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Probabilistic Approach for Determination of Thick Pressure Vessels Reliability

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Abstract: The study conducted for this paper attempts to provide a method for evaluating the integrity, condition, and behaviour of pressure vessels based on the fact that all significant parameters provide some form of stochastic distribution. It is possible to reduce the uncertainty in the assessment of the integrity of pressure equipment by evaluating the findings of the probabilistic approach in comparison to the conventional deterministic approach. Samples were taken from different parts of the vessel, different orientations of the initial notch and tests were performed in different laboratories. Testing of statistical hypotheses on the normality of the sets and the equality of the results was performed. The random variables included in the integrity assessment are considered as statistical distributions. The analysis included parameters obtained from experimental research such as yield strength, critical stress intensity factor and crack growth rate. The recording of the real characters of operating stresses depending on the production technology was performed. The analysis is carried out on thick-walled pressure vessels of the material grade 40Mn6. All results were used to define the failure function and the reliability function for two cases of load action and for hypothetical crack growth until reaching the critical length. The dependability level and reliability function limit values may be defined through experimental studies and then used to predict the remaining life, inspection intervals and fitness-for-service in the most realistic way possible.

Keywords: fracture mechanics; reliability; pressure vessel; stochastics analysis

1 INTRODUCTION

Large high-pressure vessels, in particular, fall within the category of high-risk components and as such call for additional monitoring, control, and special operating conditions. The behaviour of vessels differs from that in the case of monotonous action of the load due to the changeable character of the load as well as special exploitation conditions. As opposed to the conventional deterministic assessment, the evaluation of the integrity and remaining life of cyclically loaded pressure vessels demands for a probabilistic method, which may provide more relevant details on the reliability of these crucial components of the process equipment. In this respect, the technique presented in this study may be used to evaluate the structural integrity of such constructions in terms of failure probability.

Many of the factors and variables used in fracture control have a dissipation factor that needs to be taken into account when estimating life. All materials have variable characteristics. Structural loads are often statistical variables. The statistical control of fracture detection is also present. Due to this, while determining inspection intervals, the dispersion factor must always be taken into account in the crack growth curve.

In the last 20 years or so, there has been an increase in both the use of statistical approaches and reliability research [1-3].

The behaviour of cracks on pressurised cylindrical constructions was examined in papers [4, 5], and the focus of [6, 7] was on cracks that appeared on welded joints of complex structures.

Since all significant parameters have a stochastic distribution, the study performed in this work tries to provide a method for evaluating integrity and behaviour.

The random variables included in the integrity assessment are considered as statistical distributions. The yield stress, basic and operating durability, and material toughness curve (J_{mat}) variables have been used for the reliability research.

2 RELIABILITY OF MECHANICAL SYSTEMS

Neither theoretically nor practically, there is no absolute safety of structures against failure. The state of failure occurs due to a series of random factors that cannot be avoided even during the design, manufacture and exploitation of the structure. Since the existence and behaviour of all these affecting factors are stochastic, it is impossible to accurately describe or assume them. Due to the above causes, failure probability is the only way to represent failure safety. There are currently no defined limits for what a reasonable level of failure safety should be. Insufficiently researched areas as well as collected and systematized stochastic data indicate that the limits of the probability of construction failure are taken as an orientation and as follows [8]:

- 10^{-8} to 10^{-6} for parts or objects where a breakdown would have the greatest negative effects,
- 10^{-6} to 10^{-4} for parts or objects where a breakdown would have intermediate negative effects,
- 10^{-4} to 10^{-2} for parts or objects where a breakdown would have small negative effects.

The main difference between the widely recognised deterministic and stochastic techniques to computation is reflected in the fact that all important quantities are considered as exact values in the deterministic approach (they are usually established by regulations). The probabilistic approach, on the other hand, considers all quantities to be stochastic or random.

The deterministic method has only legal significance in terms of a safe or potentially hazardous structure, while the actual level of safety is unknown.

Because state variables in the probabilistic method are random variables, the level of safety becomes a probability of structural failure. The issue with this approach is that the real limits of acceptability, on which the design would be carried out, have not been specified.

Conventional calculation methods are based on the assumption that the maximum working load occurs at the same time as the minimum critical load, and parts are built from this starting point. Above-mentioned method represents construction under the most unsafe circumstances.

However, in the case of massive structures with damaged integrity, the most severe harm is highly likely to be located in the zone with the least load and vice versa. Such an approach to failure safety construction will give an excessively conservative result. This reason for this research is being conducted, which should show how important it is to create an approach for assessing a part's safety based on reliability.

Calculation based on probability or the probabilistic approach to part calculation begins with the understanding that it is difficult, if not impossible, to precisely identify all critical values. As a consequence of this, all parameters affecting component safety, such as load, shape, and material characteristics, are treated as stochastically variable variables with corresponding distributions. The working and critical distributions established in this manner are largely consistent.

The probabilistic approach to the calculation determines the probability that the material's strength will be greater than the working stresses. The numerical value of the possibility that the material's strength will be greater than the working stresses defines the part's reliability (R). If the reliability is R = 1, there is no failure, and for R = 0, failure occurs every time.

3 EXPERIMENTAL RESEARCHES

3.1 Material and Specimens

Material of pressure vessel is steel 40Mn6, according to DIN standard. The vessels were made using plastic deformation process. Tab. 1 shows the results of testing the chemical composition of the material [9, 10].

Table 1 Chemical composition of the tested material

Symbol	Chemical element, %				
	C	Si	Mn	P	S
40Mn6	0.29	0.40	0.66	0.018	0.021

Symbol	Chemical element, %			
	Cr	Mo	Ni	Cu
40Mn6	0.53	0.12	0.32	0.14

The present pressure vessels have been in operation for over 70 years and they were subjected to more than $15 \cdot 10^6$ load changes.

The experimental phase of the study was carried out on a number of specimens got out from the vessel's body. The specimens were cut from three different regions of the vessel, using two cutting directions that are orthogonal to each other. This form of sampling was done to investigate the effect of the anisotropy of the material caused by plastic processing, i.e. forging.

Rectangular test specimens for testing fracture mechanics parameters were cut from orthogonal regions. These test specimens had notches in the radial and circular direction of the vessel.

In order to test the material of the upper head of the vessel, test specimens were cut in circular and radial directions. Cutting the test specimens in the axial direction was impossible due to the limitation of the thickness, as shown in Fig. 1a. Fig. 1b presents specimens marked "S" that are cut from the middle of the vessel, following the circular and axial directions.

As illustrated in Fig.1c, "D" specimens were made from lower head of vessel. The orientation of these specimens is circular and axial.

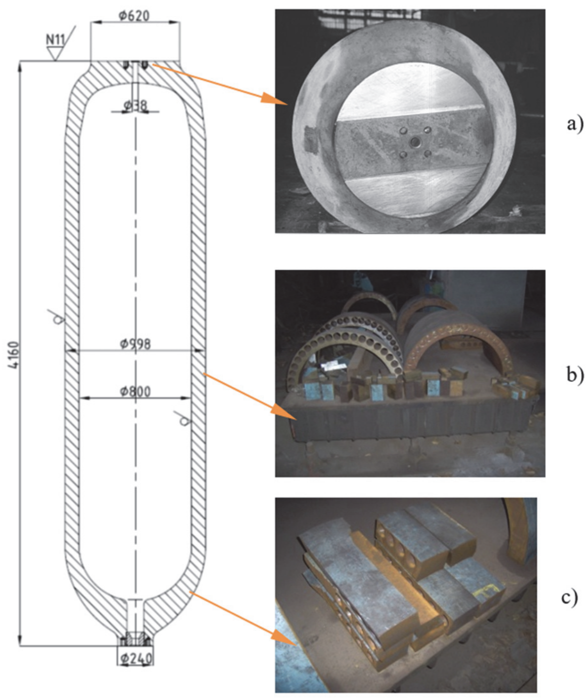

Figure 1 Cross-section of the vessel with the location of sampling for testing

3.2 Tension test

The test procedure is defined by the standard BAS EN 10002 [11], on test specimens for tensile tests. Tensile tests were performed at room temperature. Two test specimens sizes were used in accordance with the requirements of the testing laboratories. Tests were performed in three independent laboratories. The results are shown in Tab. 2 [10, 12].

Table 2 Results of tension test-average values

Samples No.	Yield strength	Tensile strength	Elong.	Young's Modulus
	R_{eH} / MPa]	R_m / MPa	A / %	E / GPa
20	321.3	586.2	27.4	207.0

3.2.1 Statistical Analysis of Tensile Test Results

3.2.1.1 Testing the Normality of the Distribution of the Basis Set

Testing the normality of the distribution was performed using the Kolmogorov-Smirnov test. Tab. 2 shows the total number of valid stress test results at the yield strength R_{eH} that was tested. A total of 20 experimental tests were conducted. The minimum and maximum values for R_{eH} are discarded as extremes, i.e. they are treated as rough errors. A total of 18 results were treated in the further analysis. The tests were carried out in three laboratories: Lab. 1 (ten samples), Lab. 2 (four samples), and Lab. 3 (six samples) [12].

The Kolmogorov-Smirnov test uses the following null and alternative hypotheses:

H_0: The data are normally distributed;

H_A: The data are not normally distributed.

