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# ESTIMATION OF AN EFFICIENT BENCHMARK PORTFOLIO FOR THE EASTERN EUROPEAN MARKET

## Abstract

**Purpose:** This paper explores the mean-variance inefficiency of cap-weighted indices based on the CECE index as a benchmark.

**Methodology:** For the period from March 2014 to September 2021, several proxies of efficient portfolios were estimated: the Global Minimum Variance (GMV) portfolio, the Maximum Sharpe Ratio (MSR) portfolio and the portfolio with equal weights of constituents (EW). Diversification of strategies was also considered by analyzing the performance of a portfolio consisting of GMV and MSR that were weighted equally. Based on monthly data, 90 out-of-sample estimations were made for each strategy in order to compare their risk-return characteristics. Furthermore, to confirm the differences in the riskiness and returns of the estimated portfolios, the F-test and the Welch test were performed, respectively.

**Results:** The results show that all analyzed portfolios achieved superior performance compared to the CECE index with the GMV portfolio leading the way.

**Conclusion:** Research findings highlight the importance of market development and liquidity when pursuing popular scientific diversification methods.

**Keywords:** Efficient portfolio estimation, risk-reward ratio, diversification strategies, "smart" beta, the CECE Composite Index

## 1. Introduction

The market capitalization-weighted indices (cap-weighted indices) are typically used for the purpose of pursuing passive investment strategies. For a long time, such approach has often been presented in the context of the modern portfolio theory (MPT) and the CAPM (Capital Asset Pricing Model), without due consideration of the appropriate application of theoretical concepts in the practice of stock market investing. However, due to the fact that the optimal market portfolio (M) introduced in the MPT is not observable in the real world, the cap-weighted indices are used as its approximation in the portfolio management process (Amenc et al., 2011). Empirical research has shown that such indices are often inefficient in terms of the risk-reward trade-off, meaning that they are not necessarily the optimal investment strategy for investors as they do not provide adequate compensation for systematic risk (Haugen & Baker, 1991; Grinold, 1992; Amenc et al., 2006).

Too high concentration leading to exposure to unrewarded risk factors and poor exposure to rewarded risk factors are often highlighted as the two main shortcomings of cap-weighted indices (Amenc et al., 2006; 2011; 2014). Therefore, alternative approaches to portfolio construction have been developed and presented in the literature to address these issues. Such approaches are called "smart" or scientific beta strategies and their aim is to construct a portfolio with better performance compared to its cap-weighted counterpart. In this paper, we focus only on efforts aimed at dealing with the first problem of cap-weighted indices, i.e., the efficient elimination of unrewarded risk in the portfolio.

The goal of this research is therefore to provide insight into the applicability of "smart" beta strategies in emerging markets. For this purpose, we focus on Eastern European markets represented by the CECE benchmark index available on the Vienna Stock Exchange. The index includes the most liquid stocks listed on the Budapest, Prague and Warsaw stock exchanges. All three countries are listed in the emerging markets category by the renowned index providers – the MSCI and the EDHEC-Risk Institute (Scientific Beta, 2022). We test the Maximum Sharpe Ratio (MSR), Global Minimum Variance (GMV) and Equal Weighting (EW) strategies.

The contribution of this paper lies in the out-of-sample testing, ensuring also the composition matching of the tested portfolios in relation to their cap-weighted benchmark. Thus, the appropriate framework for the performance analysis is set enabling the comparison of the results to other papers conducting similar research for the more and less developed financial markets. Our findings reveal that the pursued strategies outperform the cap-weighted benchmark in the case of analyzed emerging markets highlighting the importance of market liquidity.

The rest of the paper is structured as follows. A literature review is presented in the second section, while the data and methodology used in this research are presented in the third section. The empirical part is covered in the fourth section. Finally, the conclusion is given in the fifth section.

## 2. Literature review

For a developed market, it is well documented that it is possible to outperform the cap-weighted counterpart. Amenc et al. (2006) tested two weighting schemes – mean-variance optimization and equal weighting on an in-sample basis for the U.S. and European equity index markets. It was concluded that the existing stock market indices are highly inefficient compared to the mean-variance optimal portfolios. Amenc et al. (2011) and Amenc et al. (2013) provided details of the out-of-sample testing for the selected so-called "smart" or scientific beta strategies. Amenc et al. (2013) presented very thoroughly several "smart" beta strategies with weighting schemes, required parameters to be estimated and optimality conditions. Strategies tested in this paper are frequently pursued since, as presented in e.g. Amenc et al. (2013), this choice allows the examination of trade-off between optimality and estimation risk. Namely, the estimation of the MSR portfolio, which is optimal by construction, includes high estimation risk, since it requires the estimation of most parameters (the expected returns, volatilities and correlations of returns) than any other strategy. The opposite is the estimation of the EW portfolio which has no estimation risk (estimations of parameters are not needed) but the optimality risk is high.

