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The stability of Taylor vortex flow between a rotating wavy inner cylinder and a stationary circular outer cylinder

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SUMMARY

Studies on the stability problem when a viscous incompressible fluid is filled into two circular concentric cylinders are usually conducted using the following boundary conditions: the outer cylinder remains stationary, and the inner cylinder rotates with constant angular speed. In this paper, we will apply these same boundary conditions to solve the stability problem, but with a wavy inner cylinder. The main purpose of the present research is to find the effects of the asymmetric inner cylinder on the centrifugal stability problem using of the principle of analogy and superposition, which is obtained from the effort of former researchers in time-dependent Couette flow. The result shows that the critical Taylor number and critical axial wavelength are inversely proportional to the numbers of the peaks of inner cylinder, and the difference between critical Taylor numbers for inner cylinders of different peaks increases when their peak height increases.

***Key words**: centrifugal stability problem, Taylor vortex flow, incompressible fluid, wavy inner cylinder, principle of analogy.*

1. INTRODUCTION

The stability problem of a viscous fluid owing to two concentric rotating cylinders is of both academic and engineering application interest. Taylor [1] considered the stability problem both theoretically and experimentally and he obtained a perfectly good agreement. He got a criterion for the onset of a secondary motion in the form of cellular toroidal vortices spaced more or less regularly along the axis of the cylinder. Later workers, such as DiPrima [2], Chandrasekhar [3], Meksyn [4], Duty and Reid [5], used different approaches to solve this problem for m (the angular speeds of the two cylinders) very negative and large. They all solved this problem for axisymmetric disturbances with small-gap assumption, where the mean flow can be replaced by its average value. Krueger et al. [6] went to consider the fully linear Taylor problem for negative m, and found that in narrow-gap approximation, when m is less than 0.78, the most unstable disturbance is no longer axisymmetric but nonaxisymmetric. As m decreases below this value the most unstable mode changes from *m (azimuthal wavenumber)=0* to $m=1$ but then takes higher values in rapid succession. This phenomenon has also been found experimentally by Coles [7] and Snyder [8].

Experimental results from Snyder [8] showed that flow field between a rotating square inner cylinder and a stationary circular cylinder is more unstable than the system with a circular inner cylinder. Lewis [9] sketched the cellular structure between a rotating circular inner cylinder and a stationary square outer cylinder by finite difference method and Schumack et al. [10] established the incipient instability criterion for flow field between a rotating circular inner cylinder and a stationary elliptical outer cylinder by spectral element method. All their results show that asymmetric boundaries make the stability decrease.

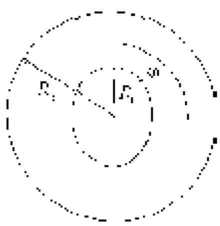
Kong & Liu [11] have solved two kinds of varied surface speeds of the inner cylinder, one is $V=V_0(1-e^{-bt})$ and the other is $V=V_0(1-e^{-bt})\cos dt$ by using the principle of analogy and superposition, where b is the factor of variable rotating speed and d is the oscillating factor. In this paper, after we transform the elliptical inner cylinder from $R=R(q)$ to $R=R(t)$ (i.e. from space coordinate to time coordinate), we can solve the present problem by Kong and Liu's method.

Pharmacy and chemical engineering usually need to stir liquid feedstock, and the proper shape of stirrer can make the feedstock well-mixed at lower stirrer rotating speed (i.e. lower energy consumption), this will lead their product to produce a higher market competition. Another application is the design of the stirrer in the washer. The cleaning effect of a washer mainly depends on the ability of stirring of the inner cylinder, proper shape of stirring blades can make the field well-mixed and enhance the cleaning effect. The authors hope all the results in this paper can be useful to this field.

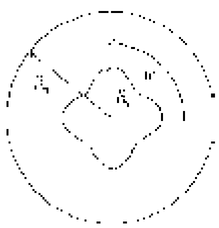
2. PROBLEM FORMULATION AND METHOD OF SOLUTION

Consider two infinite long concentric cylinders. The inner cylinder is an elliptical cylinder, and the outer cylinder is a circular one. The major axis of the inner cylinder is R_1, and the radius of the outer cylinder is R_2', the shape of the cylinders system is sketched in Figure 1 and the inner cylinder can be described by:

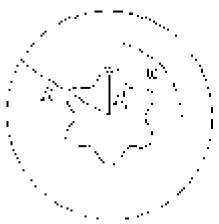
$$R=R_1(1-e\cos^2 q)=R_1(1-\frac{e}{2})-R_1(\frac{e}{2})\cos 2q \quad (1)$$



ELLIPTIC INNER CYLINDER



4-PEAKED INNER CYLINDER



6-PEAKED INNER CYLINDER

Fig. 1 Shape of the cylinders system

The cross section between the cylinders then forms an annulus region, where it is filled with a viscous fluid. Now we can establish a cylindrical coordinate system (r, q, z), with the z-axis as their common axis. If the outer cylinder remains stationary and at time $t=0$ the inner cylinder impulsively starts and reaches a fixed surface angular speed w, we can establish the relationship between q and $\hat{v}$ by:

$$q=wt \quad (2)$$

$$\hat{v}=wR=w[(1-\frac{e}{2})R_1-(\frac{e}{2}R_1)\cos 2wt] \quad (3)$$

From Eq. (3), we can decompose the velocity $\hat{v}$ into two parts: the time-independent part and time-dependent one:

$$\hat{v}=(1-\frac{e}{2})wR_1-(\frac{e}{2})wR_1\cos 2wt \quad (3.1)$$

For rotating inner cylinder, we can assume that the axisymmetry in perturbation velocities and pressure still holds, and since they are periodic in the z-direction and grow with time, we can describe the perturbation velocities and pressure by:

$$\begin{aligned} u' &= u(r)\cos a z\cdot e^{kt} \\ v' &= v(r)\cos a z\cdot e^{kt} \\ w' &= w(r)\cos a z\cdot e^{kt} \\ p' &= p(r)\cos a z\cdot e^{kt} \end{aligned} \quad (4)$$

where a denotes the wave number in z-direction, k is the growth rate of perturbation. Since the cylinders are infinitely long and axisymmetric, the Naiver-Stokes equations can be simplified to a time-dependent basic flow field. We can utilize the Hankel transformation or the so-called Fourier-Bessel transformation to solve the basic flow velocity field. First, we define the Hankel transformation and the inverse Hankel transformation respectively:

Hankel Transformation:

$$f(q)=\int_{R_1}^{R_2} f(r)rB_1(qr)dr \quad (5)$$

Inverse Hankel Transformation:

$$f(r)=\int_{R_1}^{R_2} f(q)qB_1(qr)dq \quad (6)$$

where $B_1(qr)=J_1(qr)Y_1(qR_2)-Y_1(qr)J_1(qR_2)$ in which q, r serve as weighting functions. $J_1(qr)$ and $Y_1(qr)$ are Bessel functions of the first and second kinds of order 1, respectively. From the assumptions we have made, it can be known that the basic flow velocity field is independent of z and that it is also independent of q because of circular annulus formed by rotating. If the body force can be ignored, then the equation of basic flow velocity becomes:

$$\frac{1}{m}\frac{\partial v}{\partial t} = DD^{*}V \qquad R_1<r<R_2 \quad t>0 \tag{7}$$

where:

$$D=\frac{\partial}{\partial r} \qquad D^{*}=\frac{\partial}{\partial r}+\frac{1}{r}$$

and the boundary conditions are:

$$\begin{aligned} V&=wR_1 \qquad r=R_1 \\ V&=0 \qquad r=R_2 \end{aligned} \tag{8}$$

Hankel transformation of Eq. (7) gives:

$$\frac{1}{m}\frac{d\overline{V}}{dt}=\frac{2}{r}V_0\frac{J_1(qR_2)}{J_1(qR_1)}-q^2\overline{V} \tag{9}$$

where:

$$\overline{V}=\int_{R_1}^{R_2} V(r)rB_1(qr)dr$$

Equation (9) can be solved easily:

$$\overline{V}=\frac{2}{r}\frac{V_0}{q^2}\frac{J_1(qR_2)}{J_1(qR_1)}(1-e^{-nq^2t}) \tag{10}$$

the inverse Hankel transformation of Eq. (10) is:

$$V(r,t)=pV_0\sum_q\frac{1-e^{-nq^2t}}{J_1^2(qR_2)-J_1^2(qR_1)}J_1(qR_1)J_1(qR_2)B_1(qr) \tag{11}$$

