STABILIZATION OF A DYNAMIC MODE OF A WEAKLY IONIZED PLASMA BY LARGE ELECTRON DRIFTS

B. MILIĆ

Institute of Physics, Beograd

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Abstract: Three slow and shortwave aperiodic modes with phase velocities and wavelengths smaller than electron thermal velocity and mean-free-path, respectively, and with the electric vector of the wave lying in the \( (k, E_0) \)-plane are considered in a homogeneous and infinite weakly ionized and non-magnetized plasma placed in a strong external electric d. c. field \( E_0 \). Waves propagating at right angles to \( E_0 \) are studied. Modal spectra are determined from kinetic theory using an extended Margenau-Davydov electron steady-state distribution function. It is shown that under broad assumptions regarding e-n elastic scattering, two finite critical values of electron drift can be specified for one of these modes. If electron drifts lie in this critical range, increasing amplitude waves corresponding to this mode appear; on the contrary, large electron drifts have a stabilizing effect.

1. Introduction

In an infinite uniform plasma placed in a strong external d. c. electric field \( E_0 \) oscillations with the electric vector lying in the \( (k, E_0) \)-plane can be considered separately\(^1\), since the general dispersion equation\(^10\)

\[
\left| k^2 \delta_{ij} - k_i k_j - \frac{\omega^2}{c^2} \varepsilon_{ij}(k, \omega) \right| = 0,
\]

\[ (k^2 - \frac{\omega^2}{c^2} \varepsilon_{xx}) \varepsilon_{xx} + \frac{\omega^2}{c^2} \varepsilon_{x2} \varepsilon_{xz} = 0. \]

The wave vector \( k \) is directed along the \( z \)-axis, and \( E_0 \) lies in the \( xOz \)-plane. The study of equation (2) based on linear perturbation theory has shown\(^2, 3, 4, 5\) that in a weakly ionized non-magnetized plasma there are three modes described by this equation, with phase velocities and wavelengths smaller than
electron thermal velocity and mean-free-path, respectively. Thus the collisionless kinetic equations can be used for the evaluation of the P. D. T. pertaining to these modes. Their properties depend on collisional factors through the electron steady-state distribution function only. In the papers \(^2,^3,^4\) it was assumed that this function possessed a Maxwellian isotropic part, while in the paper\(^5\) the Druyvestein function and a particular type of elastic e-n scattering were considered.

In this paper we re-consider these modes without restrictive assumptions on e-n elastic scattering. Inelastic processes are not taken into account in the present theory. To avoid arithmetically cumbersome formulae, only waves propagating at right angles to \(E_0\) are considered.

2. Electron steady-state distribution function

In the presence of an external electric field, the electron steady-state distribution function in a weakly ionized plasma with predominant elastic collisions is a functional of the e-n elastic scattering cross section; its form was determined on the ground of the Boltzman kinetic equation by Margenau and Davydov\(^6\) (cf. also\(^8\)) to the first order in the electron drift- to-thermal velocity ratio \(q\) (which is \(q \sim \sqrt{m/M}\)) in collisional plasmas. The first order approximation is generally sufficient for longitudinal waves only, whereas in equation (2) terms proportional to \(q^2\) are also important. This is readily seen from the general transformation properties of P. D. T. components of a virtually collisionless plasma\(^7\). Hence, it is necessary to retain the \(P_2\)-term in the expansion of the electron steady-state distribution function in a series of Legendre polynomials. The result is\(^5\)

\[
f_{e0} \rightarrow = f_{e0} \rightarrow (v) - u_1 (v) \frac{\partial f_{e00}}{\partial v} \cos \chi + \\
+ \frac{1}{3} u_2 (v) v \frac{\partial}{\partial v} \left[ \frac{u_1 (v)}{v} \frac{\partial f_{e00}}{\partial v} \right] (3 \cos^2 \chi - 1),
\]

where \(\cos \chi = (\vec{E}_0 \cdot \vec{v})/(E_0 v)\), and

\[
u_s (v) = \frac{e E_0}{m v_s (v)}
\]

\[
= 2\pi N_n v \int_0^\pi \sigma_{en} (v, \theta) \left[ 1 - P_s (\cos \theta) \right] \sin \theta \, d\theta;
\]
the isotropic part of this function is analogous to that of Margenau-Davydov\(^6\) (cf. also\(^1, 5, 8\)). This result was derived using the fact that Legendre polynomials are eigenfunctions of the Boltzmann collision operator\(^8, 9\).