Since the estimation of the expected return mostly presents the biggest challenge (Amenc et al., 2013), instead of the estimation of the MSR portfolio, focus should be placed on the estimation of the suboptimal portfolio. Such portfolio is the GMV portfolio as less estimation risk is involved (necessary inputs are only volatilities and correlations). However, GMV portfolios often suffer from a low-volatility bias by primarily targeting a low-volatility objective over decorrelation conditions (Clarke et al., 2011). None of the strategies mentioned above is dominant from an out-of-sample risk-adjusted perspective. Thus, DeMiguel et al. (2009) argue that the MSR strategy does not consistently outperform the EW strategy, while Amenc et al. (2013) argue that the GMV strategy typically outperforms MSR, but often at the expense of portfolio concentration. For the Asian market, Padmanaban et al. (2013) tested equal-weighted, GMV and MSR portfolios. A considerable increase in the Sharpe ratio is obtained for all alternative portfolios, except for the GMV portfolio of FTSE China 25 stocks, which ends up with a lower Sharpe ratio than the cap-weighted counterpart (Padmanaban et al., 2013, p. 8).

Whether the application of these advances in less developed markets can lead to similar results is not completely clear and the literature on the topic is scarce. Madsstuen (2015) found that the MSR and GMV strategies in emerging markets did not outperform the global cap-weighted benchmark (although an emerging market benchmark should have been used). Nowak (2016) tested EW, GMV, MSR and fundamental weighting strategies, which outperformed the cap-weighted benchmark on the Warsaw Stock Exchange in the emerging Polish stock market (but it remains unclear whether an out-of-sample estimation was conducted).

In the research focusing on the undeveloped and illiquid Croatian market, scientific beta strategies exhibited poor performance. For instance, the MSR estimation in the Croatian stock market did not outperform the cap-weighted benchmark in the out-of-sample analysis for the entire observed period regardless of market conditions (expansion or recession) (Dolinar et al., 2017). The same was found to be true regarding the GMV portfolio estimation in (Zoričić et al., 2018), although the results were better than in the case of the MSR portfolio.

## 3. Data and methodology

### 3.1 Data and data sources

In this paper, the CECE Composite Index (CECE) listed on the Vienna Stock Exchange is used as a benchmark index for the Eastern European emerging stock market. It is a "free-float" index based on market capitalization and only the most liquid stocks are used as index constituents. In other words, the CECE index incorporates only actively traded stocks in Eastern Europe's capital markets, more precisely Budapest, Prague, and Warsaw stock exchanges. Thus, the composition of the CECE index is based on the stocks included in CTX (Czech Traded Index), HTX (Hungarian Traded Index), and PTX (Polish Traded Index), which are also weighed on the "free-float" market capitalization basis. All these indices were introduced to the market for the first time in 1996. For CTX, HTX, and PTX, the maximum weight for individual stocks is set to 25%, and for the CECE index to 20%.

The CECE Composite Index is a price index, so the derived monthly returns do not include any potential dividend yields. The index is expressed in euros (EUR) and dollars (USD), respectively. For the purpose of this paper, only the CECE EUR index is observed. The index has no restrictions related to the sector or national exposure. The values of the CECE index are displayed in real time on the Vienna Stock Exchange website. Regular revisions of the CECE index are carried out twice a year, in March and September.

The observation period in this paper covers the period beginning in March 2014 and ending in September 2021. For this analysis, 15 revisions of the CECE index are taken into account, in which the number of constituents varied from 24 to 33 stocks (Vienna Stock Exchange, 2022). Altogether, 52 stocks that were included in the CECE index at any point in time were analyzed.

### 3.2 Methodology

To test whether alternative approaches to portfolio optimization can outperform the CECE index (based on market capitalization), several portfolios were estimated: the GMV portfolio, the MSR portfolio, the EW portfolio, and a portfolio consisting of the GMV and MSR portfolios (50:50%). Amenc et al. (2013) suggested that it makes sense to use the GMV portfolio as a proxy for the most desired portfolio, i.e., the optimal benchmark MSR portfolio might suffer from efficiency costs since the out-of-sample estimation of such benchmark involves a high level of estimation risk. In the case of GMV portfolio estimation, only correlations and volatilities are required as inputs for the optimization process, while the estimation of the expected returns, the dominant source of the estimation error, is not required (Amenc et al., 2013, p. 31).

