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Results on Fröhlich's Brain-wave model

Ahmet Uçar(1) and **Gerard J. Hyland**(2)
(1)Department of Electrical-Electronic Engineering, Firat University, 23119 Elazığ, Turkey
e-mail: aucar1@firat.edu.tr
(2)Department of Physics, University of Warwick, Coventry CV4 7AL, United Kingdom

SUMMARY

The study of a mathematical model, i.e. Fröhlich's Brain-wave model, pertinent to the propagation of brain waves into the Great Membrane, is the focus of interest in this paper. Some new results are presented concerning the response of the system to external stimuli - both ac and dc. In contrast to an earlier claim that a restricted version of the model (containing only 2 degrees of freedom) exhibits a chaos only when the external stimulus contains both ac and dc components we find that ac component alone is sufficient. The extensive bifurcation diagrams of the model are given and the detail of chaotic region is clearly presented for this case.

***Key words**: Fröhlich's Brain-wave model, external stimuli, electroencephalogram, deterministic chaos.*

1. INTRODUCTION

Brain-waves are minute, low frequency, electrical potentials - generally of magnitude less than 300 μV - which are produced by the brain and recorded as an electroencephalogram (EEG) using a system of electrodes attached to the scalp, Freeman [1], [2]. Although the recorded signals are highly complex - often resembling random noise - they can be broadly categorised according to their frequency and amplitude, which appear to be inversely related. Soon after their discovery by Hans Berger in the mid-1920's, it was realised that Brain-waves were a valuable electrophysiological concomitant of different levels of conscious awareness, such as attentiveness, wakefulness, sleep and various pathological and drug induced state of unconsciousness. Thus for example, the so-called α-waves, covering the frequency range of 8-12 Hz, which represents the onset of an awake, relaxed, but attentive, state with the eyes closed, give way, with the onset of drowsiness and light sleep, to the slower (4-7 Hz), larger amplitude *q* - waves, which, in turn are replaced by the even slower (0.5 - 4 Hz) *d* - waves as deep sleep is reached.

An ingenious model for the generation of brain-waves - which not only yields a frequency which is volume independent, but which also affords an understanding of the hyper-sensitivity of many aspects of brain-function to ultra-weak external stimuli - was presented by H. Fröhlich in 1974 at MIT, and subsequently published in 1977, Fröhlich [3]. This model is based on the possibility of periodic, self-sustaining, chemical reactions within an enzyme system localised in the Greater Membrane of the brain, involving its cyclical excitation and de-excitation via chemical reaction with a substrate system. The polarisation field associated with its excited enzyme state exhibits *limit cycle* behaviour which makes them highly sensitive to external electrical and (internal) chemical influences.

The interaction of external electric fields with the internal limit cycle has been the subject of continuing theoretical research since 1977, Kaiser [4], [5] and Fröhlich and Hyland [6]. One of the most interesting findings is that in the presence of an external oscillatory electric field, the model can exhibit deterministic chaos, but only - it is claimed, Kaiser [7] - provided a static field is simultaneously present.

It has been claimed that a restricted version of the model (containing only 2 degrees of freedom) exhibits a chaos *only* when the external stimulus contains both *ac* and *dc* components. In this paper it is shown that *ac* component alone is sufficient to exhibit chaotic behaviour and the detail of bifurcation diagrams are presented.

2. BRAIN - WAVE MODEL

Fröhlich's model is based upon a system of substrate molecules (of number *S*) and enzymes, *N* of which are in the highly activated state and *Z* of which are not excited. The rate of increase in the number of activated enzymes is proportional to:

(i) their concentration, *N*,
(ii) the concentration *Z* of the remaining unexcited enzymes, and
(iii) the concentration, *S*, of the substrates. Including the decrease in *N* due to spontaneous transitions back to the ground state (characterised by a rate constant *b*), the following equation for the first state variable, *N*, is obtained:

$$\dot{N} = a\,NZS - b\,N \qquad (1)$$

where *a* is a constant parameter. On the other hand, for every enzyme excited (to the highly polar state) one substrate molecule is chemically destroyed; the supply is assumed to be maintained, however, by an influx of new substrates. If the rate of attraction of new substrates is *g*, then the second state equation is:

$$\dot{S} = -a\,NZS + g\,S \qquad (2)$$

Fröhlich [3] presented a third equation:

$$\dot{Z} = -a\,NZS + b\,N + l\left(A - Z\right) \qquad (3)$$

where the constant parameter in *l* the final term on the *RHS*, $+\,l(A\text{-}Z)$, is considered to arise from selective long-range interactions, the origin of which does not concern us here.

It may be noted that if Eq. (3) is neglected (which is equivalent to assuming that the concentration *Z* of unexcited enzymes remains constant = *A*), the problem simplifies to a Lotka-Volterra "predator *(N)* - prey *(S)*" system, such as is often used to model and study biological phenomena such as biological clocks and time-dependent neural networks.

The equilibrium points of the steady-state solution of the brain-wave model are obtained by equating (1), (2) and (3) to zero; there are two sets of solutions:

$$(I.) \qquad N_0 = S_0 = 0, \quad Z_0 = A \qquad (4)$$

$$(II.) \qquad N_0 = \frac{g}{a\,Z_0}, \quad S_0 = \frac{b}{a\,Z_0}, \quad Z_0 = A \qquad (5)$$

The first solution is a saddle point (unstable) whereas the second one is a canter (stable). The trajectory of the system is illustrated in Figure 1. The arrows on the trajectories illustrated in Figure 1 represent the time evolution of the system.

The transformation of the system of Eqs. (1), (2) and (3) to new variables which deviate from the chemical equilibrium of Eq. (5), $n = N - N_0$, $s = S - S_0$ and $z = Z - Z_0$. Substituting the new variables $N = N_0 + n$, $S = S_0 + s$ and $Z = Z_0 + z$ into Eqs. (1), (2) and (3) yields a new system:

$$\dot{n} = g\,s + aAns + \frac{g\,b}{aA^2}\,z + \frac{b}{A}nz + \frac{g}{A}zs + a\,nsz \qquad (6)$$

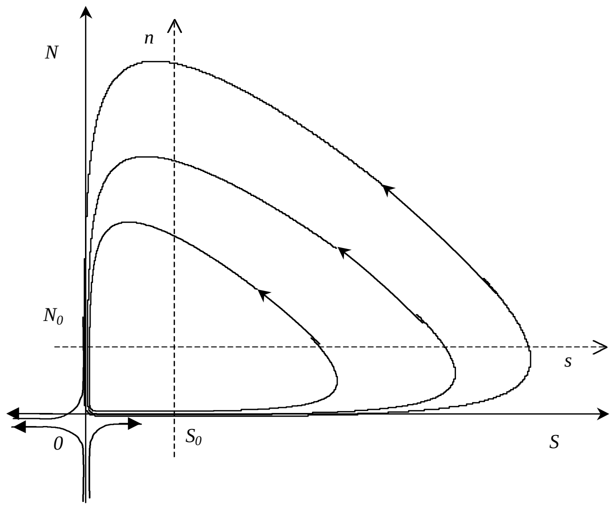

Fig. 1 The N–S phase portrait and cycles of the unforced system of (1) and (2) while the concentration of unexcited enzymes remains constant (Z º A)

$$\dot{s} = -b\,n - \frac{g\,b}{a\,A^2}\,z - \frac{b}{A}nz - a\,Ans - \frac{g}{A}zs - a\,nsz \qquad (7)$$

$$\dot{z} = -\,l\,z - \dot{n} \qquad (8)$$

where the constant parameters are all positive. The system contains Eqs. (6), (7) and (8) includes slow chemical oscillations only and its dynamical behaviour will be investigated.