3. Evaluation of plasma dielectric tensor

Let \( f_a^0 (\vec{v}) (\alpha = e, i) \) be steady state distribution functions of plasma constituents, and \( \delta f_a \sim \exp (-i \omega t + i \vec{k} \cdot \vec{r}) \) their perturbations. After some simplifications kinetic equations for these perturbations become\(^4, 10\)

\[
\delta f_a = -\frac{i e_a}{\omega m_a} \left( \vec{\delta E} \cdot \frac{\partial f_a^0}{\partial \vec{v}} \right) + \frac{\vec{v} \cdot \vec{\delta E}}{\omega - k \cdot \vec{v}} \left( \frac{\partial f_a^0}{\partial \vec{v}} \right) + \frac{i}{\omega - k \cdot \vec{v}} \vec{I}_a (\delta f_a),
\]

(6)

with \( \vec{\delta E} \) denoting the electric field of perturbation, and \( \vec{I}_a (\delta f_a) \) the Boltzmann collision integral linearized with respect to \( \delta f_a \). For the modes considered here, one can put \( \vec{I}_a \approx 0 \).

According to the usual procedure\(^{10}\), for the evaluation of P. D. T. components, one first determines the perturbation current density, \( \vec{\delta j}^{(a)} = e_a \vec{v} \delta f_a d^3 \vec{v} \), which is a linear function of \( \delta \vec{E} \); the integration is to be carried out in accordance with the Landau rule\(^{11}\). The components of conductivity tensors \( \sigma_{ij}^{(a)} \) (occurring in the relations \( \delta j_i^{(a)} = \sigma_{ij}^{(a)} \delta E_j \)) and the components of P.D.T. are connected by

\[
\varepsilon_{ij} = \delta_{ij} + \frac{4 \pi i}{\omega} \sum_a \sigma_{ij}^{(a)} = \delta_{ij} + \sum_a \delta \varepsilon_{ij}^{(a)},
\]

(7)

where \( \delta_{ij} \) is the Kronecker delta.

Using equations (3) and (6), the following results are obtained

\[
\delta \varepsilon_{xx}^{(e)} = 2 F \frac{\omega^2 L_e}{\omega^2} \frac{\langle u \rangle^2}{\langle v_e \rangle^2} + i \sqrt{\frac{\pi}{2}} \frac{\omega^2}{L_e} \frac{\omega}{k \langle v_e \rangle},
\]

(8)

\[
\delta \varepsilon_{xx}^{(e)} = \frac{k \langle u \rangle}{\omega} \frac{\omega^2 L_e}{k^2 \langle v_e \rangle^2} \left( D + i \sqrt{\frac{\pi}{2}} \frac{\omega}{k \langle v_e \rangle} \right),
\]

(9)

\[
\delta \varepsilon_{xx}^{(e)} = \frac{\omega^2 L_e}{k^2 \langle v_e \rangle^2} \left( 1 + i \sqrt{\frac{\pi}{2}} \frac{\omega}{k \langle v_e \rangle} \right)
\]

(10)
Only components of \( \delta_{ij}^{(e)} \) pertinent to equation (2) are given. Here, \( \omega_{Le} \) is the electron Langmuir (plasma) frequency, \( \omega_{Le}^2 = (4\pi e^2 N_e/m) \); \( \langle u \rangle \) and \( \langle v_e \rangle \) are electron drift and thermal velocities, respectively, defined by\(^{12,5}\)

\[
\langle u \rangle = - \frac{(2\pi \langle v_e \rangle)^3}{N_e^0} \int_0^\infty u_1(v) \frac{\partial f_{e00}}{\partial u} \, dv,
\]

\[
\langle v_e \rangle = \left\{ \frac{4\pi}{N_e^0} \int_0^\infty f_{e00}(v) \, dv \right\}^{-1/4}
\]