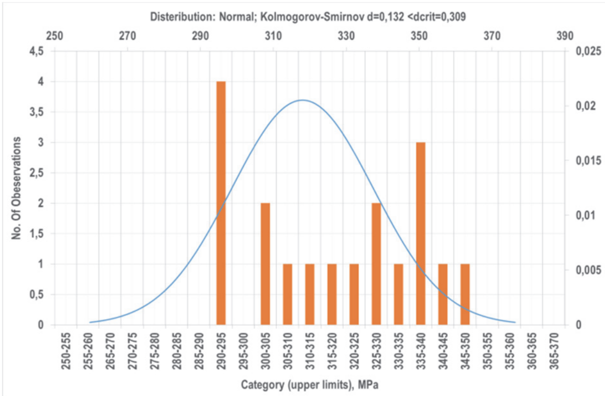

Figure 2 Statistical distribution for all yield strenght results

The maximum absolute difference between the actual values of our sample and the expected values from the normal distribution is $d = 0.1322$. This maximum value is statistically significant if it is higher than the critical value $d_{crit} = 0.309$ for $n = 18$ and $\alpha = 0.05$ according to the Kolmogorov-Smirnov table of critical values [13]. Because the computed maximum value is smaller than this critical value, we cannot reject the null hypothesis. Therefore, we may assume that our data samples are normally distributed. The results are normally distributed, and the F-test and Student's t-test may be used to further analyse them.

3.2.1.2 Testing of Samples in Three Laboratories

The statistical tool ANOVA was employed in the analysis of the effect of sample location on yield strength R_{eH}. Three series of results obtained by testing in three independent laboratories were tested. Therefore, the null hypothesis was tested: H_0: $\mu_1 = \mu_2 = \mu_3$, which affirms the equality of the mean values of these three samples, versus the alternative hypothesis: H_A: $\mu_1 \neq \mu_2 \neq \mu_3$, as shown in Tab. 3.

Table 3 ANOVA results for samples from different laboratories

Group	Quantity	Sum	Average R_{eH} / MPa	Variance
Lab. 2	3	921.8	307.26	12.643
Lab. 3	6	2000	333.33	97.466
Lab. 1	9	2815	312.77	490.02

ANOVA						
Source of variation	SS	df	MS	**F = 3.3713**	P-value = 0.06177	**F_{crit} = 3.6823**
Between groups	1992.62	2	996.31			
Within groups	4432.79	15	295.51			
Total:	6425.41	17				

Given that $F = 2.74 < F_{crit} = 3.59$ for the significance level $\alpha = 0.05$, the null hypothesis H_0 is accepted and the alternative hypothesis H_A is rejected, implying that the samples in the series have the same mean value.

As a result of the ANOVA test, there is no difference between the studied samples. The study demonstrates that all test results may be regarded as being part of the same set.

3.2.2 Testing of Samples Taken from Different Parts of the Vessel

In the second instance, three sets of results generated by assessing samples taken from the zone of the lower head (label D), the middle cylindrical section (label S), and the zone of the plate of the top head (label Z) were examined. In this case, a statistical tool ANOVA was also employed to determine the effect of sample location on the value of R_{eh}. Identical hypotheses H_0 and H_A were examined, as in the preceding example, as shown in Tab. 4.

Table 4 ANOVA results for samples from different zones of vessel

Group	Quantity	Sum	Average R_{eH} / MPa	Variance
Middle	4	1350	337.5	76.33
Lower head	5	2000	321.38	386.49
Upper head	9	2815	310.69	479.16

ANOVA						
Source of variation	SS	df	MS	***F* = 2.74**	*P*-value= 0.0925	**F_{crit} = 3.59**
Between groups	2092.3	2	1046.1			
Within groups	6473.9	17	380.81			
Total:	8566.2	19				

For the significance level of 0.05, the value $F = 2.74$ is less than $F_{crit} = 3.59$, hence the null hypothesis H_0 is accepted and the alternative hypothesis H_A is rejected. Based on the ANOVA test results, it is possible to conclude that there is no difference between the tested samples in this case as well, implying that all test results may be viewed as originating from the same set.

3.2.3 Analysis of Statistical Testing

The analysis of the variance equality of basic sets indicates that all studied annealed samples are from the same group. Furthermore, the evaluation of mean value equality reveals that there are no significant differences between the mean values of the observed basic samples, implying that one mean value, $R_{eH} = 321$ MPa, may be utilised in following analyses for the complete range of study results.

3.3 Fracture Mechanics Parameters

3.3.1 Stress Intensity Factor - K_I, K_{IC}

The fracture mechanics parameters were tested by creating an *R*-curve that illustrates the *J* integral values based on the uniform expansion of the crack Δa. All experiments were carried out in accordance with the ASTM E1152 [14] and ASTM E1820 [15] standards. Because the criterion of a plain strain defined by Eq. (1) was not satisfied in the present instance, elastic-plastic fracture mechanics concepts were applied.

$$B \geq 2.5 \cdot \left(\frac{K_{Ic}}{R_{eH}} \right)^2 \tag{1}$$

The purpose of using elastic-plastic fracture mechanics principles is to indirectly determine the value of the critical stress intensity value, K_{Ic}, by using the value of the critical *J* integral, J_{Ic}.

Due to the fracture toughness (K_{Ic}) values achieved for plane strain for this group of materials are relatively low, the steel in consideration can be characterised as poorly resistant to the presence of fractures.

