Art from Mesopotamia, figures in Mesoamerican architecture, and Native American art, and was used by the Celts of the Iberian Peninsula and Northern Europe.

The image is usually defined as a circle decorated with imagery which directs the mind of the observer (or creator) inwards from the outer rim toward deeper reflection on the meaning and purpose of life, the nature of the universe, the substance and reality of God, the true nature of the self, the underlying form of reality, cosmological truths, and, actually, any other spiritual, psychological, or emotional aspect of one's life.

In **Buddhism** the use of symbols (*pratīka*) is mainly to represent certain aspects of the Buddha's *Dharma* (teaching). Early Buddhist symbols which remain important even today, include the *Dharma wheel*, *Lotus flower*, the three jewels and the *Bodhi tree*.

Gautama Buddha (563-483 BC; *See Fig. 6.* and *7.*) founded the Buddhist faith that spread throughout India, Nepal, China, Japan and SE Asia. In Zen culture of China and Japan, an *ensō* is a circle that is hand drawn in one or two uninhibited brushstrokes to express a moment when the mind is free to let the body create.[7] The circle may be open or closed. In the former case, the circle is incomplete, allowing for movement and development as well as the perfection of all things.

In **Jainism** a mandal (circle) is considered as a sacred religious image, usually in the form of complex concentric circles. This type of mandal is a specific form in the Jain faith.

Jains believe that every soul is potentially divine and refer to beings that have achieved this, and who founded the Jain faith, as *jinas*. The hand symbol (shown above) with a wheel on the palm symbolizes *Ahinsā* (non-injury), which represents the *dharma chakra* that stands for the resolve to halt the world (*samsara*) rough the relentless pursuit of *Ahinsā* (*See Fig. 8*).

Circle is a pre-Christian symbol and its original meaning has been adopted by **Christianity** widely. It is universally known as the symbol of eternity and never-ending existence. Extremely common on gravesites, its usual representation is a cross surrounded by circle. Two circles, one above the other, represent earth and sky. Three interconnected circles represent the Holy Trinity. Latin cross surrounded by circle or oval representing eternity or never-ending existence.

The definition and meaning of the *circle* as a **Catholic Christian** symbol in art provides a clear graphic illustration that represents people or items of religious significance. It symbolizes eternity as it has no beginning or end. Because of this many early Christians believed that there was something divine in circles. Early astronomy and astrology were connected to the divine for most medieval scholars, the circular shape of the sun, moon and the planets were related to God's act of Creation (*See Fig. 9.*).

World around, in most churches, we find art works, depicting Jesus Christ, Mother Marry, Christian Priests, and holy cross etc. in decorated circles on many church walls.

In ancient Celtic religion, commonly known as **Celtic paganism** – the religion of the ancient Celtic peoples of Europe, there were some "broad structural similarities" and "a basic religious homogeneity".

As far as is known, the Celts had no temples before the Gallo-Roman period. Their ceremonies took place in forest sanctuaries, and the trinity symbol has been found in ancient manuscripts, and sighted stones dating as far back as 1000 AD with carvings of the *triquetra*.

Many believe that Celtic symbols represents the pillars of early Celtic Christian teachings of the Holy Trinity (God the Father, the Son and the Holy Spirit). It also represents the unity of spirit when enclosed in a circle. The circle protects it, so the symbolic spirit cannot be broken.

Scholars suggest that the Celtic circle may represent the celestial sphere, i.e., the idea of the heavens surrounding the earth. Deeply ingrained in the heart and soul of Celtic Christian spirituality is the mystery of one God in three persons, a truth that is taught clearly in the Bible, even though the word *trinity* is not found in the Scriptures.[8]

In **Judaism** a circle symbolizes eternity. It starts nowhere and it finishes nowhere. The circle is also a symbol of equality. There is an ancient Jewish custom to dance for hours around the *bima* (lectern) on *Simchat Torah* -- the festival on which people celebrate the completion of the yearly Torah cycle. This circle of dancing symbolizes solidity, stability, and physicality. Pairings of circles and squares are sometimes used

to represent heaven and earth or spiritual and material. Circles are commonly seen as spiritual because these are unending and, thus, eternal.

