UDC 519.21: 691: 519.61
Original scientific paper
Received: 03.10.2000.

Probabilistic first-ply failure analysis of a laminate in composite material

Andrea Zucchelli, Giangiacomo Minak and **Francesco Cesari**
DIEM, University of Bologna, Engineering Faculty, via Risorgimento 2, I-40136 Bologna, Italy
e-mail: andrea.zucchelli@mail.ing.unibo.it

SUMMARY

A general procedure to develop a probabilistic analysis of composite materials is presented. The second order of the Taylor series has been used in order to determine the statistics of the first ply failure for a symmetric equilibrate composite. The Monte Carlo Method has been used to validate the accuracy of the procedure.

***Key words**: composite first ply failure, probabilistic analysis, Taylor series method.*

1. INTRODUCTION

Failure of a structural element occurs when it cannot perform its intended function. Material fracture is the obvious type of failure but not the only one. Excessive deflection and partially damaged materials may be considered types of failure if the performance of the structure is compromised [1, 2]. Damage and fracture of composite materials may occur in a variety of failure modes. Fiber breaking, matrix crazing, matrix cracking, fiber debonding and delamination are some failure modes examples. It is difficult to incorporate these many modes of failure into design strategy [2, 3]. A simpler way is to use empirical criteria, similar to the failure criteria used in metal design, but customized for composites. The most common criteria to predict failure of a single ply are the maximum stress, the maximum strain, the Tsai-Hill and Tsai-Wu criteria [4, 5]. A classical approach, for the prediction of laminate failure, consists in using one of the failure criteria to predict the first ply failure (FPF) load, which is the load at which the first layer failure occurs [1, 2, 3, 4, 6]. A number of researchers have studied the failure probability of composite laminates, and an extended review may be found in Refs. [7, 8, 9]. Regarding the macro-mechanic probabilistic failure criteria a comparative study is presented in Ref. [10]. A micro-mechanic approach for probabilistic failure criteria can be found in Ref. [11].

Kam and his associates investigated the FPF probability considering the elastic properties of the material, the fiber orientation and the lamina thickness as random variables [12, 13, 14].

In this paper the probabilistic prediction of the FPF, for the symmetric and equilibrate laminates, is developed considering a simple plane stress problem as shown in Figure 1. The lamina stiffness and strength properties have been considered as random variables [15].

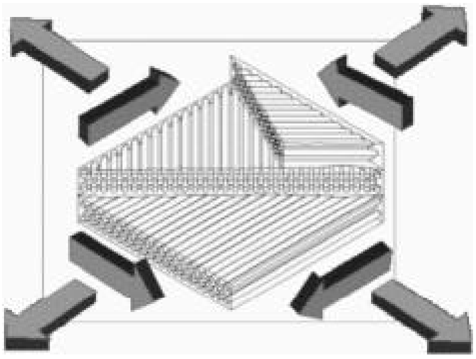

Fig. 1 In-plane load condition for a laminate in composite

2. THE PHYSICAL PROBLEM

This section describes the physical problem and the probabilistic mathematical formulation of the first ply-failure criteria. The aim of the probabilistic analysis [16] is the calculus of the first and the second statistical moment of the FPF load.

In order to calculate the lamina stress status, the Tsai-Hill and Tsai-Wu criterion has been used.

Using the in-plane lamina longitudinal s_1, transverse s_t and shear stress t_{lt}, the Tsai-Hill criterion can be written as follows:

$$G_{TH} = \left[s_l \quad s_t \quad t_{lt} \right]^k \begin{bmatrix} 1/s_{l_R}^2 & -1/2 s_{l_R}^2 & 0 \\ -1/2 s_{l_R}^2 & 1/s_{t_R}^2 & 0 \\ 0 & 0 & 1/t_{lt_R}^2 \end{bmatrix} \begin{bmatrix} s_l \\ s_t \\ t_{lt} \end{bmatrix}^k =$$

$$= \left[s_L^k \right]^T \boldsymbol{M}_{TH} s_L^k = 1 \quad (k = 1, \ldots, N_S) \qquad (1)$$

where k is the lamina number, s_{l_R} is the strength in the fiber direction, s_{t_R} is the strength in the transverse direction and t_{lt_R} is the in-plane shear strength. The Tsai-Hill failure theory uses corresponding strength, tensile or compressive, as shown in Table 1.

In Table 1 X_T and Y_T are the longitudinal and transversal tensile strength, X_C and Y_C are the longitudinal and transversal compressive strength, T is the shear strength.

s_{l_R}	$s_l > 0$	X_T
	$s_l < 0$	X_C
s_{t_R}	$s_t > 0$	Y_T
	$s_t < 0$	Y_C
t_{lt_R}	$t_{lt} < 0$	T
	$t_{lt} > 0$	

Table 1. