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Modelling and simulation of a series excited synchronous motor

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SUMMARY

This study attempts to build up a mathematical model, for a type af series excited synchronous motor. This model, when used for simulation, is expected to yield the dynamic behaviour of the system for typical cases such as asynchronous start-up, synchronising and synchronous operation. The model consists of a set of differential equations involving semiconductor switching with constant topology and variable parameters and utilises phase variables. But an inter-conversion between phase variables and space phasors is required to take into account the magnetic saturation. The model is then used for a sample system, to enable a comparison between experimental and computed results. This comparison is used to assess the accuracy, validity and usefulness of the model. Furthermore, steady-state behaviour of the scheme is explained by means of a simple analysis and illustrated by test results.

Key words*: synchronous motor, mathematical model, magnetic saturation.*

1. INTRODUCTION

For fixed and variable-frequency a.c. drivers, a synchronous motor is usually preferable, but in many cases induction motors are used because of their relatively low cost and simplicity. This is especially true for smaller drives, where the cost of a separate excitation system would be prohibitive, and it has been the motivation behind recent research into reluctance motors, permanent magnet excitation and other possibilities. Use of slip-ring induction motors, through a suitable excitation scheme, as synchronous motor have been considered since the pioneering attempts by Danielson [1]. The aim of the similar studies has been to merge the popular and relatively cheap induction motor with the improved power factor, constant speed and controllable load angle properties of the synchronous motor, especially in the reshaping of motor drives in industry. Several excitation systems for such applications have been described by Danielson [1], Brosan and Hayden [2], Griffin [3] and Williamson [4].

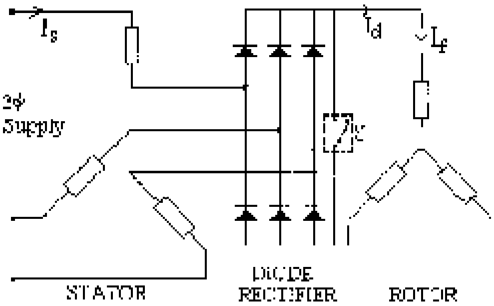


Fig. 1 The series excited synchronous motor

The scheme proposed by Williamson [4] is interesting for it provides self excitation by feeding the rotor circuit via a rectifier bridge connected in series with the stator windings (Figure 1). Williamson [4] provided a simplified mathematical model to explain the steady state performance of this system. A more accurate model is needed to take into consideration the transient behaviour of the system especially to observe the synchronisation behaviour, factors effecting this behaviour and the influence of several machine parameters on this behaviour. An attempt utilising the

PSPICE package was described previously [9] which was in fact a digitally applied analogue simulation. This paper puts forward a completely digital solution approach with a specially derived mathematical model not requiring a package like PSPICE.

2. STUDY OF STEADY-STATE PERFORMANCE

Series exited synchronized induction motor is operated as an induction motor having cylindrical rotor. The system avoids the requirements of a separate excitation supply, transformers or additional machine windings. However, the steady state operation caused by the series excitation differs from the behavior and study of well known separately-excited synchronous motor. For excitation current in series excitation shows variation with the stator current which changes with the load. Therefore, for general performance study a method has been used by considering the wide variations of the excitation.

In the round-rotor synchronous machine the excitation current for Danielson connection is given by the effective stator phase current as:

$$I_e = \frac{1}{\sqrt{2}} \frac{n_r}{n_s} I_f = k_1 I_f \qquad (1)$$

In this equation, the rotor windings with n_r turns has Danielson connection and carry the excitation current, I_f. This current has been represented by the equivalence of the stator current which flows through the stator windings with n_s turns and having effective phase current value of I_e. Here, k_1 is the ratio of the excitation current to its stator equivalent value. The excitation current, if it is constant in the separately excited motor, but, it is dependent on the stator current in the series excited motor. The relationship between the input and output currents of the rectifier in series excited motor becomes as:

$$I_f = k_2 I_s \qquad (2)$$

In order to vary the relation between the stator and field current (i.e., the value of k) and modifications in the design of the machine, the field winding can be shunted by a resistor, as shown in Figure 2:

$$I_f = I_d \frac{R_p}{R_p + R_f} = k_3 I_d \qquad (3)$$

and the stator equivalence of the series excitation current is given as:

$$I_e = k_1 \times k_2 \times k_3 \times I_s = k\, I_s \qquad (4)$$

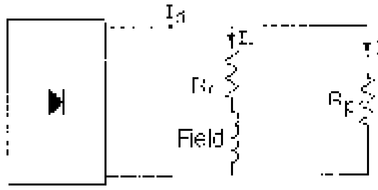


Fig. 2 Changing the coefficient, k by connecting a resistor to excitation circuit in parallel

The analysis of the rectifier bridge is well known, and much information is available with respect to input impedance, power factor and waveforms. However, the case considered here is unusual in the sense that, for typical machine parameter values, the bridge will be almost short-circuited on the d.c. Consequently, it has been necessary to analyse the bridge for these conditions throughout the complete range of modes of operation, including that of near-short- circuit with four devices conducting simultaneously.

The stator equivalence of the excitation in the separately-excited machine is only found by coefficient, k_1 in Eq. (l). In the series excitation the bridge rectifier behaves as an impedance from the point of the a.c side of the rectifier. One phase impedance of the bridge rectifier is taken (R_b+jX_b) and added to the series circuit. This impedance varies with the total rotor resistance [4].

ln this method, the air-gap voltage corresponding to any value of the excitation current is calculated by using the magnetization curve and the total voltage expression is found by using the equivalent circuit shown in Figure 3b. For this purpose, a certain value of the stator current is taken as real and the initial value is chosen. The excitation current is transformed into the stator current. The summation of the stator current and the stator equivalence of the excitation current gives the magnetization current as shown in Figure 3a.

a)

b)

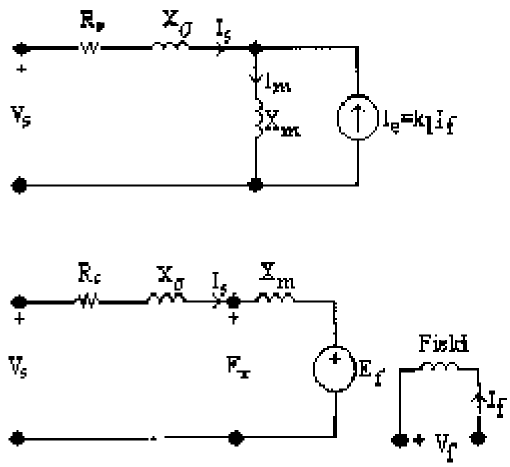


Fig. 3 Current (a) and voltage (b) equivalent circuits of round-rotor synchronous motor

The saturation eflect is taken into consideration by calculating the air-gap voltage the magnetization curve. The total voltage expression is:

$$\overline{V} = V_r + jV_i = \overline{E}_r + I_s R_s + jI_s X_s \qquad (5)$$

The angle between the stator current and the stator equivalence of the excitation current, took variable and repeated it until Eq. (5) is obtained. As a result of this repetition, the power factor, load angle, torque losses and power variables are calculated by using the equations given in Ref. [4].