Applying the basic fracture mechanics Eq. (2) and using the conventional yield stress, i.e. $R_{p0.2}$ = σ and assuming that the shape factor is equal to Y = 1.12, approximate values for the critical crack length were calculated, employing Eq. (3).

$$K_{\mathrm{Ic}} = \sigma \cdot Y \cdot \sqrt{\pi \cdot a_c} \tag{2}$$

$$a_c = \frac{1}{\pi} \cdot \left(\frac{K_{\mathrm{Ic}}}{Y \cdot \sigma_{\max}} \right)^2 \tag{3}$$

The tests were carried out in two laboratories for a total of 32 samples, with the lowest and highest results being excluded from further research [10, 11]. Fig. 3 illustrates a typical force-displacement diagram for samples tested in Laboratory No. 1.

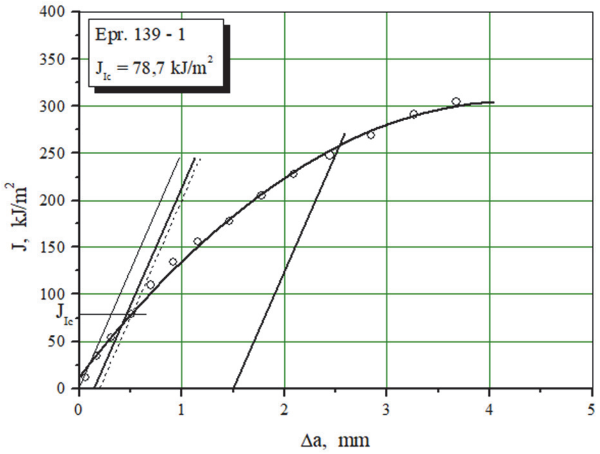

Figure 3 J-Δa diagram for specimen 139-1

The force-displacement graphs for the specimens from the middle of the vessel are shown in Fig. 4.

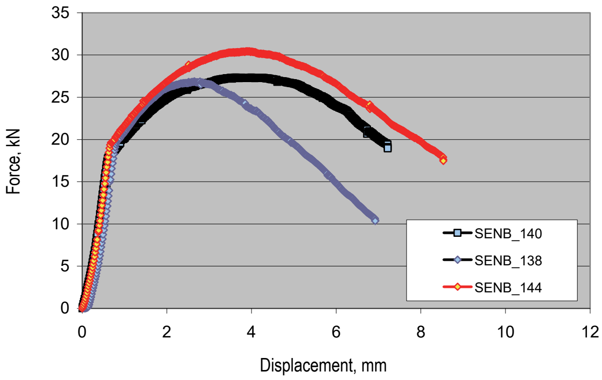

Figure 4 Force-displacement dependence for specimens from the middle of vessel

3.3.1.1 Statistical Testing of Fracture Mechanics Parameter Test Results

The fracture toughness, K_{Ic}, was employed in this analysis as a fracture mechanics parameter. The Kolmogorov-Smirnov test [13] is used to determine the normality of the distribution, as shown in Fig. 5. A total of 30 valid K_{Ic} test results were tested.

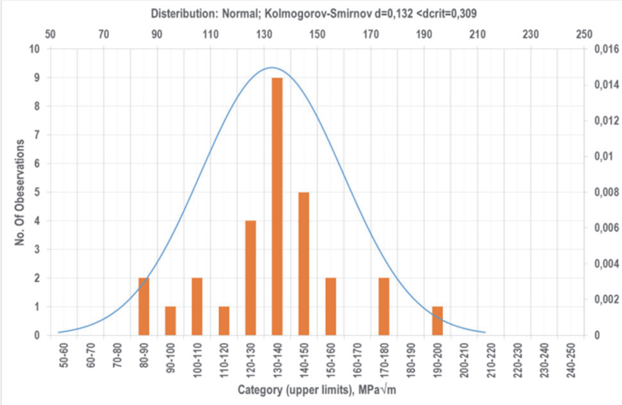

Figure 5 Statistical distribution for all fracture toughness results

The highest absolute difference between the observed sample's actual values and the predicted values from the normal distribution is d = 0.139. According to the Kolmogorov-Smirnov table [13], the critical value for 30 samples and a precision of α = 0.05 is d_{crit} = 0.242. With regard to $d < d_{\mathrm{crit}}$, the null hypothesis cannot be rejected, as it was in the preceding two examples. As a result, it is reasonable to conclude that the data samples are normally distributed, and the F-test and Student's t-test may be used in the following analysis.

3.3.1.2 Testing of Samples Examined in Two Laboratories

The samples tested in two different laboratories were analysed. The total number of test results carried out in Laboratory No. 1 is 22, and the overall amount of test results performed in Laboratory No. 2 is 8. The study was performed using the "F-test: Two samples for Variance", in which the hypothesis of the equality of variances of two samples is evaluated, with the objective of proving that the test results obtained in two separate laboratories came from the same set, as shown in Tab. 5.