and \( C, D, F, G \) are four functionals of \( f_{e00}(v) \)

\[
C = \frac{(2\pi \langle v_e \rangle)^3}{N_e^0} f_{e00}(0),
\]

\[
D = - \frac{4\pi \langle v_e \rangle^2}{N_e^0 \langle u \rangle} \int_0^\infty u_1(v) v \frac{\partial f_{e00}}{\partial v} \, dv,
\]

\[
F = \frac{4\pi \langle v_e \rangle^2}{3N_e^0 \langle u \rangle^2} \int_0^\infty v^2 u_2(v) \frac{\partial}{\partial v} \left[ \frac{u_1(v)}{v} \frac{\partial f_{e00}}{\partial v} \right] \, dv,
\]

\[
G = (2\pi)^{1/4} \frac{\langle v_e \rangle}{N_e^0} \int_0^\infty v f_{e00}(v) \, dv.
\]

For the Maxwellian \( f_{e00}(v) \), all these coefficients become equal to unity and it is reasonable to expect that they should be of this order of magnitude for any \( f_{e00}(v) \); in general \( F \) may also be a function of \( E_0 \).

The drift and heating of ions in an external d. c. field are negligible in linear stability theory, since they are at least \( M/m \) times smaller than the corresponding quantities for electrons. It is, therefore, possible to take a Maxwellian \( f_i(v) \) and express the results of integration in equation (6) in terms of the plasma dispersion function\(^{10}\)

\[
\mathcal{J}_+(z) = \frac{z}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-x^2/2} \, dx
\]

(17)
with the integration contour chosen in accordance with the Landau rule\textsuperscript{11).} The relevant results are

\begin{equation}
\delta \varepsilon_{xx}^{(i)} = - \frac{\omega_{Li}^2}{\omega^2} \mathcal{I} + \left( \frac{\omega}{k \langle v_i \rangle} \right), \tag{18}
\end{equation}

\begin{equation}
\delta \varepsilon_{xx}^{(i)} = \delta \varepsilon_{xx}^{(i)} = 0, \tag{19}
\end{equation}

\begin{equation}
\delta \varepsilon_{xx}^{(i)} = \frac{\omega_{Li}^2}{k^2 \langle v_i \rangle^2} \left[ 1 - \mathcal{I} + \left( \frac{\omega}{k \langle v_i \rangle} \right) \right], \tag{20}
\end{equation}

\( \omega_{Li} \) is the ion Langmuir (plasma) frequency, and \( \langle v_i \rangle = (T_i/M)^{1/2} \) is the ion thermal velocity with temperature expressed in ergs.

4. General discussion of spectra

We first consider very slow modes with phase velocities smaller than the ion thermal velocity \((\omega/k \ll \langle v_i \rangle \ll \langle v_e \rangle)\). Using an adequate approximation for \(\mathcal{I}^+\), as well as obvious inequalities \(\omega, k \langle u \rangle \ll ck\), the following dispersion equation results from Eq. (2)

\begin{equation}
\left[ c^2 k^2 - i \sqrt{\frac{\pi}{2}} \omega \left( G \frac{\omega_{Le}^2}{k \langle v_e \rangle} + \frac{\omega_{Li}^2}{k \langle v_i \rangle} \right) \right] \left[ 1 + \frac{\omega_{Le}^2}{k^2 \langle v_e \rangle^2} + \frac{\omega_{Li}^2}{k^2 \langle v_i \rangle^2} \right] = \omega_{Le}^2 \frac{\langle u \rangle^2}{\langle v_e \rangle^2} \left[ 2F + (2F - D^2) \frac{\omega_{Le}^2}{\omega_{Li}^2} \langle v_i \rangle^2 \right]. \tag{21}
\end{equation}