On *Simchat Torah*, the custom is to take the Torah scrolls out of the Ark and to encircle the reader's platform and throughout the synagogue with great joy, singing, and dancing[9]. Circular *Hakafots* are symbols of perfection and unity, or sometimes a symbol of communal cooperation. At the height of celebration, when confident in own piety, people are reminded of the power of repentance. Lest one sins again, one is reminded to never despair. This is why the Jews dance in a circle on *Simchat Torah*. The scholar and the ignoramus, the pious and the not-so-pious, all in one equal circle. A circle has no beginning and no end, no high point and no low point; all are equal in a circle.

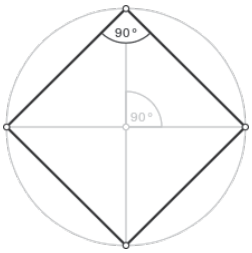
In **Islam** the centre of a circle is an apt symbol of a religion that emphasizes one God, and symbol of the role of Mecca, the centre of Islam, toward which all Muslims face in prayer. The rays of a star reach out in all directions, making the star a fitting symbol for the spread of Islam.

The geometric designs in Islamic art are often built on combinations of repeated squares and circles, which may be overlapped and interlaced, as can *arabesques* (with which they are often combined), to form intricate and complex patterns, including a wide variety of tessellations. These may constitute the entire decoration, may form a framework for floral or calligraphic embellishments, or may retreat into the background around other motifs. The complexity and variety of patterns used evolved during 9th to 16th century.

Hindus and Buddhists believe that in **human body**, there exist 7 circles (chakras), and these are actually the energy centres known to regulate one's emotions. Both the religions talk about the shifting nature of our body *chakras*.

The Square

In science of geometry, scholars have developed the theoretical foundations of these different shapes. In Euclidean geometry, a *square* (Sans. चतुष्कोण) is a regular quadrilateral, which means that it has four equal sides and four equal angels (90-degree angles, $\pi/2$ radian angles, or right angels). It can also be defined as a rectangle with two equal-length adjacent sides. It is the only regular polygon who's internal, central and external angels are all equal (90°), and whose diagonals are all equal in length. A square with vertices *ABCD* would be denoted as *ABCD*.



Square is one of the oldest and most universal symbols in the world. It is found in art, architecture, and nature all over the globe. Though the spiritual meaning and

symbolism of a square may vary depending upon the culture and context, but there are some common themes that emerge.

Generally, the square represents strength, stability, and security. It is a symbol of order and organization, as well as restraint and moderation. In spiritual contexts, it often symbolizes purity, honesty, and integrity. The square is associated with the number four and alludes to the four elements of the physical world — earth, air, water, and fire. Following are some highlights in the history of the circle: in 1700 BC – The Rind papyrus gives a method to find the area of a circular field. The result corresponds to 256/81 (3.16049...) as an approximate value of π; the book *Euclid's Elements*, deals with the properties of circles; Plato (427-347 BC), in his 7th *Letter*, defines and details the explanation of a perfect circle, as to how it is different from any drawing, words, definition or explanation.

Ancient Egypt

Egyptian geometry refers to as it was developed and used in Ancient Egypt. Theirs was a necessary outgrowth of surveying to preserve the layout and ownership of farmland, which was flooded annually by the Nile. We only have a limited number of problems from ancient Egypt that concern geometry. Geometric problems appear in both the *Moscow Mathematical Papyrus* (MMP) and in the *Rhind Mathematical Papyrus* (RMP). The examples demonstrate that the ancient Egyptians knew how to compute areas of several geometric shapes and the volumes of cylinders and pyramids.

Mesopotamia

A *Ziggurat* to protrude, or to build high, is a type of massive *square* structure that were built in ancient Mesopotamia (*See Fig. 10*). It has the form of a terraced compound of successively receding levels. Notable ziggurats are of Ur, and Aqar Ouf. These are destroyed now. The Sumerians believed that the Gods lived in the temple at the top of the Ziggurats, so only priests and other highly respected individuals could enter. Society offered them many things such as music, harvest, and creating devotional statues to live in the temple against the manner in which bowls thwart the demons and curses, or as a component of another elaborate ritual (*See Fig. 11*). It involved ritual drawings with a variety of powdered cereals to counter different threats and is accompanied by the gloss: *sag.ba sag.ba* (Akkadian: *māmīt*).