Neglecting any terms *non*-linear in *n*, *s*, and *z*, we obtain:

$$\ddot{n} + g\,b\,n = 0 \qquad (9)$$

which is a simple harmonic oscillator of natural frequency $w_0 = (bg)^{1/2}$.

Associated with the oscillatory electric polarisation is an electric current which, being subject to resistance, gives a negative contribution to *dn/dt* (see below). On the other hand, a positive non-linear contribution arises from the interaction between the excited (dipolar) enzyme states which favours a non-zero average polarisation. Including these effects, Eq. (6) generalises to:

$$\dot{n} = g\,s + a\,Ans + \frac{g\,b}{aA^2}\,z + \frac{b}{A}nz + \frac{g}{A}zs + a\,nsz + \left(c^2 e^{-G^2 n^2} - d^2\right)n \qquad (10)$$

where *c*, *d* and *G* are constant parameters. The model containing Eqs. (10), (7) and (8) is a superposition of slow chemical and slow electric oscillations. This model is also called the 'coherent oscillation model', with the frozen *z* concentration [4]. It should be emphasised that this system is not just a mathematical model. The basis is completely within profound physical considerations given in Kaiser [4, 5]. The system represented by Eqs. (7), (8), and (10) exhibits point attractor for d^2 ³ $c^2 > 0$; conversely it exhibits limit cycle behaviour for c^2 ³ $d^2 > 0$, where the system parameters were chosen as $G^2 = 0.01$, $A = 100$, $aA = 0.1$, $b = g = 4$, and $l = 60$. It has been shown that the limit cycling frequency is *w* » *4.8 rad/s* and is preserved when different sets of the c^2 and d^2 are taken for the initial conditions of the concentration *n*, *s* and *z* are respectively *4*, *0* and *0*.

The dynamic behaviour of the two component *(n,s)* system in the case c^2 ³ $d^2 > 0$ has been extensively investigated by Kaiser [4]. In this study $c^2 = 5$ and $d^2 = 1$ are taken for the second order system.

It has been shown that the dependence of the maximum value of the fluctuating z variable on the value of the parameter l is significantly greater than zero only for $1 < l < 20$; for $l > 500$, z is effectively zero, Uçar [8], and we retrieve the two component *(n,s)* system which has already been extensively investigated by Kaiser [5].

3. EFFECTS OF EXTERNAL STIMULUS

Of particular interest is the response of the system to an external electric field stimulus, $F(t)$, which interacts directly (*via RHS* of Eq. (10)) with the dynamic polarisation associated with the activated enzyme state:

$$\dot{n} = g\,s + a\,Ans + \frac{g\,b}{a\,A^2}z + \frac{b}{A}nz + \frac{g}{A}zs + a\,nsz + \left(c^2 e^{-G^2 n^2} - d^2\right)n + F_{ext}(t) \qquad (11)$$

where $F_{ext}(t)$ donates the effect of an external time-dependent electric field.

In this paper particular emphasis is given to dynamic behaviour of Fröhlich's Brain-wave model with the external stimulus of $F_{ext} = F_0 + F_1 coswt$. The response of the system has been studied so far, only when z is *neglected* (*coherent oscillation model,* [4]). Within this restricted *(n, s)* model, it has been claimed, Kaiser [4], that *chaos* arises *only* when the external field contains a *dc* component, F_0, Figure 2. The bifurcation diagram shown in Figure 2 was obtained with a static electric field of $F_0=100$ and $w = 2.45$; these parameter values which have been used by Kaiser, and this diagram confirm his results. For the bifurcation diagram, the stroboscopic amplitude n consultation was obtained such that n sampled with the dynamic electric force frequency w and depicted as a function of amplitude F_1.