Here only the relevant imaginary term in \(\varepsilon_{xx}\) has been retained. The ratio \(\langle \omega_{Le}^2/\omega_{Li}^2 \rangle \langle \langle v_i \rangle/\langle v_e \rangle \rangle^2\) in the square brackets on the right-hand side of this equation may be important in isothermal plasma only, as it is of the order of \(T_i/T_e\). Thus it can be omitted for a plasma in a strong electric field. Furthermore, one usually has \(\omega_{Le}^2/k^2 \langle v_e \rangle^2 \gg 1\), so that the following spectrum is obtained\textsuperscript{2, 8)}

\begin{equation}
\omega = -i \sqrt{\frac{2}{2\pi}} \frac{c^2 k^2}{\omega_{Le}^2} \frac{k \langle v_i \rangle}{G \langle v_e \rangle} + \frac{m}{M} \left( 1 - 2F \frac{\langle u \rangle^2}{\langle v_e \rangle^2} \frac{\omega_{Le}^2}{c^2 k^2} \right) \tag{22}
\end{equation}

This mode becomes aperiodically unstable for

\begin{equation}
\frac{\langle u \rangle^2}{\langle v_e \rangle^2} > \frac{1}{2F} \frac{c^2 k^2}{\omega_{Le}^2}. \tag{23}
\end{equation}
It should be kept in mind that in some cases $F$ may be a function of the applied field $E_0$, so that Eq. (23) defines the critical drift only implicitly.

For modes with larger phase velocities but still not exceeding the thermal velocity of electrons ($\langle v_i \rangle \ll \omega/k \ll \langle v_e \rangle$), Eq. (2) yields

$$
\left( c^2 k^2 + \omega_{Li}^2 - i \sqrt{\frac{\pi}{2}} \frac{\omega_{Le}^2}{k} \frac{\omega}{\langle v_e \rangle} \right) \left[ 1 - \frac{\omega_{Li}^2}{\omega^2} + \frac{\omega_{Le}^2}{k^2 \langle v_e \rangle^2} \left( 1 + i \sqrt{\frac{\pi}{2}} \frac{C}{k} \frac{\omega}{\langle v_e \rangle} \right) \right] + \omega_{Le}^2 \frac{\langle u \rangle^2}{\langle v_e \rangle^2} \left[ 2 F \left( \frac{\omega_{Li}^2}{\omega^2} - \left( 2 F - 1 \right) \frac{\omega_{Le}^2}{k^2 \langle v_e \rangle^2} \right) \right] = 0. \tag{24}
$$

Irrelevant imaginary terms have again been omitted. This equation describes two dynamically (aperiodically) unstable modes.

One of these modes is in the low-frequency region, and exists for $\omega \ll (\omega_{Li}/\omega_{Le})^2 k \langle v_e \rangle$. Its spectrum is$^{3, 5}$

$$
\omega^2 = \omega_{Li}^2 \left( 1 - 2 F \frac{\langle u \rangle^2}{\langle v_e \rangle^2} \frac{\omega_{Le}^2}{c^2 k^2 + \omega_{Li}^2} \right). \tag{25}
$$

The aperiodically growing instability of this mode sets on for drifts determined by Eq. (23). If Eq. (23) does not hold, the mode is kinetic and always stable$^0$.

The other mode lies in the region of high frequencies, $\omega/k \langle v_e \rangle \gg \omega_{Li}/\omega_{Le}$, $c^2 k^2/\omega_{Le}^2$; the spectrum is$^{4, 5}$

$$
\omega = i \sqrt{\frac{2}{\pi}} \frac{\langle u \rangle^2}{\langle v_e \rangle^2} \frac{2 F - 1}{G} k \langle v_e \rangle. \tag{26}
$$

In this case the aperiodical instability sets on for $F \gg 1/2$.

In a very strong external electric field, the average kinetic energy of electrons is much larger than that of heavy particles, so that one can put $T_n = 0$ in the Margenau-Davydov function$^0$. Hence, we approximately have

$$
f_{e^{00}}(v) = A \exp \left[ - \int_0^\infty \frac{3 \delta m^2 v^2}{2 e^2 E_0^2} d\nu \right], \tag{27}
$$

where $A$ is a normalization factor, and $\delta = 2m/M$. It has been shown by Gurevich$^{14}$ that from a large variety of functions given by this equation for various cross sections $\sigma_{en}$, only two functions can be actually encountered. For strong fields below some critical value $f_{e^{00}}$ will always be Maxwellian,
and above this critical value always Druyvesteinian\(^{13,8}\). This is a consequence of scarce e–e collisions taken into account in the Gurevich theory by an iteration process. These two particular distribution functions are formally obtained from Eq. \((27)\) by letting \(v_1 (v) = v_{10}\), and \(v_1 (v) = v_{10} \cdot v\), respectively, where the latter case corresponds to the »billiard-ball« model; \(v_{10}\) is a constant in both cases.