Equation (10) is the solution of the basic flow velocity field. With simple transformation, the basic velocity can be rewritten and divided into two parts:

$$V(r,t)=\frac{V_0R_1}{r}\frac{r^2-R_2^2}{R_1^2-R_2^2}- \\ -p\,V_0\sum_q\frac{J_1(qR_1)J_1(qR_2)}{J_1^2(qR_2)-J_1^2(qR_1)}B_1(qr)e^{-nq^2t} \tag{12}$$

The time-dependent basic flow velocity distribution Eq. (12) was done by Tranter [12]. The first term of the right hand side in Eq. (12) is the steady part of the basic velocity profile, while the second term is the unsteady part of the basic velocity profile. The summation symbol Σ in Eq. (12) indicates that all roots of Eq. (6) must be calculated.

Suppose that the perturbed flow is of the form:

$$\vec{u}=u'V+v'w' \text{ and } P=p' \tag{13}$$

If we substitute the velocity terms in the Naiver-Stokes Eqs. by the relationship in Eq. (13) it gives:

$$\frac{\partial u'}{\partial t}-\frac{2v'V}{r}=-\frac{1}{r}\frac{\partial p'}{\partial r}+n(\nabla^2u'-\frac{u'}{r^2}) \tag{14}$$

$$\frac{\partial v'}{\partial t}+u'D^{*}V=n(\nabla^2v'-\frac{v'}{r^2}) \tag{15}$$

$$\frac{\partial w'}{\partial t}=-\frac{1}{r}\frac{\partial p'}{\partial z}+\nabla^2w' \tag{16}$$

where:

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$$

and the continuity equation is:

$$\frac{\partial(ru')}{\partial r}+\frac{\partial(rw')}{\partial z}=0 \tag{17}$$

Now, introducing the nondimensional coordinate radial distance:

$$x=\frac{r}{R_2-R_1}-\frac{1}{2}\frac{R_2+R_1}{R_2-R_1}$$

axial wave number:

$$a=(R_2-R_1)a$$

perturbation growth rate:

$$s=(R_2-R_1)^2\frac{k}{n}$$

Reynolds number:

$$Re=\frac{R_1V_0}{n}$$

Taylor number:

$$Ta=\frac{4(R_2-R_1)^4V_0^2}{(R_2^2-R_1^2)U^2}=4Re^2\frac{(1-h)^4}{(1-h^2)h^2}$$

ratio of the half length of major axis of inner cylinder and the radius of the outer cylinder:

$$h=\frac{R_1}{R_2} \tag{18}$$

can be obtained, so that the boundary surfaces are now at $x=\pm\frac{1}{2}$. Then we eliminate p' and w' from Eq. (14) to Eq. (17), drop the primes and substitute Eq. (4) into them to get the linearized nondimensional perturbation equations of motion:

$$[DD^{*}-a^2-s][DD^{*}-a^2]u=Ta\cdot a\cdot V_1\cdot n\frac{(R_1+R_2)}{2r} \tag{19}$$

$$[DD^{*}-a^2-s]v=uD^{*}V_1 \tag{20}$$

where:

$$D=\frac{d}{dx} \qquad D^{*}=\frac{d}{dx}+\frac{1}{r(x)}t$$

and the boundary conditions are:

$$u=v=D^{*}u=0 \quad \text{at} \quad x=\frac{1}{2} \tag{21}$$

Eqs. (19) to (21) are the governing equations of flow field which result from a constant velocity $V_0=wR_1$, but from Eq. (3), we can find that an additional time-dependent part will also have influence on the flow field. If we let:

$$\hat{V}=wR_1(1-\frac{e}{2})\cos 2wt=U_1-U_2\cos 2wt \tag{22}$$

then the disturbed velocities and pressure which results from *cos2wt* can be described by:

$$u' = u(r)\cos \alpha z \cos 2wte^{kt} = u(r)\cos \alpha z\, Re[\, e^{(k+2iwt)}\,]$$
$$v' = v(r)\cos \alpha z \cos 2wte^{kt} = v(r)\cos \alpha z\, Re[\, e^{(k+2iwt)}\,]$$
$$w' = w(r)\sin \alpha z \cos 2wte^{kt} = w(r)\sin \alpha z\, Re[\, e^{(k+2iwt)}\,]$$
$$p' = p(r)\cos \alpha z \cos 2wte^{kt} = p(r)\cos \alpha z\, Re[\, e^{(k+2iwt)}\,] \tag{23}$$

where: Re[...] of [...].