These words were used as a defensive measure and drawn on the ground around prophylactic figurines as part of a Babylonian ritual to thwart the evil spirits, around a patient's bed to protect against the manner in which bowls thwart demons and curses, or as a component of another elaborate ritual.

China, Japan, S. Korea

The square with its straight lines and sharp corners, in ancient Chinese culture, represent laws and regulations, therefore, the realm of man. It is a stable structure and firmly grounded, therefore, it represents the earth.

It's worth mentioning that the Chinese concept of a *square* is quite different from that in other countries. While in the Latin world a *square* is a shape with all four sides of the same length, and similar shapes with sides of varied lengths are called *rectangles*. In Chinese, all of those shapes are part of the same family, 方 (fāng). Under that, squares are 正方 (zhèng fāng, *perfect square*) and rectangles 长方 (cháng fāng *stretched square*). Therefore, when talking about *squares* in context of Chinese culture, we mean all four-sided shapes with four right angles, regardless of height-length ratios.

India

The design principle of Hindu temples extensively uses squares - both in their horizontal layout and vertical spire design. Designs that use harmonic ratios other than perfect square are known, but these too are precise mathematical series and ratios embedding squares and circles inside rectangles (1:2, 1:3, 2:3, 2:5, etc.).

In **Hinduism**, square is an important symbol. It represents earth. Two squares one within the other, symbolize heaven and earth together. Square can be found in various *yantras* and *chakras*. It is also an important element in tantric puja and worship. The *sanctum sanctorum*, which houses the main deity in a temple, is a square. In Tantric concept, the four corners of the square depict the four elements that surround and protect. They are *Prithvi* (earth), *Agni* (Fire), *Varuna* (water) and *Vayu* (wind). In Tantric worship, square is drawn inside a circle and a triangle. Square's outer perimeter represents male. Fire pits or *havan kunds* (worship pits) are square. These square contains two vertical and two horizontal lines. The horizontal lines represent stability. Vertical lines represent activity. Thus, the square is both inward and outward looking. Both materialistic and spiritual. Through a square *havan* (worship) ritual, one can wish for *moksha* (insurrection) or *fulfilment of desire* (*See Fig. 12*. and *13*.).

Ancient Greece

Evidence for the geometric culture that has come down to us in the form of Greek epic poetry, artistic representation, and the archaeological record. Studying geometry was considered the gold standard of their mathematical and scientific pursuits. From Hesiod (*Erga,* 639–40 AD), we assume that most 8th century BC Greeks lived off the land.

The epic poet describes the difficult life of the farmer. There are, however, few archaeological remains that describe everyday life, including square images during this period. Monumental craters, originally used as grave markers, depict funerary rituals and heroic warriors. The presence of fine metalwork attests to prosperity and trade. In the earlier Geometric period, these objects, weapons, fibulae, and jewellery are found in graves—most likely relating to the status of the deceased. By the late 8th BC, however, the majority of metal objects are small bronze figurines—votive offerings associated with sanctuaries.

The Romans

The square *Basilicas* were created by the Christian church but was conceived by the Romans as a place for any large gathering, with the most common use being law courts. They were usually built along one side of the forum, which was enclosed on all sides by colonnades supported by columns and piers on all sides. A typical example is the Serena Basilica (216 CE). The *Roman Baths* too, display the typical Roman ability for creating marvellous interior space. The largest of these often huge complexes were built symmetrically along a single axis and heating, and sometimes inter-wall heating through terracotta piping.

In Religions

There are three dominant ancient religions in India – Hinduism, Buddhism, and Jainism. Geometric symbols are an important part of Hinduism. They represent creation, union, male and female principle, and teaching, etc.

Early Buddhist symbols (the three jewels) which remain important today include the *Dharma wheel*, the *Lotus flower*, and the *Bodhi tree*. Buddhist symbolism is intended to represent the key values of the Buddhist faith. The popularity of certain symbols has grown and changed over time as a result of progression in the follower ideologies.