GMV and MSR portfolio estimation is always performed for the actual CECE index composition. This means that for each revision of the CECE index, it is necessary to create a new set of inputs (estimation of covariances from the period before the revision) and a new set of outputs (estimation of an out-of-sample performance) of the GMV and MSR portfolios. For the estimation of the ex-post covariance matrix, we observe a 3-year period of monthly returns for each constituent stock before each CECE index revision. Such estimated covariance matrix is then used in the optimization process to estimate the optimal weights of the constituents for each portfolio separately. The estimation of optimal weights of constituents for GMV portfolios is performed using the following formula:

$$\boldsymbol{w}^* = arg\min_{\boldsymbol{w}} \frac{\boldsymbol{\Sigma}^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}}, \tag{1}$$

where $\boldsymbol{w}^*$ is the vector of weights (i.e. optimal weights), $\boldsymbol{1}$ is the vector of ones, and $\boldsymbol{\Sigma}$ is the covariance matrix for expected returns of the constituents. Matrix elements (covariances and variances) are estimated using the following formula:

$$\sigma_{ij}^2 = \frac{1}{T-1}\sum_{t=1}^{T}(r_i - \overline{r_i})\left(r_j - \overline{r_j}\right), \tag{2}$$

where $T$ represents the number of in-sample observations, $r_i$ are monthly returns of stocks, and $\overline{r_i}$ represents the arithmetic mean of stock returns.

An MSR portfolio is then estimated that should best mimic the optimal market portfolio (M) from the MPT. Such a portfolio is optimal by its construction; however, it requires the highest number of estimated parameters (expected returns, volatilities, and correlation of stocks) than any other strategy with the estimation of the expected return presenting the biggest challenge (Amenc et al., 2013). Risk parameters are estimated in the same way as for the GMV portfolios, while the expected returns are estimated using the following formula:

$$E(r_i) = \frac{1}{T}\sum_{t=1}^{T} r_i \, , \tag{3}$$

where $T$ represents the number of in-sample observations and $r_i$ are monthly returns of stocks. In addition to the arithmetic mean, the median is also used to estimate the expected return, which yielded better results compared to the arithmetic mean (Table 1). The following formula is used to estimate the optimal weights of the constituents for the MSR portfolios:

$$\boldsymbol{w}^* = arg\max_{\boldsymbol{w}} \frac{\boldsymbol{w}'\boldsymbol{\mu}}{\sqrt{\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}}}\, , \tag{4}$$

where $\boldsymbol{w}^*$ is the vector of weights (i.e. optimal weights), $\boldsymbol{\mu}$ is the vector of the expected returns, and $\boldsymbol{\Sigma}$ is the covariance matrix for the expected returns of the constituents.

Furthermore, to achieve greater portfolio deconcentration in the GMV and MSR portfolios, the constraint on the minimum weight for a constituent is imposed in the optimization process by defining the lower limit as follows:

$$w_i^* \geq \frac{1}{\lambda N}\, , \tag{5}$$

where $w_i^*$ represents the optimal weight of stock $i$ in the GMV and MSR portfolios, $N$ is the number of constituents in each revision, and lambda (λ) represents a flexibility parameter (Amenc et al., 2011). A higher lambda implies a weaker constraint leading to a higher concentration of the GMV and MSR portfolios. In this paper, lambda is set arbitrarily to 4 and 1.5. Setting lambda to 1 reduces a GMV portfolio to an EW portfolio. The use of weaker constraints pronounces a serious concern regarding minimum variance portfolios as they are typically heavily concentrated in assets with the lowest volatility (Amenc et al., 2013, p. 31).

The rolling window of 36 months referring to 36 in-sample observations is used to estimate both the GMV and MSR portfolio out-of-sample. The out-of-sample performance of the GMV and MSR portfolios is assessed on a monthly basis. Namely, 15 revisions of the CECE index are covered (March 2014 - September 2021), so the rolling window of out-of-sample periods is carried over 7.5 years. In this way, the GMV and MSR portfolios are estimated 90 times, i.e., for each GMV and MSR portfolio, time series of 90 monthly returns are obtained.