2.1 Results for steady state

Experiments have been performed in a three phase wound rotor induction motor having the parameters of; *P=1.1 kW, U=220 V, f=50 Hz, I=2.5 A, P=1.* The magnetization curve is experimentally obtained by the relationship between the excitation current and induced voltage in the stator when the machine is operated in the synchronous speed as generator. In addition, from the no-load test, the variation of the core losses with the magnetization current has been considered in the calculations. The output torque has been measured by a foucault break coupled to the rotor shaft. The load angle or position angle of the rotor is obtained by a scaled disk coupled to the shaft. The stator/rotor effective turns ratio for the motor used is *0.967.* Coefficient *k=1.1* is obtained for which the bridge rectifier is directly connected to the rotor by using Danielson connection. Coefficient *k* is *1.23* if the one phase of the rotor is open-circuit, *k* is *0.78* if an additional resistor of *18 ohm* in parallel is used in Danielson connection and finally *k* is approximately obtained *1.0* as in the one phase of the rotor open-circuited with a parallel connected resistor of *55 ohm*. The curves showing effects of *k* for series excitation is given in Figure 4. Experimental results are given using symbols. The very good agreement shown between the calculated and measured performance justifies the assumptions and approximations made in the analysis with respect to harmonics, representation of the rectifier bridge, commutating reactance and magnetic saturation. In order to assess the performance obtained from the machine in the synchronous motor mode, it was tested as a conventional induction motor with the slip rings short-circuited.

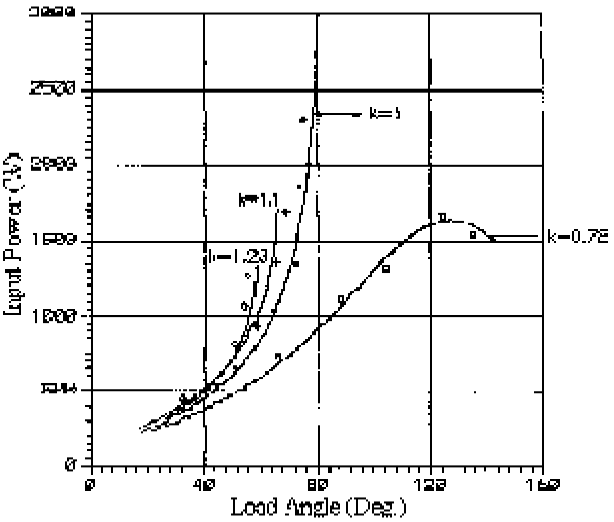


Fig. 4 Variations of the input power for different k's

The machine was also tested more efficient in the synchronous mode as shown by Figure 5, where the variations of input power are compared. The reduction in total loss is a result of a reduction in the stator ohmic loss associated with the improvement in the power factor, and indicates that no significant additional losses are caused by harmonics.

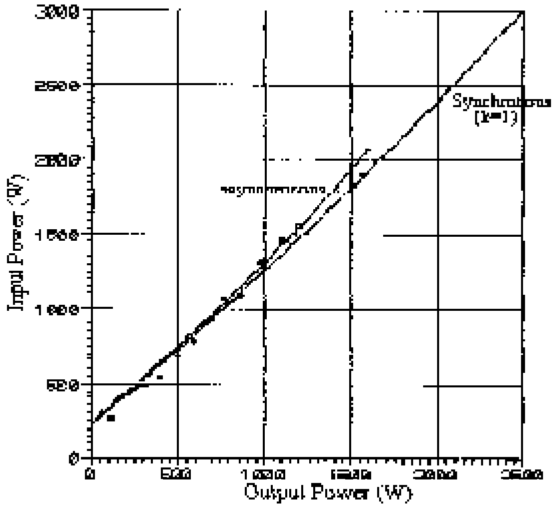


Fig. 5 Variations of input powers at the same output powers for synchronous and asynchronous operations

In Figure 6, the measured variation of input current with coupling torque for the induction mode is compared with that for the synchronous mode with the inherent value of *k=1*.

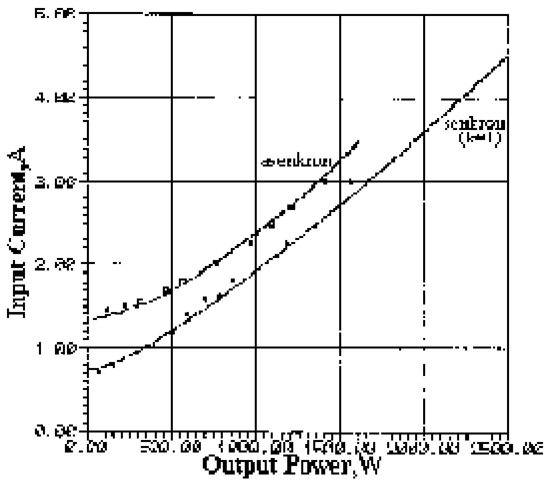


Fig. 6 Measured variations of input current with output for induction and synchronous modes (k=1)