Research has shown that the aesthetic perception of the Buddhist gesture symbol has positively influenced the perceived happiness and life satisfaction.

The square symbolism in Buddhism is well represented in *stupa* structures showing balance, logic, and law and order (*See Fig. 14., 15.* and *16.*). These do represent the laws of nature that exist in the physical realm, and how the laws do with number 4, and relates to the four elements of the physical world***: earth, air, water* and *fire.***

Jainism was founded by saint Mahavira (599-527 BC) a contemporary of Buddha. Square images in Jainism, are fairly common in caves, temples and art of ancient Indian civilization. Mahavira taught that observance of the vows of non-violence, *satya* (truth), non-stealing, chastity, and non-attachment that are necessary for the spiritual liberation. He taught the principles of (many-sided reality) (*See Fig 17.*).

It must be emphasized that the early **Christians** and all peoples living at that time were much more attuned to symbols and arcane meanings than people of the 21st century. The Greco-Romans, Christians and pagans, were steeped in myths, mysteries, metaphors, omens, divinations and allegories. Everything that happened meant something and everything that had a root of meanings around it. To enter the world of early Christian symbols is to enter a very rich, deeply intellectual and poetic world of meanings.

One of the most interesting and ingenious early 1st century Christian symbols is what is called the *Sator* or *Rotas* square (*See Fig. 18*.). The existence of the square was long recognized from early medieval times, and it has been found on the continents of Europe (and Byzantium in Asia Minor), North Africa (mainly Coptic settlements), and the Americas. Many medieval European churches and castles have Sator square inscriptions cross, the principal symbol of the Christian religion, recalling the Crucifixion of Jesus Christ and the redeeming benefits of his passion and death. The earliest Sator squares were found at several Roman-era sites, all in Rotas - form, with the discovery at Pompeii (and also likely pre-AD 62). The earliest square that included explicit additional Christian-associated imagery dates from the sixth century, and by medieval times Sator squares had been found across Europe, Asia Minor, and North Africa.

The 1st century Christians were a persecuted minority forced to identify themselves to each other by secret signs and actions, the most common of which were the primitive *fish sign* and the *spilling of a little wine on the ground*. The obvious meaning of the Sator Square to a Christian would have been Jesus' Parable of the Sower (in Matthew 13:3-9), when He likened the spreading of the word of the kingdom of God to a farmer who sows seeds. In Celtic society the square or shield knots tend to be much more elaborate and complex. Some of the commonly held meanings include the four seasons, the four directions, the four elements of nature (earth, wind, fire, water), the four Celtic festivals of Samhain, Bealtaine, Lughnasadh and Imbolc, or St. Brigid's four branches of wisdom; hand, hearth, head and heart. In any case, they were most likely used as a form of protection and good fortune.

The designs created by ancient Celts, both on the Continent of Europe and on the islands of Britain and Ireland, are typically characterized by two-dimensional graphics, featuring the use of complex patterns based on a number of recurring motifs. These motifs include knots, zoomorphic imagery, interlace patterns, spirals, and crosses. In their heyday of the 3rd and 4th centuries BC, the Celts straddled the whole continent of Europe. As a result, Celtic art and design of this period, drew on a huge range of artistic traditions, from the Balkans, the Greeks, Etruscans, and the Romans, Egyptians and many others. During the later Christian period (5th -10th c. CE), Celtic artists and metal workers reviewed upon motifs, techniques and stylistic forms from Anglo-Saxon and Germanic artists, as well as Christian figurative imagery.

In **Islam**, the square represents the physical experience or materiality. The Kaaba (meaning cube in Arabic) is a square building, elegantly draped in a silk and cotton veil, located in Mecca. It is the holiest shrine in Islam.

In Judaism, The Tree of Life (*See Fig. 19.*) is most widely recognized as a concept within the Kabbalah, which is used to understand the nature of God and the manner in which he created the world. The Kabbalists developed this concept into a full model of reality, using the tree to depict a map of creation. The nodes represent *spheres* and the lines the *paths*. The nodes usually represent encompassing aspects of existence, God, or the human psyche. The nodes are also associated to deities, angels, celestial bodies, and values. The columns symbolize as pillars representing different kinds of values, or types of ceremonial magic. The squares are also seen as particularly stability and orderly, standing for firm foundations, both literally, and metaphorically. The square can be seen as a symbol of civilization.