Let the perturbation growth rate be:

$$s' = \frac{(R_2 - R_1)^2 (k + 4wi)}{n} \tag{24}$$

Repeat the operation described above, we can obtain the governing equation results from the varied velocity $cos2wt$:

$$Re\{[DD^* - a^2 - s'][DD^* - a^2]u = = Ta \cdot a^2 \cdot V_2 \cdot n \frac{R_2 + R_1}{2r}\} \tag{25}$$

$$Re\{[DD^* - a^2 - s']v = u \cdot D^* V_2\} \tag{26}$$

If we rewrite Eqs. (19), (20) and (12) in form of:

$$[DD^* - a^2 - s][DD^* - a^2]u = Taa^2 V_1 v \frac{R_2 + R_1}{2r} \tag{19'}$$

$$[DD^* - a^2 - s]v = uD^* V_1 \tag{20'}$$

$$V(x,t) = \frac{\eta x^2 - 1}{x\,\eta^2 - 1} + \sum_{i=1}^{n} q\,l_i t \left[J_1(l_i \frac{x}{h}) Y_1(\frac{l_i}{h}) - Y_1(l_i \frac{x}{h}) J_1(\frac{l_i}{h}) \right] e^{-l_i^2 t} \tag{12'}$$

where:

$$x = \frac{r}{R_2} \quad l_i = qR_1 t = \frac{nt}{R_1^2} Q(\frac{l_i h}{J_1}) = = pJ_1(\frac{l_i}{h}) \left[\frac{J_1(l_i)}{J_1(\frac{l_i}{h^2}) - 1} \right]^{-1}$$

and:

l_i is the roots of $J_1(\frac{l}{h}) Y_1(l) - J_1(\frac{l}{h}) = 0$.

For Eqs. (19), (20) and (25), (26) are subject to the same boundary conditions, we can apply the principle of analogy and superposition to obtain the exact form of V_2 in a quasi-state sense:

$$V_2 = \frac{U_2}{U_1} \cos 2wt \tag{27}$$

If we introduce another nondimensional parameter $W = wR_1^2$, then Eq. (27) can be rewritten to:

$$V_2 = \frac{U_2}{U_1} \cos 2Wt \tag{27'}$$

Hence, the final governing equations are Eqs. (19') to (25) $\times \frac{U_2}{U_1} \cos 2Wt$, and Eqs. (20') to (26) $\times \frac{U_2}{U_1} \cos 2Wt$:

$$[DD^* - a^2 - s][DD^* - a^2]u - - \frac{U_2}{U_1} \cos 2Wt\, Re[[DD^* - a^2 - s'][DD^* - a^2]u] = = Ta \cdot a^2 V_1 v \frac{R_1 + R_2}{2r} - \frac{U_2}{U_1} \cos 2Wt\, Re[Ta \cdot a^2 V_2 v \frac{R_2 + R_1}{2r}] \tag{28}$$

$$[DD^* - a^2 - s]v - \frac{U_2}{U_1} \cos 2Wt\, Re[[DD^* - a^2 - s']v] = = uD^* V_1 - \frac{U_2}{U_1} \cos 2Wt \cdot u\, Re[D^* V_2] \tag{29}$$

and the boundary conditions are still Eq. (21).

The derivation for the cases of four-peaked and six-peaked inner cylinders is still Eq. (1) to Eq. (28), but the shape of the inner cylinders in Eq. (1) should be changed into the form of $R = R_1(1 - e c\cos^2 2q)$ and $R = R_1(1 - e\cos^2 3q)$, respectively.

3. RESULT AND DISCUSSION

As mentioned before: what we try to find out is the Taylor number that contents governing equations, and the axial wave number that corresponds to the governing equations. But it doesn't indicate that each of the Taylor number is the borderline of the stability and instability of the flow field. Actually, the angular speed of inner cylinder is increased step by step, and when it reaches a certain level, the instability occurs; the so-called critical Taylor number is corresponded by the lowest angular speed that makes the flow field unstable, and it is also the lowest one among the crowded Taylor number which can make the flow field unstable. This is the way that we find out the critical Taylor number at a specific time, but we may not predict the corresponding specific time of the critical Taylor number onset, so after we keep down the critical Taylor number of the specific time, we have to change the time and repeat the steps stating above, until we obtain the "real" critical Taylor number that can judge the whole process of time.