Table 5 F-test data for variances for two laboratories

	Variable 1 Lab. No. 1	Variable 2 Lab. No. 2
Middle value K_{Ic}, MPa√m	154.2	125.2273
Variance	987.5543	417.5402
No. of samples	8	22
df	7	21
$\boldsymbol{F_o}$ - calculated	**2.365**	
P (F ≤ f) one-sided	0.059	
$\boldsymbol{F_{crit}}$ (one-sided)	**2.487**	

In this case, the null hypothesis H_0 was tested: *There is no difference in the measured values of the critical stress intensity factor* K_{Ic}, *i.e.* $\sigma_{L1}^2 = \sigma_{L2}^2$. For defined degrees of freedom O_A = 22 – 1 = 21 and O_B = 8 – 1 = 7, table value of limit value F_2 for significance level of 5 % is F_2 = 2.487.

Considering that calculated value F is less than critical value, null-hypothesis was accepted with error less than 5%. In this case, similarly, every result might be considered to be from the same set.

3.3.2 Crack Growth Δ*N*/Δ*a*
3.3.2.1 Experimental Testing S-Curve

According to the ASTM E647 standard [16], the parameters of the Paris equation for crack growth rate were calculated in two different laboratories denoted as Laboratory No. 1 (7 samples) and Laboratory No. 2 (5 samples). This standard specifies the computation of the stress intensity factor range, ΔK, and the measurement of the fatigue crack development rate da/dN that develops from an existing crack. A total of 12 samples cut from the vessel body were tested. Fig. 6 shows a typical diagram. [10, 11].

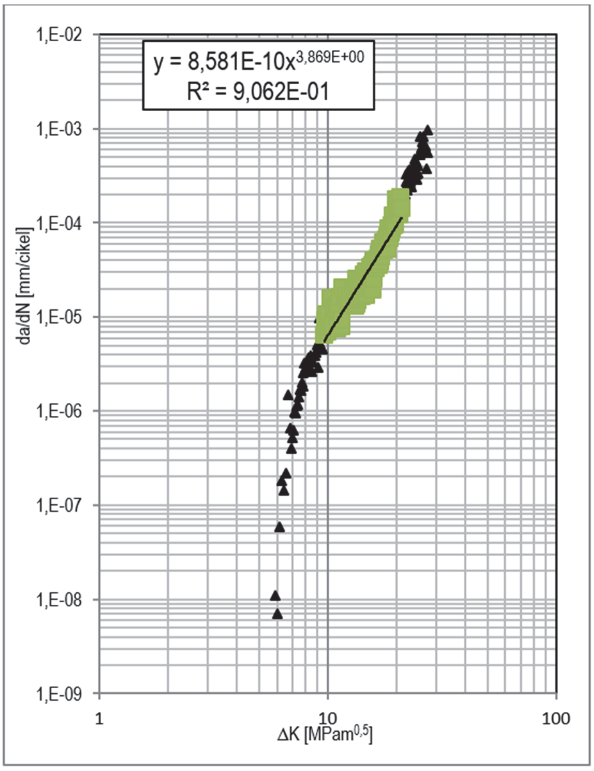

Figure 6 S-curve da/dN - ΔK for specimens from the middle of vessel

The Paris equation represented by Eq. (4) is used to calculate the number of cycles to failure.

$$\Delta N = \frac{1}{C_p \cdot \left[Y \cdot \Delta\sigma \cdot \sqrt{\pi} \right]^{m_p}} \cdot \frac{a_0^{\left(1-\frac{m_p}{2}\right)} - a_d^{\left(1-\frac{m_p}{2}\right)}}{\frac{m_p}{2} - 1} \tag{4}$$

In Eq. (4), N refers to the number of cycles required for crack growth from initial a_0 to the critical a_c or acceptable crack length a_d. It is assumed that the correction factor Y is independent of fracture length and that $\Delta\sigma$ = const.

3.3.2.2 Testing the normality of the distribution

The normality of the distribution is additionally assessed in this instance using the Kolmogorov-Smirnov test [13], as illustrated in Figure 7. The fatigue limit ΔK_{th} is examined with a total of 10 valid test results. Because there were relatively few tests, statistical analyses are limited to the total number of results, which were mostly done without repetition.

The highest absolute difference between the actual values of the tested sample and the predicted values from the normal distribution is more than the critical limit, as illustrated in Fig. 7. The null hypothesis is similar accepted here, which means that the results have a normal distribution and may be analysed further using the Student's or F-test.

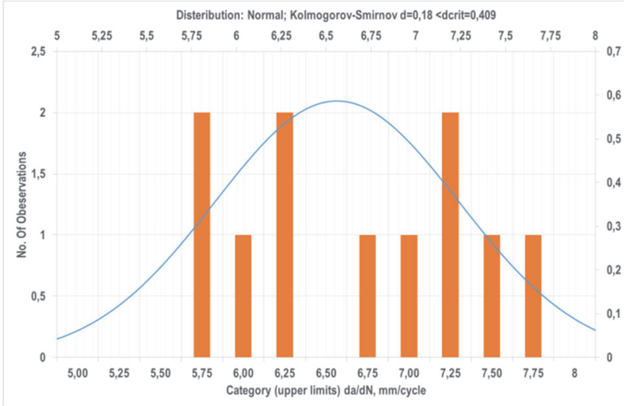

Figure 7 Statistical distribution of all crack growth rate test results

3.3.2.3 Testing of Samples Examined in Two Laboratories

Crack growth testing was also carried out in two distinct laboratories. The results of this evaluation are shown in Tab. 6.