Declarations

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Conflicts of interest/Competing interests

There is no conflict of interest/competing interests.

Availability of data and material

Data will be available upon request.

Code Availability

Not applicable.

Authors' Contributions

Not applicable.

NOTES:

[1] The Egyptian units of length are attested from the Early Dynastic Period. Although it dates to the 5th dynasty, the Palermo stone recorded the level of the Nile River during the reign of the pharaoh Djer, when it was estimated as 6 cubits and 1 palm (about 3.217 m). An III rd Dynasty diagram shows how to construct a circular vault using body measures along an arc. If the area of the square is 434 units, the area of the circle is estimated at 433.7.
From an ostracon found near the pyramid of Saqqara, it is evident that a curve is divided into five sections and the height of the curve is given in cubits, palms, and digits in each of the sections. At some point, lengths were standardized by cubit rods. These cubits are 52.5 cm long and are divided into palms and hands: each palm is divided into four fingers from left to right and the fingers are further sub divided into row from right to left. The rules are also divided into hands so that e.g. one foot is given as three hands and fifteen fingers and also as four palms and sixteen fingers. Surveying and itinerant measurement were undertaken using rods, poles, and knotted cords of rope.
[2] The ancient philosophers, mathematicians, architects have greatly contributed to the development of geometry. Some great names of the science are: Thales (624-548 BC), Pythagoras (570-495 BC), Euclid (c. 300 BC), Archimedes (287-212 BC), and Hippocrates (c. 470-410 BC).
[3] Problem 50 of the Ahmes papyrus (written in hieratic, and probably originated from the Middle Kingdom) i.e. 1 to the square of 8/9 of the circle's diameter. This assumes that π is $4 \times (8/9)2$ (or 3.160493...). Ahmes knew of the modern 22/7 as an approximation for π, and used it to split a hekat (an ancient Egyptian unit of measurement app. 4.8 lit.), hekat x 22/x 7/22 = hekat; however, Ahmes continued to use the traditional 256/81 value for π for computing his hekat volume found in a cylinder.
Problem 48 involved using a square with side 9 units. This square was cut into a 3x3 grid. The diagonal of the corner, squares was used to make an irregular octagon with an area of 63 units. This gave a second value for π of 3.111. The two problems together indicate a range of values for π between 3.11 and 3.16.
Problem 14 in the MMP gives the only ancient example finding the volume of a frustum of a pyramid.
An Indian priest-mathematician, Baudhayana (800-740 BC), has formulated, in his Sulbasutra (I-48), the so-called Pythagoras theorem, centuries before Pythagoras. He stated that the perimeter of the circle depends only on its radius or diameter and that it is actually proportional to the radius or diameter. Though it has not been stated explicitly, it is clear from various sutras that they were well aware that for similar figures, the ratio of the areas equals the square of the ratio of the lengths of the corresponding sides. It was also known that the area of the circle depends only on its radius or diameter and that it is actually proportional to the square of the radius. That is, for a circle, it was known that: Area = Ka x r2 and the Perimeter P = Kp X (2r).
[4] Indian priest-mathematician, Baudhayana (800-740 BC), has formulated, in his Sulbasutra (I-48), the so-called Pythagoras theorem, centuries before Pythagoras. He stated that the perimeter of the circle depends only on its radius or diameter and that it is actually proportional to the radius or diameter. Though it has not been stated explicitly, it is clear from various sutras that they were well aware that for similar figures, the ratio of the areas equals the square of the ratio of the lengths of the corresponding sides. It was also known that the area of the circle depends only on its radius or diameter and that it is actually proportional to the square of the radius. That is, for a circle, it was known that: Area = Ka x r2 and the Perimeter P = Kp X (2r)
In another sutra (I-51) he has given a general rule for finding the square root of any number, both geometrically and arithmetically. In his Sutra (I-61) he found the value of $\sqrt{2}$ to a great accuracy and has given the procedure for the same. This Indian mathematician could construct a circle almost equal in area to a square and vice versa. He has described such procedures in his sutras (I-58 and I-59). As Baudhayana was designing a religious altar for performing the Hindu rites, he constructed a square within a square. Collection point of universal forces. Man (the microcosm), by mentally entering the mandal and proceeding toward its centre, is by analogy guided through the cosmic processes of disintegration and reintegration.
In Sanskrit, the word chakra (Sans. चक्र), means disk or wheel and refers to the energy centres in our body. These wheels or disks of spinning energy correspond to certain nerve bundles and major organs. To function at their best, one's chakras need to stay open, or balanced. If they get blocked, our experience physical or emotional symptoms related to a particular chakra is disturbed. There are 7 main chakras that run along our spine. They start at the root, or base, of our spine and extend to the crown of our head.
[5] Yantra (Sans. यन्त्र), is a geometrical diagram, mainly from the Tantric traditions of the Indian religions. Yantras are used for the worship of deities in temples or at home; as an aid in meditation; used for the benefits given by their