As shown in Figure 2, it is the comparison of stablility for five ellipses with the same major axes but different minor axes, the larger e indicates the greater difference in length of the major axis and minor axis. According to the figure, we know that: the greater difference of the major axis and minor axis is, the lower the corresponding stability should be. The corresponding nondimensional time for instability onset will be getting less by increasing the value of e, but generally they distribute among $t=3.0$ to 4.0. In Figures 3 and 4, we can find that for four-peaked and six-peaked inner cylinders, there are similar phenomenon occuring. In Snyder's stability experiment of the asymmetric inner cylinder, he found that the instability onset in his cylinders system is similar to the toroidal, axisymmetric structure between circular cylinder; but its corresponding critical Reynolds number is almost ten-times less than the one which appears in circular cylinders system. In Figure 2 to Figure 4, we can find that the stability is indeed reduced by the asymmetric inner cylinders, but its effect is not as large as Snyder asserted. We conclude that the causes are:

(1) The values of h in the two cases are different; the h of Snyder's instrument is about 0.5, which is larger than h we adopt to calculate, 0.1. This is because the flow field with narrow gap width is more unstable than wider one; and the added disturbing source will have larger effect on the former one.

(2) After Snyder's conversion of square inner cylinder, the value of e is only about 2.9, thus, this kind of stability drop absoultely is not caused by e. Besides the cause mentioned above, the other is that the boundary of inner cylinder is not a smooth curve, so this discontinuous boundary makes the stability far less than our estimate.

In Figures 5 to 7 they are plotted as functions of their geometric shape for $h=0.1$, and $e=0.05$, 0.1 and 0.3. As seen in each of figures, although the maximum depth of the concavity of inner cylnder (e) is the same, yet the stability of the flow field decreases as the number of peaks increases, and the stability difference between cylinders becomes significant as e increases. From a physical point of view, the increasing in number of peaks enhances the frequency of disturbing in flow field, and larger e means enlarging in surface area of "blades". Therefore, besides the original centrifugal effect, irregular configuration of the inner cylinders makes the flow field more unstable.

It has been proved by both theoretical and experimental methods that in the narrow gap system, if we keep the outer cylinder stationary, the value of nondimensional critical wave number a_c that corresponds to the onset of instability is about p, this suggests that the cross section for Taylor vortex is nearly square; however, with the increasing of the gap width, the critical wave number a_c will increase. In Figure 8, we can find out that no matter what kind of ellipses are, their corresponding critical wavelength is about 4. To convert into a general type of nondimensional critical wavelength, we divide dimensional wavelength by gap width (R_2-R_1) instead of the radius of inner cylinder (R_1) and we obtain its critical wavelength that is about 0.44; that is, its nondimensional critical wave number is about $4p$. In circular cylinders system, such a short critical wavelength may not occur even if we reduce the h to 0.1. We think that when the asymmetric cylinder rotates, besides the supply of the centrifugal force, and the energy which the "blades" wipe out the fluid particles, the vortices will gather around the outer cylinder; it is also the reason why the wavelength of vortices is so short. Thus, in Figures 8 to 10, we can find out that as the number of peaks increases, the critical wavelength reduces; and as the e increases, the decreasing of wavelength becomes more apparent.

4. CONCLUSIONS

In the case of co-axial cylinders, if outer circular cylinder is kept stationary, the geometric shape of the rotating cylinder will indeed have influence on the stability of flow field, generally speaking:

1. The stability of the flow field will reduce as the number of peaks increases, that is, with the same ratio of the length of the inner cylinder (R_1) and the length of the outer cylinder (R_2), and the same maximum depth of concavity of inner cylinders (e), $Ta_{c(circular)} > Ta_{c(ellipse)} > Ta_{c(four\_peak)} > Ta_{c(six\_peak)}$ and as the maximum depth of concavity of inner cylinders increases, the difference of the stability between each geometric shape will become more significant.
2. With the increasing number of peaks, the nondimensional critical wavelength will be reduced; and with the increasing of the maximum distance of each concavity of inner cylnders, the decreasing of wavelength becomes more apparent.