## 4. Research findings

The out-of-sample performance of the estimated GMV and MSR portfolios for the whole observation period is compared to the CECE index, i.e., the benchmark index that is the market-cap counterpart. The results are shown in Table 1. In addition, the EW portfolio is observed as an additional portfolio for the purpose of performance comparison since it presents a naïve diversification strategy. The risk-reward ratio is used as a key performance measure. Returns are calculated based on monthly returns and refer to the geometric mean, while risk refers to their standard deviation. The estimation of the MSR portfolio based on the arithmetic mean and median is reported separately.

***Table 1 Performance of the estimated portfolios and the CECE index***

| | CECE | GMV (λ = 4) | GMV (λ = 1.5) | MSR* (λ = 4) | MSR* (λ = 1.5) | MSR** (λ = 4) | MSR** (λ = 1.5) | EW |
|---|---|---|---|---|---|---|---|---|
| Return | 0.00%[1] | 0.67% | 0.68% | 0.64% | 0.70% | 0.86% | 0.93% | 0.56% |
| Risk | 5.88% | 3.21% | 3.61% | 4.46% | 4.34% | 4.63% | 4.48% | 4.48% |
| Risk-reward ratio | 0.001 | 0.209 | 0.187 | 0.144 | 0.161 | 0.186 | 0.207 | 0.125 |

* Arithmetic mean used as a measure of expected returns.
** Median used as a measure of expected returns.

*Source: Authors' calculation*

As depicted in Table 1, all estimated portfolios managed to outperform the cap-weighted benchmark, including the EW portfolio. As expected, the EW portfolio achieved a higher return compared to the cap-weighted counterpart, surprisingly with lower risk. Research has shown that EW portfolios have significantly higher risk compared to cap-weighted and price indices, and that a higher risk-reward ratio is a result of the increase in the return due to the exposure to rewarded risk factors in spite of the accompanying increase in risk (Plyakha et al., 2012).

The F-test is performed to confirm the differences in the riskiness of the estimated portfolios, the EW portfolio, and the benchmark index. These differences in variances (riskiness) of all tested strategies and the benchmark index are statistically significant at the 1% level of significance. In addition, differences in variances (riskiness) between the GMV and MSR portfolios and the GMV and EW portfolios are statistically significant at the 1% level of significance, while the differences in variances (riskiness) between the MSR and EW portfolios are not statistically significant even at the 10% level of significance.

The Welch test was performed to test the differences in the returns of the estimated portfolios, the EW portfolio, and the benchmark index. These differences in returns of all tested strategies and the benchmark index are not statistically significant at the 10% level of significance. Furthermore, with the same level of significance, the differences in returns between the GMV and MSR portfolios, the GMV and EW portfolios, and the MSR and EW portfolios are not statistically significant.

***Table 2 Performance of the estimated portfolios and their combinations with the arithmetic mean used for the estimation of the expected returns***

| | GMV (λ = 4) | MSR (λ = 4) | 50% GMV + 50% MSR (λ = 4) | GMV (λ = 1.5) | MSR (λ = 1.5) | 50% GMV + 50% MSR (λ = 1.5) |
|---|---|---|---|---|---|---|
| Return | 0.67% | 0.64% | 0.67% | 0.68% | 0.70% | 0.69% |
| Risk | 3.21% | 4.46% | 3.58% | 3.61% | 4.34% | 3.89% |
| Risk-reward ratio | 0.209 | 0.144 | 0.186 | 0.187 | 0.161 | 0.177 |

*Source: Authors' calculation*

In addition to the performance of the estimated GMV and MSR portfolios that has already been presented (Table 1), Table 2 presents portfolios estimated as a combination of the optimization techniques for GMV and MSR portfolio estimation with the arithmetic mean used as an estimator for the expected return. The results suggest that the equally weighted portfolio of the GMV and MSR portfolios dominates over the MSR portfolio due to risk reduction yielding a higher risk-reward ratio (for both constraints set for the minimum weight for the constituents). However, due to higher risk and an (almost) equal return to the GMV portfolio, its performance is not superior to the GMV portfolio. Regardless of the constraint set for the minimum

1 More specifically, the average return of the CECE index was 0.00332%. All results in the tables are rounded to two decimal places for the sake of readability.

weight for the constituents, the performance of the portfolio with equal weights of the GMV and MSR portfolios is between the performance of the GMV and MSR portfolios.