supposed occult powers based on Hindu astrology and tantric texts. They are also used for adornment of temple floors, due mainly to their aesthetic and symmetric qualities. Specific yantras are traditionally associated with specific deities and/or certain types of energies used for accomplishment of certain tasks, vows that may be materialistic or spiritual in nature. It becomes a prime tool in certain sadhanas performed by the *sadhaka* (the spiritual seeker). Yantras hold great importance in Hinduism, Jainism and Buddhism.

[6] In Sanskrit, the word chakra (Sans. चक्र), means disk or wheel and refers to the energy centres in our body. These wheels or disks of spinning energy correspond to certain nerve bundles and major organs. To function at their best, one's chakras need to stay open, or balanced. If they get blocked, our experience physical or emotional symptoms related to a particular chakra is disturbed. There are 7 main chakras that run along our spine. They start at the root, or base, of our spine and extend to the crown of our head.

[7] Zen practitioners relate the idea to *wabi-sab*i (transience and imperfection). Their symbol *ensō* illustrates an absolute enlightenment, a strength, elegance, the universe, and *mu* (the void). It is characterized by a minimalism born of Japanese aesthetics.

Drawing *ensō* is a disciplined-creative practice of Japanese ink painting, (*sumi-e*). Usually, a person draws the *ensō* in one expressive fluid stroke. When drawn according to the *sosho* (cursive) style of Japanese calligraphy, the brushstroke is especially swift.

Once the *ensō* is drawn, one does not change it. It evidences the character of its creator and the context of its creation in a brief, continuous period of time. Drawing *ensō* is a spiritual practice that one might perform as often as once per day. This spiritual practice of drawing *ensō* for self-realization exemplifies various dimensions of the Japanese perspective and aesthetic: *fukinsei (asymmetry,* irregularity), *kanso* (simplicity), *koko* (basic; weathered), *shizen* (without pretence; natural), *yugen* (subtly profound grace), *datsuzoku* (freedom) and *seijaku* (tranquillity).

[8] The image of three in one is frequently found in Celtic art and poetry. A popular 5th century Christian poem, *Carmen Paschale*, uses that sort of imagery, and it survives in over 400 medieval manuscripts. It including the past, the present and the future.

Over the years, the meaning of the symbol has changed depending upon the group of people that used it. Their ancient Celtic goddess *Brigit*, was one of three goddesses of the same name, daughters of *Dagda*, the great god of that country. Her two sisters were connected with healing and with the craft of the smith. Other Celtic deities found in many regions include *Lugus*, the tribal god *Toutatis*, the thunder god *Taranis*, the horned god *Cernunnos*, the horse and fertility goddess *Epona*, the divine son *Maponos*, as well as, *Belenos*, *Ogmios*, and *Sucellos*