Table 6 F-test data for variances for two laboratories

	Lab. No. 1	Lab. No. 2
Middle value, da/dN / mm/cycle	7.066	5.95
Variance	0.135	0.047
No. of samples	7	5
df	6	4
F_0 - calculated	**2.866**	
P ($F \le f$) one-sided	0.164	
F_{crit} (one-sided)	**6.163**	

The null hypothesis, as established in the preceding situation, claims the equivalence of the variances of the two sets, i.e. $L_1 = L_2$. The null hypothesis is accepted since the estimated value of F is higher than the critical value [13]. This means that at the level of significance = 0.05, there is no statistically significant difference.

3.4 Operating Loads and Stress

3.4.1 Measurement of the Actual Characteristics of the Stress Variation

Pressure vessels are constructed for continuous development of the forging process on the press and are subjected to an effect of internal pressure change, the nature of which is unknown. Strain-gauge measurements of strain states on the outer shell have been performed in this regard to identify not only the type of change (static or dynamic), but also the frequency of changes to which the vessels are subjected throughout the forging process. In such cases, the load change is always unidirectional. Furthermore, it is important to emphasise that strain-gauge measurements are taken at the middle of the cylindrical component.

Five special procedures have been recorded, three of which required substantial participation, based on the types of products and operations conducted on the press. Tab. 7 shows forging techniques and their statistical characteristics, as well as overall participation in the production process.

Data was acquired at a frequency of 10 Hz, and a distinct amount of details were collected for each recorded processing method. A huge quantity of data was collected, and discretization was performed using the time interval approach. The time period was chosen as Δt = 1/100 since it was determined that applying so will divide each stress change into at least 10 divisions while keeping the change's nature.

Fig. 8 represents a typical example of a diagram showing the dependence of the stress change in the transverse direction (x-axis) on real collected data and the form of the discretized diagram for 1/100 of the collected values [12].

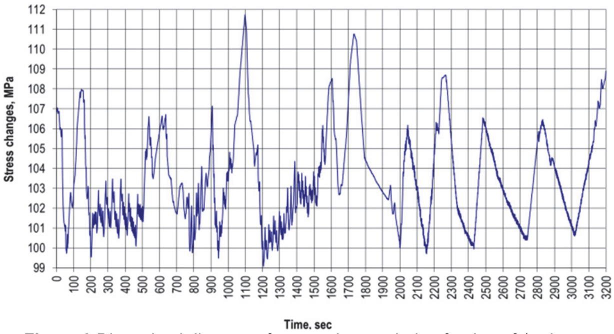

Figure 8 Discretized diagram of stress change during forging of 15t ingots

3.4.2 Selection of a Representative Distribution of Operating Stress and Hypothesis Testing

To further simplify the analysis, the measurement results must be statistically processed. The Gaussian curve consists of a continuous random variable x that has a normal parameter probability distribution if its probability distribution density function is expressed by Eq (5).

$$f(x)=\frac{1}{\sigma\cdot\sqrt{2\pi}}\cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, x\in R \quad (5)$$

The essential parameters for the mathematical expression of stochastic stresses that occur during forging were determined by taking into consideration the representation of individual operations in the overall activity of the press and the results of statistical processing. Tab. 7 illustrates the typical sizes and mathematical equations that characterise the function of the frequency of occurrence of the transverse component of the stress S_{1-x}, which is also the most intense, in the press's whole production programme.

Because the functions of the stress distribution on the vessel's shell are varied, a total of five different spectra were detected. Three representative spectra (serial numbers 1, 2, and 3) were chosen in this case, whereas the remaining two spectra (serial numbers 4 and 5) always occur as an integral component of the first three, i.e. they are unavoidable operations inside these three representative spectra.

The parameters of the combined stress distribution function S_{1-x} in the middle of the vessel may be calculated using Eqs. (6) and (7), taking into consideration the frequency of their occurrence (series number from 1 to 3) over the working life of the press.

Table 7 The characteristics and forms of the stress distribution density function