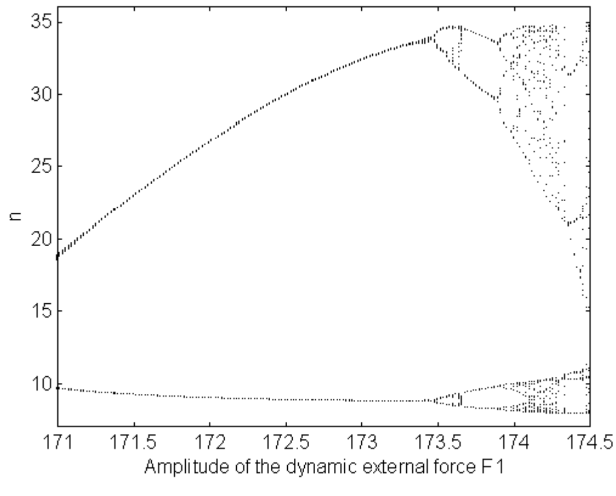

Fig. 2 Bifurcation diagram of the second order system: concentration, n, as a function of the stimulus field F_1 of frequency w =2.45, and static field F_0=100

The bifurcation diagram is depicted in Figure 3 corresponding to $F_0 = 0$ and $w = 2.45$ shows that the presence of a small chaotic region is difficult to detect. In the vicinity of $128.1<F_1<128.35$, the system variable n bifurcates after *jumping phenomena*, Moon [9], and has a narrow chaotic bound. The system again settles down onto two limit cycles after chaotic region. The bifurcation diagrams depicted in Figures 2 and 3 are relevant only to $w = 2.45$. It is therefore necessary to investigate the system's dynamic behaviour for a range of both amplitudes and frequencies of the input stimulus. The bifurcation diagram depicted in Figure 4 as a function of frequency, w, shows that the system exhibits chaotic behaviour in the same frequency region for $F_1=60$, *without* the *dc* input component in the external stimulus.

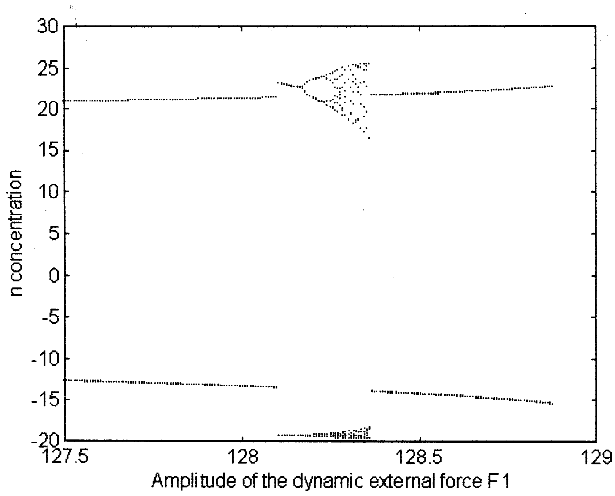

Fig. 3 Bifurcation diagram of the second order system: concentration, n, as a function of the stimulus field F_1 of frequency w =2.45, and F_0=0

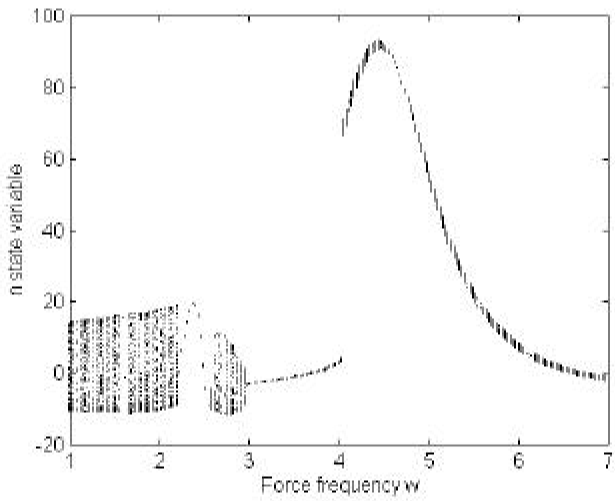

Fig. 4 Bifurcation diagram of the second order system: concentration, n, as a function of the stimulus frequency, w of F_1 = 60, and F_0 = 0