3. MATHEMATICAL MODEL OF THE MOTOR

The proposed connection by Williamson [4] for the series excited synchronous motor is shown in Figure 1. The machine is first started as a short-circuited rotor induction motor with the synchronising switch *S* closed. After run up to speed *S* is opened so that the machine is synchronised and runs as a synchronous motor thereafter. The simulation process requires the provision of two models namely for *S-closed* position and *S-open* position in the asynchronous and synchronous operation modes, respectively. The diodes are modelled by a commonly used logical equivalent circuit as in Figure 7 [8], where the logical switch *SW* is dependent on the branch currents and voltage of the diode at any time: *SW=1* for forward conduction and $\overline{SW} = 1$ for reverse blocking conditions.

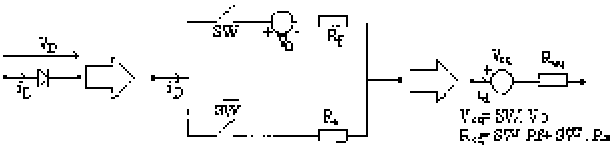


Fig. 7 Diode equivalent circuit model

Figure 8 shows the general circuit outline of the system to be modelled.

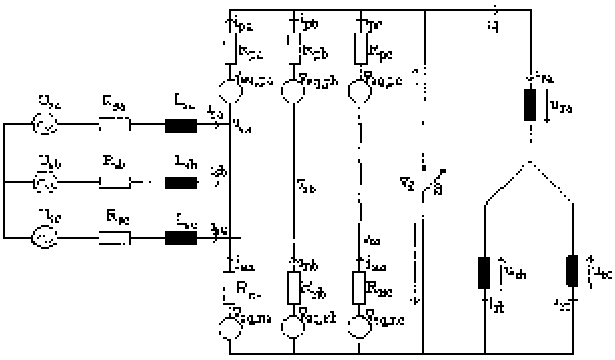


Fig. 8 General circuit outline of the system

With switch *S-closed* the stator equations can be derived as:

$$[V_{sc}] = [U_s] - [R_s][i_s] - [L_{ss}]\frac{d}{dt}[i_s] - [L_{sr}]\frac{d}{dt}[i_r] - \left[\frac{\partial L_{sr}}{\partial q}\right]\frac{dq}{dt}[i_r] \qquad (6)$$

along with the rotor equations:

$$[0] = [R_r][i_r] + [L_{sr}]\frac{d}{dt}[i_s] + [L_{rr}]\frac{d}{dt}[i_r] + \left[\frac{\partial L_{sr}}{\partial q}\right]\frac{dq}{dt}[i_s] \qquad (7)$$

With switch *S-opened* the stator equations can be derived as:

$$[V_{so}] = [U_s] - [R_s][i_s] - [L_{ss}]\frac{d}{dt}[i_s] - [L_{sr}]\frac{d}{dt}[i_r] - \left[\frac{\partial L_{sr}}{\partial q}\right]\frac{dq}{dt}[i_r] \qquad (8)$$

along with the rotor equations:

$$[V_d] = [R_r][i_r] + [L_{sr}]\frac{d}{dt}[i_s] + [L_{rr}]\frac{d}{dt}[i_r] + \left[\frac{\partial L_{sr}}{\partial q}\right]\frac{dq}{dt}[i_s] \qquad (9)$$

The voltage at the output of the rectifier in Figure 8:

$$[V_d \downarrow] = -[R_p][i_p] - [R_n][i_n] \qquad (10)$$

is obtained, where the values of $[i_p]$ and $[i_n]$ can be written separately:

$$[i_n] = -\left[\frac{R_p}{R_p + R_n}\right][i_s] - \left[\frac{1}{R_p + R_n}\right][V_d \downarrow] \qquad (11)$$

$$[i_p] = -\left[\frac{R_n}{R_p + R_n}\right][i_s] - \left[\frac{1}{R_p + R_n}\right][V_d \downarrow] \qquad (12)$$