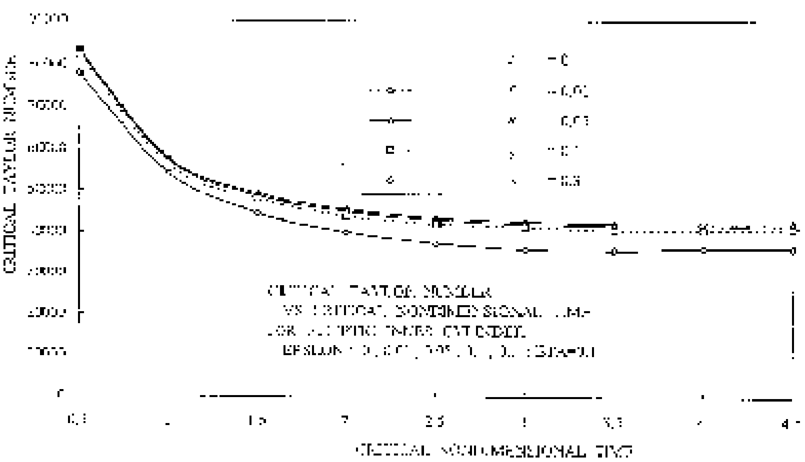


Fig. 2

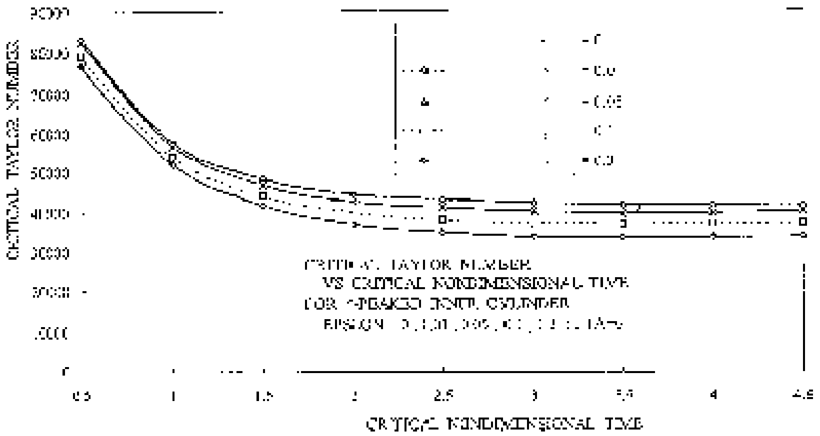


Fig. 3

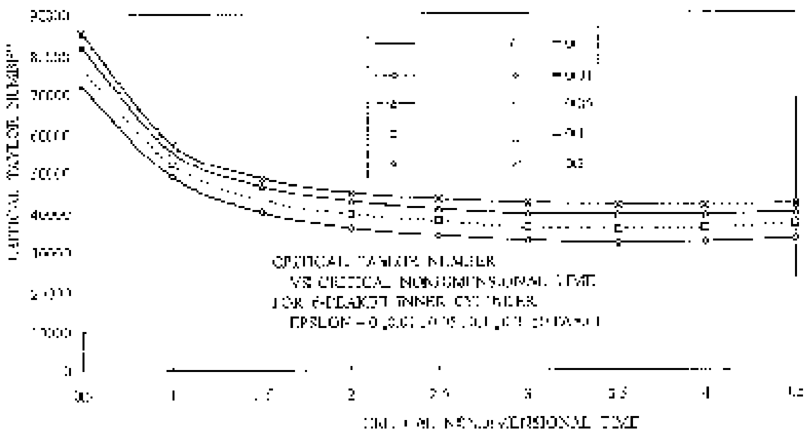


Fig. 4

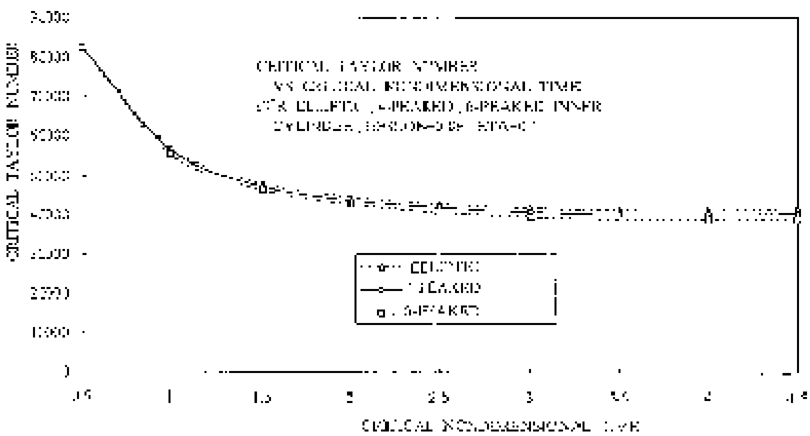


Fig. 5

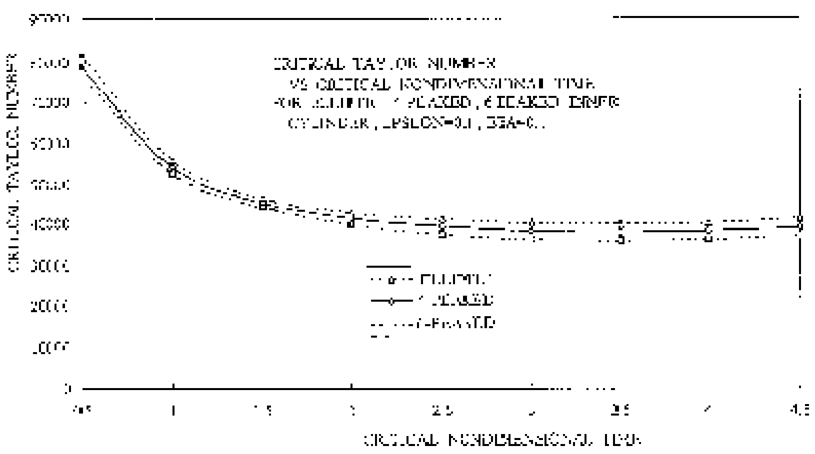


Fig. 6

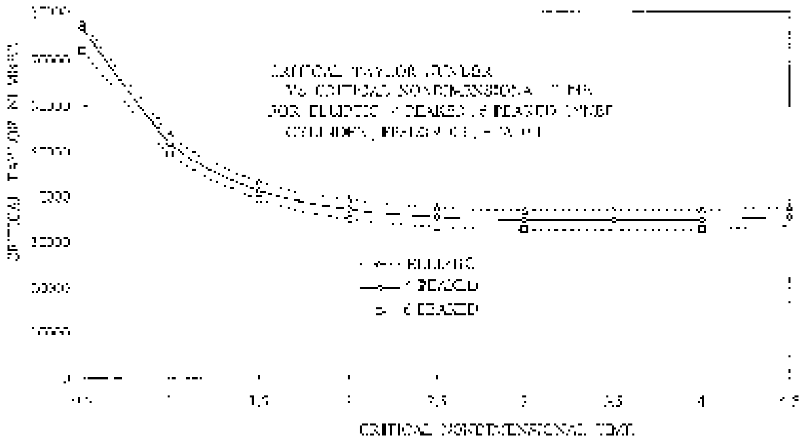


Fig. 7

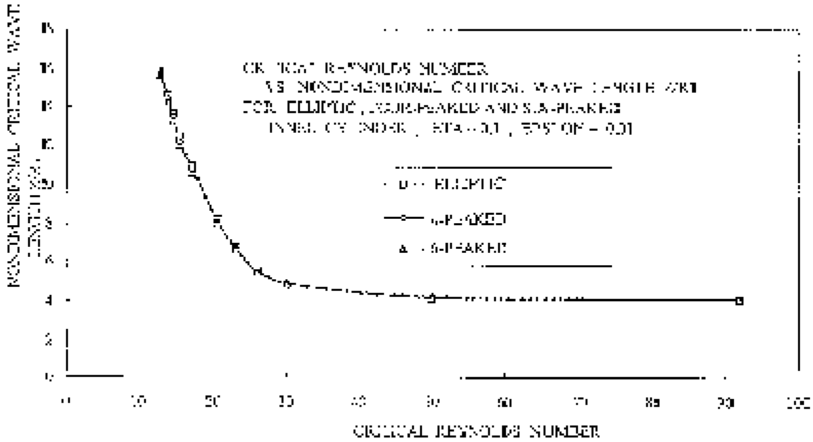


Fig. 8

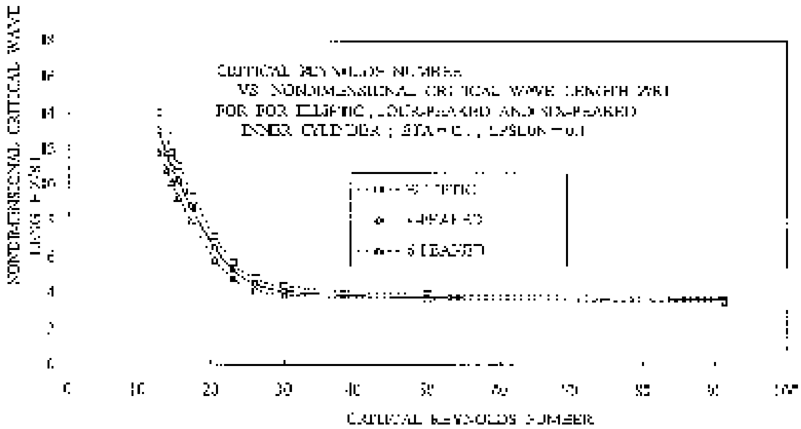


Fig. 9

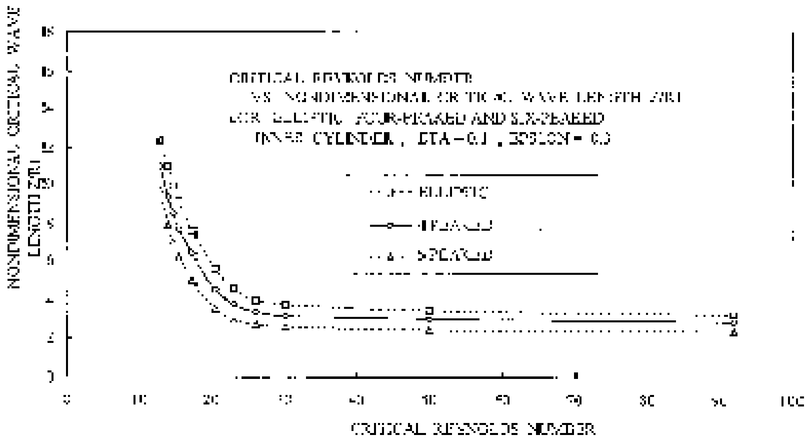


Fig. 10

3. For elliptical inner cylinders, the greater the difference in length of the major axis and minor axis is, the lower the corresponding flow field stability is; and with the increase of each maximum depth of concavity of inner cylnders, the nondimensional critical time for instability onset will be reduced.