No.	Forging operation	Main value of stress / MPa	Standard deviation	Life / %	Normal distribution function $f(S)$
1	Ingot 15t	103.51	2.46	75	$f(S)=\frac{1}{2.46\sqrt{2\pi}}\times e^{-\frac{1}{2}\left(\frac{S-103.51}{2.46}\right)^2}$
2	Ingot 25t	97.28	4.92	23	$f(S)=\frac{1}{4.92\sqrt{2\pi}}\times e^{-\frac{1}{2}\left(\frac{S-97.28}{4.92}\right)^2}$
3	Ring	106.34	2.46	2	$f(S)=\frac{1}{2.46\sqrt{2\pi}}\times e^{-\frac{1}{2}\left(\frac{S-106.34}{2.46}\right)^2}$
4	Ingot cutting	108.64	5.24	-	$f(S)=\frac{1}{5.24\sqrt{2\pi}}\times e^{-\frac{1}{2}\left(\frac{S-108.64}{5.24}\right)^2}$
5	Circular flatten	101.49	2.94	-	$f(S)=\frac{1}{2.94\sqrt{2\pi}}\times e^{-\frac{1}{2}\left(\frac{S-101.49}{2.94}\right)^2}$
6	**Sum function**	**102.13**	**3.02**	**100**	$f(S)=\frac{1}{3.02\sqrt{2\pi}}\times e^{-\frac{1}{2}\left(\frac{S-102.13}{3.02}\right)^2}$

$$\mu=\frac{1}{100}\cdot\left[75\cdot103.51+23\cdot97.28+2\cdot106.34\right]=102.13 \quad (6)$$

$$\sigma^*=\frac{1}{100}\cdot\left[75\cdot2.46+23\cdot4.92+2\cdot2.46\right]=3.02 \quad (7)$$

Fig. 9 shows the stress distribution curves S_{1-x}, which demonstrate a representative spectrum of stress variations in the circular direction in the middle of the vessel. The combined frequency function substitutes all of the individual functions and has a normal distribution.

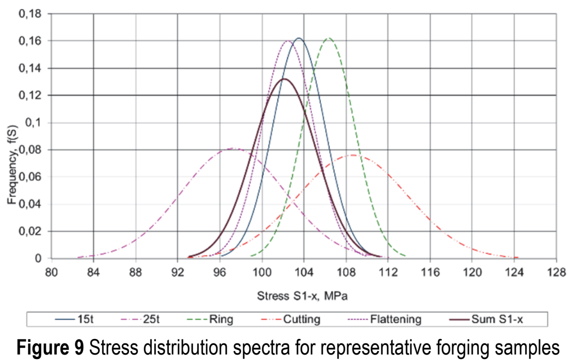

Figure 9 Stress distribution spectra for representative forging samples

As a result, it may be assumed in future research that the largest amount of predicted stresses is between the lowest and highest stresses, which have values of 93 MPa and 111 MPa, respectively, as shown in Fig. 9.

4 REALIBILITY OF RESULTS AND DISCUSION

4.1 Assessment of Reliability Indicators

Finding an appropriate mathematical model that can describe the predictability of this system's behaviour from the perspective of failure probability is one of the fundamental parts of system state prediction and optimisation. Maintenance procedures, the schedule of essential inspections, and the service life may all be predicted using the established reliability distribution model. Decisions concerning the causes of failure can also be made based on the defined method of distribution.

The data for the temporal representation of the condition are derived from the real indicators of failure status. Failure may occur due to the construction's specifics in the occurrence of vessel fracturing, vessel wall thickness loss, or reaching a critical fracture size (average value approx. 40 mm) [12]. Given the lack of data for a specific system, they may be represented hypothetically by assuming that the vessels can have any beginning fracture size ranging from 1 mm to 39 mm. Operating stresses, on the other hand, vary, as indicated by measurements taken on the exterior shell during operation.

As seen in Fig. 8, the highest stress change was approximately $\Delta\sigma = 20$ MPa. The appearance of any range of stress is possible in the range from $\Delta\sigma_{\min} = 0$ to $\Delta\sigma_{\max} = 20$ MPa, and using a random selection of sizes, utilising Excel randomization functions, may obtain the number of cycles to failure for all sizes of cracks from 1 to 40 mm with an increment of 1 mm. Form (4) is used to determine the number of cycles.

A statistical data processing procedure was carried out, taking into account that one ingot is processed with an average of 100 cycle changes and the assumption that the determined data can be represented by a Gaussian distribution (confirmed by the Kolmogorov-Smirnov test). The results of the process are reported in [12] as shape parameters (σ) and scale (μ) of the normal standard distribution.

The final equations that define the reliability functions of the vessels in consideration for the chosen values of the operational stress range and crack size are given in the form of the three functions shown below [17]:

Reliability function:

$$R(n) = 1 - P(n) = 1 - \frac{1}{\sqrt{2 \cdot \pi}} \int_{-\infty}^{n} \mathrm{e}^{-\frac{n^2}{2}} \mathrm{d}u \tag{8}$$

Failure density function:

$$f(n) = \frac{1}{\sqrt{2 \cdot \pi}} \cdot \mathrm{e}^{-\frac{n^2}{2}} \tag{9}$$