For the rectifier output current the following equation is written:

$$i_{ra} = \frac{1}{2}[1 \quad 1 \quad 1][i_p] + \frac{1}{2}[1 \quad 1 \quad 1][i_n] \qquad (13)$$

Combinations of these equations with the rectifier circuit equations provide a state equation of the form:

$$\left[\frac{di}{dt}\right] = [G^{-1}][u] - [G^{-1}]\left[R + 3w\frac{\partial L}{\partial q}\right][i] \qquad (14)$$

with the entries in the coefficient matrices depending on the operation mode of the machine. These equations are accompanied by the torque equation:

$$\begin{aligned} Te = &-\frac{p}{2}M_{sr}\left[(i_{ra}i_{sa} + i_{rb}i_{sb} + i_{rc}i_{sc})\sin q\right] - \\ &-\frac{p}{2}M_{sr}\left[(i_{ra}i_{sb} + i_{rb}i_{sc} + i_{rc}i_{sa})\sin\left(q - \frac{2p}{3}\right)\right] - \\ &-\frac{p}{2}M_{sr}\left[(i_{ra}i_{sc} + i_{rb}i_{sa} + i_{rc}i_{sb})\sin\left(q + \frac{2p}{3}\right)\right] \end{aligned} \qquad (15)$$

equation of motion:

$$\frac{dw_{m1}}{dt} = \frac{1}{J}(T_d - T_y) \qquad (16)$$

and rotor position equation as:

$$\frac{dq_1}{dt} = w_{m1} \qquad (17)$$

to form a differential equation model to be integrated.

To account for the saturation behaviour of the machine space phasor machine model devised by Kovacs [6] in the manner modified for solid rotor induction motor by Önbilgin [5] is utilised. During each interval of numerical integration of the differential equation set formed by Eqs. (14), (15), (16), (17), stator and rotor current space phasors $(\bar{i}_s, \bar{i}_r)$ are computed from the corresponding phase currents which will yield a magnetising space phasor:

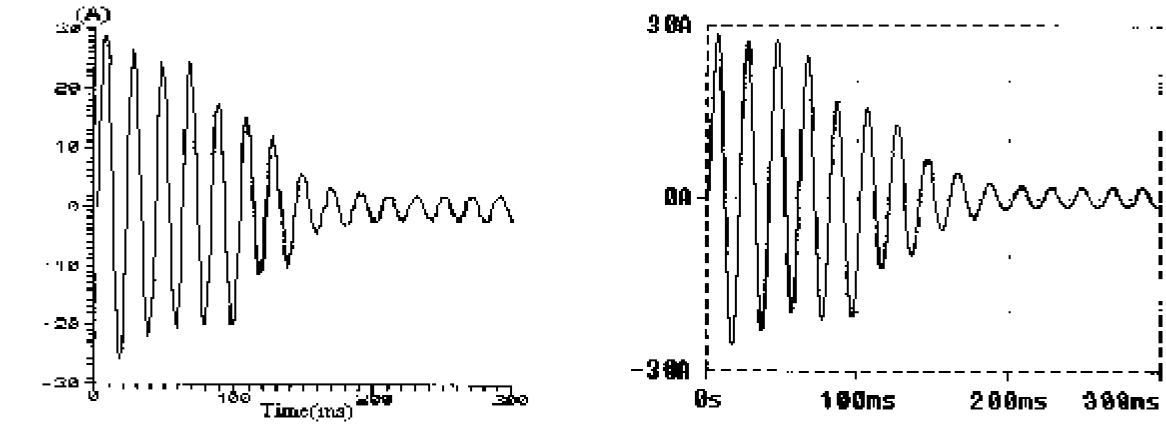
$$\bar{i}_m = \bar{i}_s + \bar{i}_r \qquad (18)$$

It is then possible to use the magnitude of this i_m to iterate until convergence at each integration step, over an M_{sr}-i_m curve which can easily be determined by a no-load test or by magnetic field computations.