***Table 3 Performance of the estimated portfolios and their combinations with the median used for the estimation of the expected returns***

| | **GMV** (λ = 4) | **MSR** (λ = 4) | **50% GMV + 50% MSR** (λ = 4) | **GMV** (λ = 1.5) | **MSR** (λ = 1.5) | **50% GMV + 50% MSR** (λ = 1.5) |
|---|---|---|---|---|---|---|
| Return | 0.67% | 0.86% | 0.77% | 0.68% | 0.93% | 0.81% |
| Risk | 3.21% | 4.63% | 3.71% | 3.61% | 4.48% | 3.95% |
| Risk-reward ratio | 0.209 | 0.186 | 0.208 | 0.187 | 0.207 | 0.204 |

*Source: Authors' calculation*

Similarly to Table 2, Table 3 presents portfolios estimated as a combination of the optimization techniques for GMV and MSR portfolio estimation but with the median used as an estimator for the expected return. As depicted in Table 3, an equally weighted portfolio of the GMV and MSR portfolios with λ = 4 dominates over the MSR portfolio due to risk reduction yielding a higher risk-reward ratio. In comparison to the GMV portfolio, the portfolio with equal weights of the GMV and MSR portfolios achieved a higher return with slightly higher risk. When a tighter constraint is set to the minimum weights for the constituents, the portfolio with equal weights of the GMV and MSR portfolios does not dominate over the MSR portfolio.

The F-test is performed to confirm the differences in the riskiness of the GMV and MSR portfolios and portfolios estimated as a combination of the optimization techniques for GMV and MSR portfolio estimation. These differences in variances (riskiness) of the MSR portfolio and a portfolio estimated as a combination of the GMV and MSR portfolios with λ = 4 are statistically significant at the 5% level of significance. However, this difference in riskiness is not statistically significant at the 1% level of significance. Regarding the GMV portfolio, the differences in variances (riskiness) of the GMV portfolio and a portfolio estimated as a combination of the GMV and MSR portfolios with λ = 4 and the median used as an estimator for the expected return are statistically significant at the 10% level of significance. However, this difference in riskiness is not statistically significant at the 5% level of significance. In other cases, the differences in the riskiness are not statistically significant at the 10% level of significance.

The Welch test was performed to test the differences in the returns of the estimated GMV and MSR portfolios and portfolios estimated as a combination of the optimization techniques for the GMV and MSR portfolios. The differences in returns of these strategies are not statistically significant at the 10% level of significance.

## 5. Discussion

In contrast to the results for the Croatian stock market (Dolinar et al., 2017), the results given in Table 1 suggest that it is possible to estimate the MSR portfolio with a higher return compared to the EW portfolio and the benchmark index. In addition, when the arithmetic mean is used as an estimator for the expected return, it can be noticed that the risk of the MSR portfolio is lower than the risk of the benchmark index and approximately equal to the risk of the EW portfolio, which could be interpreted as a result of a better diversification of unrewarded risk factors. Namely, Amenc et al. (2014) define scientific methods as a tool to achieve superior results compared to cap-weighted counterparts since they solve the problem of excessive exposure to unrewarded risk factors. These superior results (a higher return and lower risk compared to the EW portfolio and the benchmark index) are achieved when the limit for the minimum weight for the constituents of the MSR portfolio is set to 1.5 (λ = 1.5), which presents a stronger constraint on the minimum weight leading to higher deconcentration compared to the MSR portfolio with λ = 4. When the median is used for the estimation of the expected return instead of the arithmetic mean, the average return is higher compared to results when the arithmetic mean is used; however, the risk is also higher and closely resembles the risk of the EW portfolio if lambda is 1.5. Based on the presented results, the median proved to be a better estimator of the expected return than the arithmetic mean. It is more robust and can be more easily adapted to the data used for estimation.

As expected, the GMV portfolios achieved the lowest risk but also a higher return compared to the benchmark index, the EW portfolio, and the MSR portfolio, with weaker constraints on the minimum weight for the constituents ($\lambda = 4$) and the arithmetic mean used for the estimation of the expected return. As opposed to the MSR portfolio, the GMV portfolio achieved better performance with higher lambda ($\lambda = 4$), which is in line with the research of Zoričić et al. (2018), where the GMV portfolio performance is improved when a higher concentration in the portfolio is allowed. Generally, in this research, the GMV portfolio dominates the MSR portfolio, especially in the case when weaker constraints on the minimum weights for the constituents are used.