4. Plasma with the Druyvestein distribution

Inserting \(v_1 (v) = v_{10} \cdot v\) into Eq. \((27)\), one obtains\(^{13,8}\)

\[
\rho_0 (v) = \frac{N_0}{\pi v_D^3} \exp \left( - \frac{v^4}{v_D^4} \right). \tag{28}
\]

Here, \(v_D = (8e^2 E_0^2/3m^2 \delta v_{10}^2)^{1/4}\) is the characteristic velocity of the distribution. The normalization factor is expressed in terms of the \(I\)-function.

For this \(\rho_0 (v)\), equations \((11)-(16)\) yield

\[
\langle v_e \rangle = v_D \left[ I' (0.75)/I' (0.25) \right]^{1/2} \approx 0.57 \ n_D ,
\]

\[
\langle u \rangle = 2 \sqrt{2\pi} \left[ I' (0.75)/I' (0.25) \right]^{3/2} \frac{eE_0}{m v_{10} n_D} \approx 0.96 \ \frac{eE_0}{m v_{10} n_D} ,
\]

\[
C = 2 \sqrt{2\pi} \left[ I' (0.75)/I' (0.25) \right]^{3/2} \Gamma^{-1} (0.75) \approx 0.78 ,
\]

\[
D = \frac{2}{\sqrt{2\pi}} \left[ I' (0.25)/I' (0.75) \right]^{3/2} \Gamma^{-1} (0.25) \approx 1.10 ,
\]

\[
G = \sqrt{2\pi} \left[ I' (0.25)/I' (0.75) \right]^{1/2} \Gamma' (0.5) \approx 1.05 .
\]

The evaluation of \(F\), Eq. \((15)\), requires a more detailed knowledge of the scattering process. Unfortunately, reliable data on absolute differential cross sections are practically unavailable at present. It is, however, physically admissible to represent the cross section in a series of Legendre polynomials

\[
\sigma_{en} (v, \phi) = \sum_{r=0}^{\infty} \sigma^{(r)} (v) P_r (\cos \phi) ; \tag{29}
\]

the coefficients of this expansion are restricted only by the requirement that the integrals
\[ 2\pi \int_0^{\pi} \sigma_{en}(v, \vartheta) \sin \vartheta d\vartheta \quad \text{and} \quad 2\pi \int_0^{\pi} \sigma_{en}(v, \vartheta) (1 - \cos \vartheta) \sin \vartheta d\vartheta \]

be positive, since they have an immediate physical meaning as total and momentum transfer cross sections. From Eqs. (5) and (29) one deduces

\[ \nu_1(v) = 4\pi N_n v \left[ \sigma^{(0)}(v) - \frac{1}{3} \sigma^{(1)}(v) \right], \quad (30) \]

\[ \nu_2(v) = 4\pi N_n v \left[ \sigma^{(0)}(v) - \frac{1}{5} \sigma^{(2)}(v) \right]. \quad (31) \]

For the Druyvestein distribution, \( \sigma^{(0)} \) and \( \sigma^{(1)} \) ought to be constants. Furthermore, \( \sigma^{(0)} \) is necessarily positive, whereas \( \sigma^{(1)} \) can be negative as well and is restricted by \( \sigma^{(1)}/\sigma^{(0)} \leq 3 \) only. As for \( \sigma^{(2)}(v) \), we shall assume it in the form \( \sigma^{(2)}/v^2, 0 \leq s \leq 2 \). We re-write Eq. (31) as

\[ \nu_2(v) = 4\pi N_n v \sigma^{(0)}(1 - \gamma/v'), \quad (32) \]

where \( \gamma = \sigma^{(2)}/5\sigma^{(0)} \) can be both positive and negative. Inserting Eq. (32) into Eq. (15), one obtains

\[ F = \frac{2}{3\pi} \sqrt{\frac{T'(0.25)}{T'(0.75)}} \left( 1 - \frac{1}{3} \frac{\sigma^{(1)}}{\sigma^{(0)}} \right) \int_0^\infty \frac{\xi^{s+2}}{\xi^{s} - (\gamma/v')^2} \frac{d\xi}{d\xi} (\xi e^{-\xi'}) d\xi \quad (33) \]