Differential equation solution has been realised by the LSODE solver based on the Gear algorithms [7] due to the stiff nature of the equations.

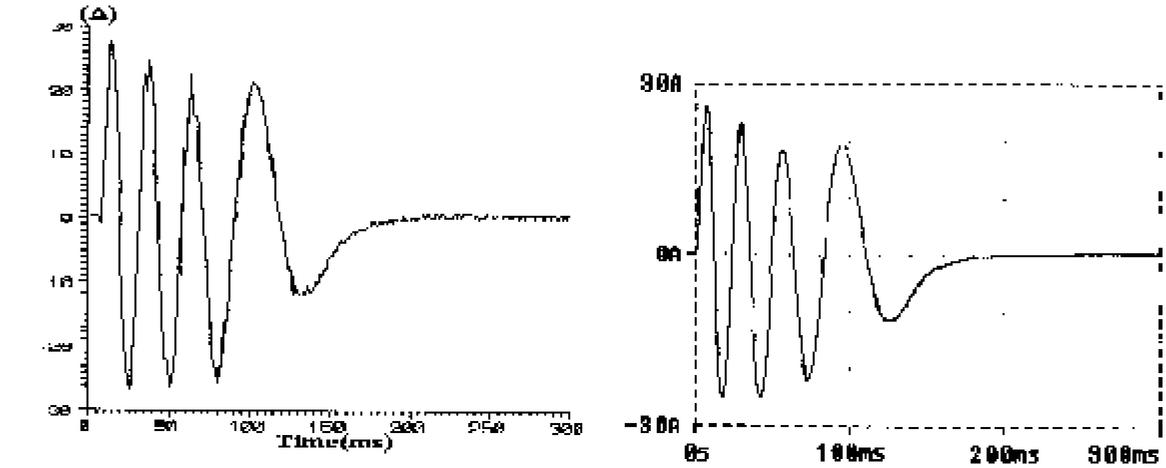
4. RESULTS

The model has been applied to a sample machine to simulate the starting, synchronising and synchronous running behaviour. The results of the computation are compared to the experimental recordings. Figure 9 displays the results for stator current in starting up and Figure 10 displays the results for rotor current in starting up. Figure 11 displays the results for stator current during synchronisation and Figure 12 displays the results for rotor current during synchronisation. Figures 9, 10, 11 and 12 include these computational variations each accompanied by experimental recordings. The waveforms are recorded by a digital storage oscilloscope which loads the measured data into a diskette which are then processed by the Grapher routine on the PC.



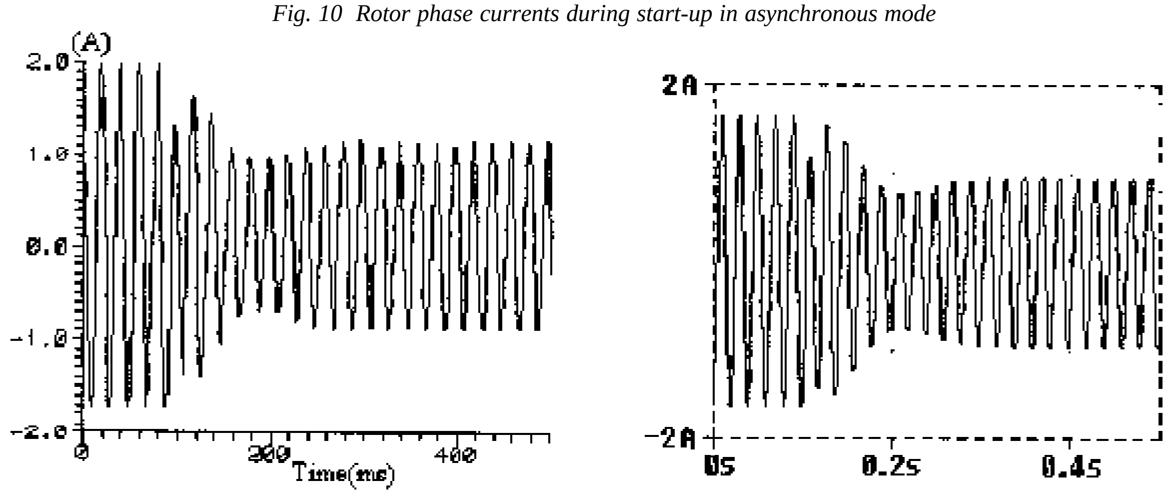
Experimental Simulation (computed)