Failure intensity function:

$$\lambda(n) = \frac{f(n)}{R(n)} \tag{10}$$

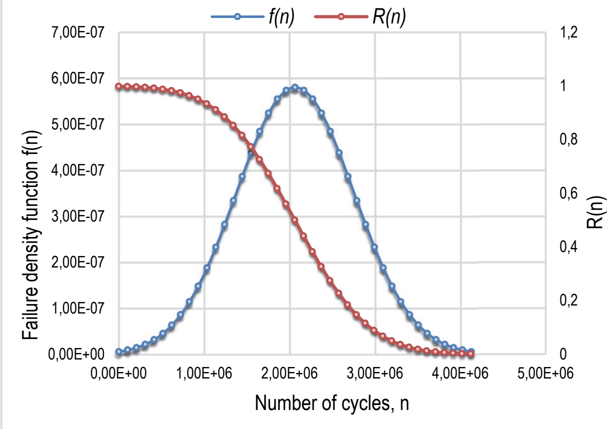

Figure 10 Reliability and failure density function for random stress range

Figs. 10 and 11 present a graphical representation of the functions $R(n)$, $f(n)$, and $\lambda(n)$ for the random range of the stress change from $\Delta\sigma = 0$ to $\Delta\sigma = 20$ for the cycle intervals of changes $\Delta n = \Delta N/100$.

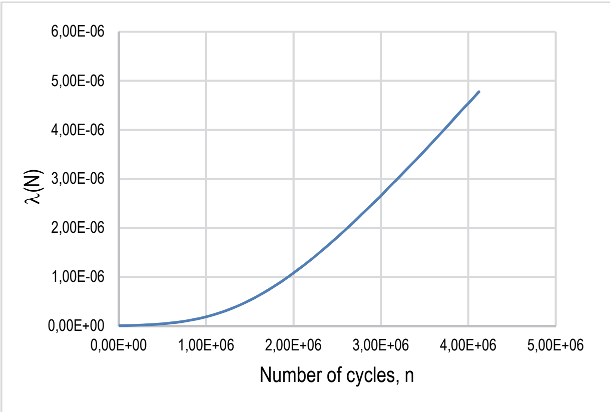

Figure 11 Failure intensity function for random stress range

The functions for the case of a constant maximum load range of $\Delta\sigma = 20$ MPa are calculated in the same way as the preceding ones [12].

Figs. 12 and 13 show a graphic representation of the functions $R(n)$, $f(n)$, and (n) for the considered vessel under the effect of a constant maximum load range for the intervals of the cycle of changes $\Delta n = \Delta N/100$.

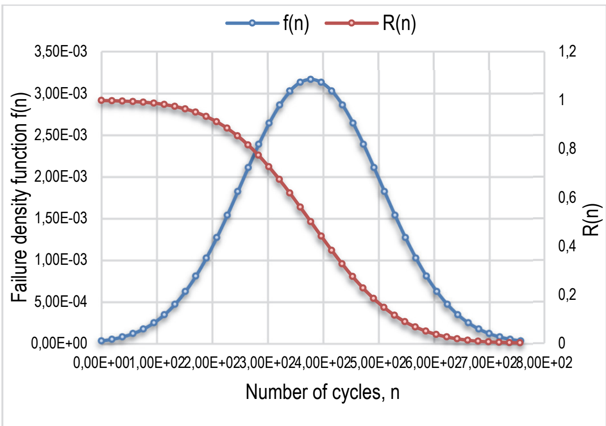

Figure 12 Reliability and failure density function for stress range $\Delta\sigma = 20$ MPa

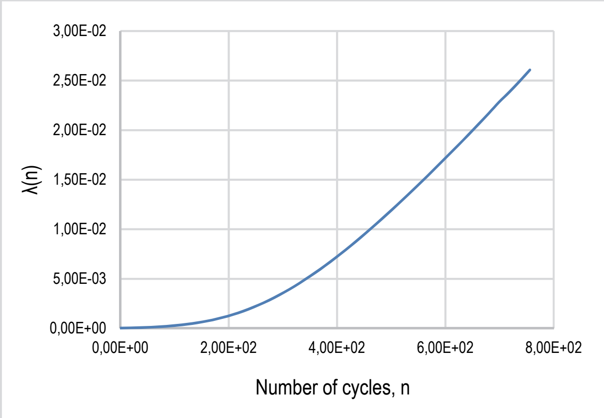

Figure 13 Failure intensity function for stress range $\Delta\sigma$ = 20 MPa

5 CONCLUSION

It is possible to determine the limit values of the reliability level, reliability function, and failure intensity through experimental study, which can then be utilised to calculate the remaining life, inspection period, and preparation for use. Based on the proven results presented in this study, the following conclusions about evaluating the dependability of pressure vessels may be established:

- A Gaussian normal distribution can be used to characterise the observed vessel's behaviour in terms of potential failures over time.
- The vessel's test results for different ranges, as well as the maximum range, show that the reliability functions and distribution parameters are quite comparable. This shows the set's homogeneity and the acceptability of the results.
- The determined reliability functions of the observed vessels can be used to establish the suitable number of cycles "*n*" or time "*t*" after which preventative inspections, general audits, and assessment of the analysed system's suitability for use should be carried out.
- The failure intensity results indicate that carrying out the analysis using the highest ranges resulted in a significantly higher level of failure intensity, giving incorrect representation of behaviour.
- The failure intensity estimates determined in this manner can be used in the process of probabilistic vessel calculations based on reliability.

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