Obviously, the inner cylinders of arbitrary shape can be expected by different kinds of transformation. By adopting the concept of transformation from space coordinate to time coordinate, the authors hope the present work can provide valuable information to the researchers in similar field.

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STABILNOST TAYLOR-OVOG VRTLOŽNOG TOKA IZMEĐU ROTIRAJUÆEG VALOVITOG UNUTRAŠNJEG CILINDRA I NEROTIRAJUÆEG OKRUGLOG VANJSKOG CILINDRA

SAŽETAK

Prouèavanje problema stabilnosti kada su dva okrugla koncentrièna cilindra ispunjena nestlaèivom viskoznom tekuæinom obièno se vrši pomoæu slijedeæih rubnih uvjeta: vanjski cilindar ostaje nepomièan, a unutrašnji cilindar rotira konstantnom kutnom brzinom. U ovom radu primijenit æemo iste rubne uvjete za rješavanje problema stabilnosti, ali s valovitim unutrašnjim cilindrom. Osnovna namjera ovog istraživanja je pronalaženje djelovanja asimetriènog unutrašnjeg cilindra na centrifugalni problem stabilnosti pomoæu naèela analogije i superpozicije što su uz napor postigli bivši istraživaèi u vremenski ovisnom Conette-ovom toku. Rezultat pokazuje da su kritièan Taylor-ov broj kao i kritièna osovinska valna duljina obrnuto proporcionalni brojevima najviših toèaka unutrašnjeg cilindra, a razlika između kritiènih Taylor-ovih brojeva za unutrašnje cilindre razlièitih najviših vrhova poveæava se kada se poveæava i visina njihovog vrha.

***Kljuène rijeèi**: centrifugalni problem stabilnosti, Taylor-ov vrtložni tok, nestlaèiva tekuæina, valoviti unutrašnji cilindar, naèelo analogije.*