A combination of the optimization techniques for GMV and MSR portfolio estimation did not yield a portfolio with superior performance compared to the GMV or MSR portfolios. Moreover, the median proved to be a better estimator for the expected return for the MSR portfolio allowing for a higher return and a higher risk-reward ratio of the portfolio estimated as a combination of the GMV and MSR portfolios compared to the same portfolio when the arithmetic mean is used. Since only MSR estimation requires expected return estimation, this is due to better MSR estimation, which leads to the conclusion that the performance of a combined portfolio would be even better if enhanced techniques for the covariance matrix and expected return estimation were used. Since this was not tested in this paper, it could be considered as a limitation of the research.

The findings demonstrate that for emerging markets (in contrast to the undeveloped and illiquid Croatian market), the out-of-sample estimation results corroborate the findings for the developed markets, such as in Amenc et al. (2011; 2013). The results suggest that the selection of the most liquid shares traded on the stock exchanges leads to the possibility of outperforming the cap-weighted benchmark. Furthermore, to find a trade-off between optimality risk and estimation risk, strategies are often combined (i.e. diversification of strategies). This also exploits a low correlation of parameter estimation errors among strategies. Additionally, the performed tests showed that the differences in the riskiness of all estimated portfolios and the benchmark index are statistically significant at 1%, while the differences in the returns of the tested strategies and the benchmark index are not statistically significant at 10%.

## 6. Conclusion

This paper presents the out-of-sample testing of modern strategies available to research-driven investors for the developed markets, demonstrating the possibility to estimate mean-variance efficient portfolios that outperform market cap-weighted counterparts. In this paper, the CECE Composite Index (CECE) listed on the Vienna Stock Exchange is used as a benchmark index. The CECE index includes the most liquid stocks traded on stock exchanges in Eastern Europe, more precisely in Budapest, Prague, and Warsaw. As an alternative to weighting by market capitalization, scientific methods of diversification – GMV and MSR portfolio estimations – are used. Motivation for GMV and MSR portfolio estimation originates from the research conducted for the Croatian stock market for which the failure of the estimation of the MSR portfolio relative to the cap-weighted benchmark (Dolinar et al., 2017) can be attributed to poor estimation of stocks' expected returns since GMV portfolio estimation outperformed the MSR portfolio (Zoričić et al., 2018).

In addition to GMV and MSR portfolio estimation, the EW portfolio and a portfolio composed of a combination of the GMV (50%) and MSR (50%) portfolios were estimated. For the estimation of the expected return needed for the MSR portfolio, two different measures were used – arithmetic mean and median. The median was found to achieve superior results. In addition, to achieve greater portfolio deconcentration in the GMV and MSR portfolios, the constraint on the minimum weight for a constituent was imposed in the optimization process.

All the analyzed portfolios achieved better performance than the benchmark CECE index, including the EW portfolio, which is the result of a naïve diversification method. Therefore, investors in the Central Eastern Europe (CEE region) stocks have alternative strategies at their disposal that can be used to estimate more efficient portfolios using constituents of the benchmark CECE index. As expected, the EW portfolio achieved a higher return compared to the cap-weighted counterpart. However, surprisingly, this was accompanied by lower risk in contrast to research that found EW portfolios to have significantly higher risk compared to their cap-weighted counterparts. In this research, the GMV portfolio dominates the MSR portfolio, especially in cases when weaker constraints on the minimum weights for the constituents are used. The GMV portfolio achieved better performance with higher lambda (λ = 4), which is in line with the research of Zoričić et al. (2018), where GMV portfolio performance is improved when a higher concentration in the portfolio is allowed. This could be explained by the fact that it is better to estimate a sub-optimal portfolio, i.e., a portfolio with certain optimality risk but decreased estimation risk (GMV), than the MSR portfolio. However, in this case, even the MSR portfolio achieved superior results compared to its cap-weighted counterpart. In contrast to the GMV portfolio, the MSR portfolio achieved better results with lower lambda (λ = 1.5, i.e., a tighter constraint on the minimum weight for the constituents) and when the median is used for the estimation of the expected returns.

In this paper, a combination of the GMV and MSR portfolios did not result in a portfolio with superior performance compared to the GMV or MSR portfolio. Since MSR portfolio estimation was improved with the median used as an estimator for the expected return, an open question remains whether the performance of the combined portfolio would be even better if advanced techniques for the covariance matrix (such as Martellini & Milhau (2017)) and expected return estimation were used, which could be tested in future research.

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