Fig. 9 Stator phase currents during start-up in asynchronous mode



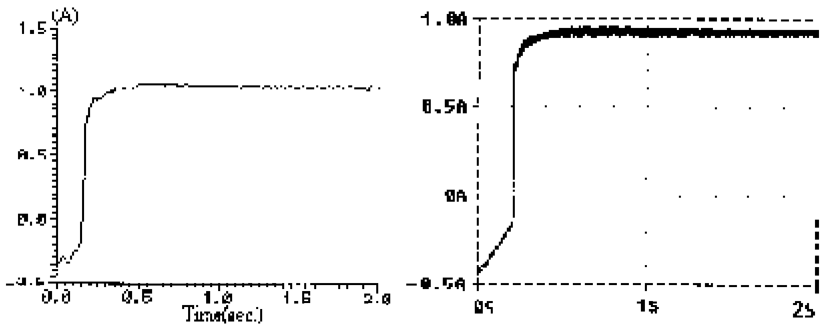
Experimental Simulation (computed)

Fig. 10 Rotor phase currents during start-up in asynchronous mode



Experimental Simulation(computed)

Fig. 11 Stator phase currents during synchronisation



Experimental — Simulation (computed)

Fig. 12 Rotor phase current during synchronisation

5. CONCLUSION

This paper attempts to develop a transient simulation model for a type of a series excited synchronous motor. The contribution is in the from of deriving the model equations along with a novel consideration of saturation, via a phase value to space phasor value conversion. The model is assessed by comparing computed results to the experimental ones. Model promises a wide usage area in the design of such machines where it can take into account the effect of various design parameters.

6. APPENDIX

A typical two-pole slip-ring induction motor of 1.1 kW is used in this simulation and the experiments. The motor parameters are:

R_s=5.2 Ω	R_r=4.85 Ω	B=0.00292 Nms/rad
L_s=0.650458 H		L_r=0.72073 H
$M_{s,r}$=0.449044 H		J=0.00732 kgm^2

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MODELIRANJE I SIMULACIJA SERIJSKI POKRENUTOG SINKRONOG MOTORA

SAŽETAK

Ova studija pokušava napraviti matematièki model za tip sinkronog motora serijski pokrenutog. Kada se ovaj model koristi za simulaciju, oèekuje se postignuæe dinamièkog ponašanja sistema za tipiène sluèajeve poput asinkronog pokretanja, sinkronizirajuæeg i sinkronog rada. Model se sastoji od skupine diferencijalnih jednadžbi koje ukljuèuju poluvodièko paljenje s konstantnom topologijom i parametrima varijabli, te koristi i fazne varijable. Međutim, za ukljuèivanje magnetskog zasiæenja, potrebno je uzeti u obzir međukonverziju između faznih varijabli i prostornih fazora. Model se stoga koristi za sistem uzorka da omoguæi uspoređivanje eksperimentalnih i izraèunatih rezultata. Usporedba se koristi da se potvrdi toènost, vrijednost i korisnost modela. Nadalje, ponašanje stabilnog stanja sheme objašnjava se jednostavnom analizom, a ilustrira pomoæu rezultata testova.

***Kljuène rijeèi**: sinkroni motor, matematièki model, magnetsko zasiæenje.*