Let us first consider the case \( s \neq 0 \). For very strong electric fields one has \( \gamma/v' \ll 1 \), so that with first-order terms we obtain

\[ F = 0.18 \left( 1 - \frac{1}{3} \frac{\sigma^{(1)}}{\sigma^{(0)}} \right) \left[ 1 + \frac{\gamma}{v_D'} A(s) \right], \quad (34) \]

where \( A(s) = (1 - s/2) I'(0.75 - s/4) I^{-1}(0.75) \). The correction in the square brackets is important for the mode given by Eq. (26) only. The instability sets on if \( F > 1/2 \) or

\[ 1 + \frac{\gamma}{v_D'} A(s) > \frac{2.79}{1 - \frac{1}{3} \frac{\sigma^{(1)}}{\sigma^{(0)}}}. \quad (35) \]

A detailed analysis of this condition shows that the obtained inequality can hold with \( \sigma^{(1)} < 0 \) only. For \( \gamma < 0 \), inequality (35) is possible only if \( |\sigma^{(1)}|/\sigma^{(0)} > 5.37 \), and is equivalent to
\[ \nu_D' > \left| \gamma \right|, \left| \gamma \right| A(s) \frac{3 + (|\sigma^{(1)}| / \sigma^{(0)})}{5.37 + (|\sigma^{(1)}| / \sigma^{(0)})} \] (36)

If, however, \( \gamma > 0 \) and \( 3 < |\sigma^{(1)}| / \sigma^{(0)} < 5.37 \) one obtains

\[ \gamma < \nu_D' < \gamma A(s) \frac{3 + (|\sigma^{(1)}| / \sigma^{(0)})}{5.37 - (|\sigma^{(1)}| / \sigma^{(0)})}. \] (37)

Hence, if the conditions leading to inequality (36) are fulfilled, the instability corresponding to the mode considered takes place for \( E_0' \)’s or, alternatively, for electron drifts above some critical value. On the contrary, if the conditions leading to inequality (37) are valid, a gradual increase of \( E_0 \) first destabilizes the mode since the lower limit in (37) is attained, and then again stabilizes the mode for \( E_0' \)'s and drifts above the value given by the upper limit. Which of the mentioned conditions will be fulfilled depends on the nature of collisions.

For \( s = 0 \), Eq. (33) yields simply

\[ F = \frac{0.22}{1 - \gamma} \left( 1 - \frac{1}{3} \frac{\sigma^{(1)}}{\sigma^{(0)}} \right) \] (38)

Since \( F \) is independent of \( E_0 \), the mode will be either stable or unstable for any drift. From Eq. (38) and the requirement \( F > 1/2 \), one easily deduces the conditions under which these alternatives take place.

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Razmotrene su tri spore kratkotalasne aperiodične mode, sa faznim brzinama i talasnim dužinama manjim od termalne brzine i srednje slobodne putanje elektrona respektivno, i sa električnim vektorom talasa u \((\mathbf{k}, \mathbf{E}_0)\)-ravni, kod homogene i beskonacne, slabojonizovane i nemagnetoaktivne plazme koja je smeštena u jako spoljašnje konstantno električno polje \(\mathbf{E}_0\). Njihovi spektri su određeni na osnovu kinetičke teorije, uz korišćenje generalisane Margenau-Davydov-ijeve distribucije za elektrone u stacionarnom stanju. Pokazano je da pod vrlo opštim fizičkim predpostavkama u pogledu elastičnog e-n rasejanja u plazmi, za jednu od ovih moda se mogu odrediti dve konačne kritične vrednosti drifta. Talasi rastuće amplitude koji odgovaraju ovoj modi nastaju samo ako elektronski drift leži u tom kritičnom intervalu; velike vrednosti drifta ih stabiliziraju.