A large chaotic region may be seen in the bifurcation diagram, Figure 5, which was obtained for $w = 5$. It is, of course, possible - as shown in Figures 3, 4 and 5 - to preserve a limit cycle behaviour, for appropriate choices of amplitude of the external stimulus and the frequency, w. Figure 5 indicates the presence of a chaotic region between $F_1 = 144.5$ and $F_1 = 146.9$ for $w = 5$, beyond which the system becomes unstable.

Thus for F_{ext}(º $F_1 \cos wt$) = $146 \cos 5t$, and the same parameter values as used above, the time response of the *n* and *s* concentrations is shown in Figure 6 and the associated phase portrait and Poincaré section in Figures 7 and 8, respectively; these clearly indicate the presence of a deterministic chaos.

Thus, contrary to the implication of Kaiser [4], it has been found that the second order system with the dynamic external stimulus exhibits chaotic behaviour *without* the necessity of a *dc* component.

In view of this difference, it is necessary to re-examine the case F_0 ¹ 0. For $F_0=100$; it can be seen from Figure 2 that chaos indeed exists, but does not set in until F_1 » 174.5; evidently, the effect of the *dc* component, F_0 (as may be seen by comparing Figure 2 to Figure 3) actually *stabilises* the system *against* chaos!

4. CONCLUSIONS

Fröhlich's model for Brain-waves (EEG) in its second order form has been analysed and it has been found to exhibit deterministic chaos. In particular, the response of the model to weak external, time-periodic stimuli has been investigated, thus extending the existing analyses, Kaiser [5], based on the restricted *(n,s)* model.

So far Fröhlich's Brain-waves model is analysed where z concentration is frozen. However, it is considerably important to investigate the chaotic behaviour without external stimuli in the general form of the model.

The study of the dynamic behaviour of the Brain-waves model not only facilitates understanding dynamic behaviour of the Brain-wave but also may stimulate the development of new control strategies based on the given model. There exist already some contributions to the controll of the chaos in the brain, Schiff et al. [10].

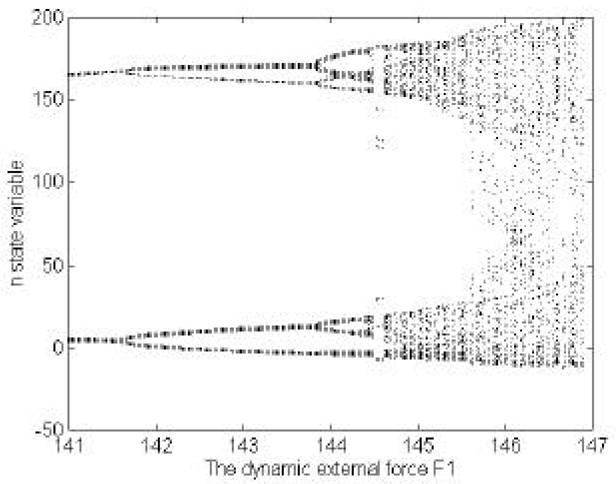

Fig. 5 The bifurcation diagram of the second order system in which z and F_0 are neglected