[9] The High Holiday season actually begins on the 17th of Tammuz, the day that marks the beginning of the destruction of the ancient Jewish Temple in Jerusalem. On this day Jewish people mourn the lost glory of past, and yearn for the restoration of the Holy Temple. The date also marks the day that Moses destroyed the first set of Tablets. Concluding that they were no longer worthy of their divine mandate, he hurled the Tablets to the ground. This sin was the beginning of a long slide that culminated with the second tragedy marked on this day—the destruction of the Temple. Mindful of these two tragedies, observe a period of repentance that extends till Rosh Hashanah, the first day of the New Year. On this day God sits in judgment, and people beseech Him to judge us favourably. Jewish fate remains uncertain till Yom Kippur, the day deemed by history as the Day of Atonement. On this day God forgave the ancestors for the sins and consented to provide Moses with a new set of Tablets. The circle closes. What began on the day the Tablets were shattered ends on the day the Tablets were replaced? What began as a drive for repentance ends with absolute atonement? This positive conclusion is a cause for celebration, and people do indeed rejoice. The Jews launch into the holiday of Sukkot, a festive time of joy and celebration.

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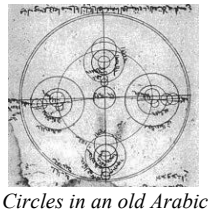
IMAGES RELATED TO ABOVE TEXT

(Fig.1.a): A ram headed falcon representing the Ban of Ra with Shen rings in his grasp

(Fig.1.b: Goddess *Nekhbet's staff with Shen ring

(Fig. 2.): Mongol images— Astronomical Drawing

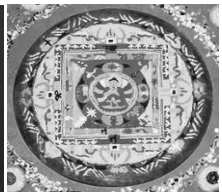
Circular piece of silk with some Mongol images



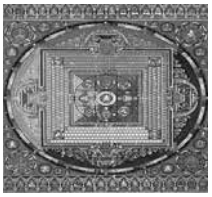
Circles in an old Arabic drawing

(*Fig. 3*): *Some images of the famous mandals*

Aztec Mandal, Mexico



Mandal from Nepal, Tibet and China



The Diamond Realm Japan

A Chines Silk Hanging

A Ilkhanate Silk Mandal

A Hindu Sand Mandal

Mandal with Ganesh

Mandal of Vishnu

A Floor Mandal

(*Fig. 4*): *Greek Pottery Image showing Wheel*

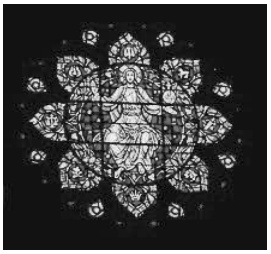
(*Fig. 5*): *The Colosseum, Rome*

(*Fig. 6*): *Gautama Buddha in Yoga*

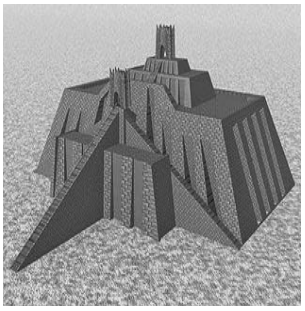
(*Fig. 7*): *Buddha Image in Tibet*

(*Fig. 8.*): *The Jain Ahinsā Symbol (Ahinsā written in circle)*

(*Fig. 9.*) : *The Holy Cross inside the circle; and Mandal at the Marsh Chapel*



(*Fig. 10.*) : *Ziggurat of Ur (2100 BC)*



(*Fig. 11.*): *The Sun God Shamash in his Shrine (Notice the Circle)*

(*Fig. 12.*) :*Sq. in Mongol Image*

(*Fig. 13.*): *Sq. in a Circle with Buddha in the Centre (9th c. AD)*

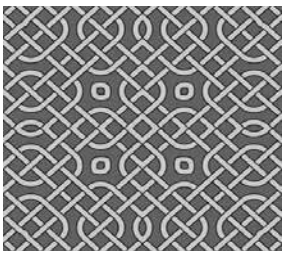
(*Fig. 14.*): *Lotus motif from Sanchi*

(*Fig. 15.*):*Avlokiteshvara at Ajanta Caves, India*

(*Fig. 16.*): *The Buddhist stupa in Sanchi, India*

(*Fig.17*): *Squares in Jainism (Note in the centre)*

(*Fig.18*): *Celtic Sator Sq.*



*The Tree of Life (*עֵץ חַיִּים*) – diagram*