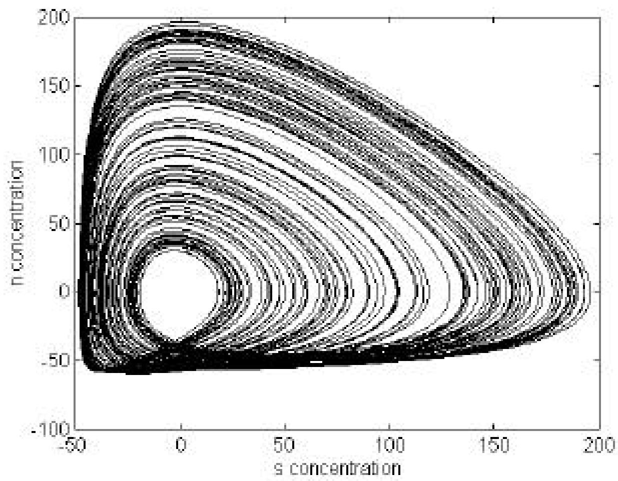

Fig. 7 The phase portrait of n-s system (Z=A) when driven by an external stimulus F_{ext} = 146cos5t applied to the variable n

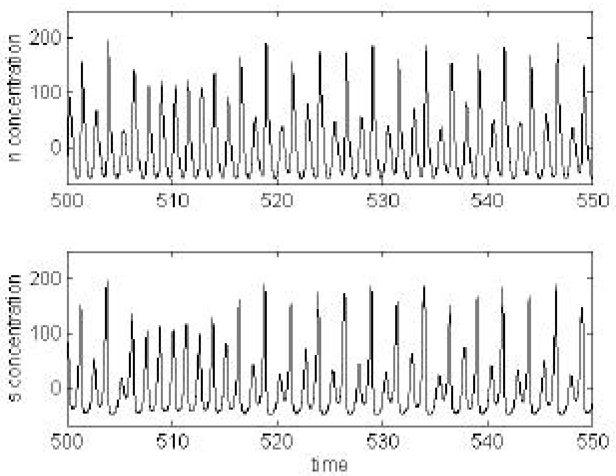

Fig. 6 The time response of enzyme concentrations, n and s, when n is driven by an external stimulus F_{ext} = 146 cos5t

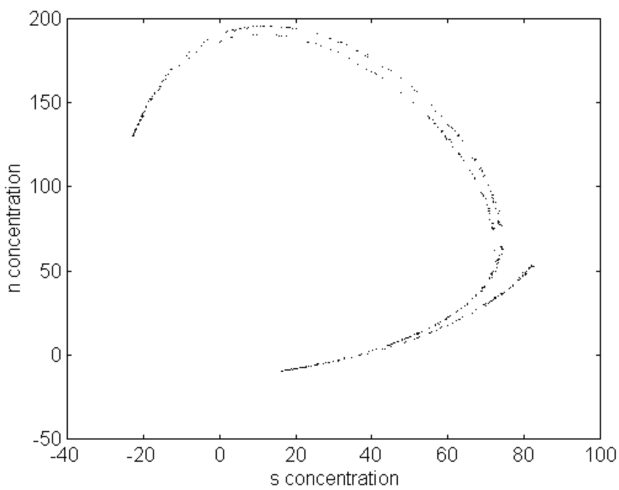

Fig. 8 The Poincaré section of the forced system (F_{ext} = 146 cos5t)

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REZULTATI FRÖHLICHOVOG MODELA MO@DANIH VALOVA

SA@ETAK

Glavna tema ovog rada je prou~avanje matemati~kog modela, tj. Fröhlichovog modela mo`danih valova, koji se odnosi na širenje valova do velike membrane. Iznose se neki novi rezultati koji se odnose na reakciju sustava na vanjske poticaje, odnosno na ac i dc stimulanse. Za razliku od ranijeg mišljenja da ograni~ena verzija modela (koja ima samo dva stupnja slobode) dovodi do kaosa samo onda kad vanjski poticaj sadr`i i ac i dc komponente, autor ovog rada smatra da je dovoljna i samo ac komponenta. U radu su prikazani detaljni dijagrami bifurkacije ovog modela kao i detalji kaoti~nog podru~ja za ovaj slu~aj.

Klju~ne rije~i: *Fröhlichov model mo`danih valova, vanjski poticaji, elektroencephalogram, determinizirani kaos.*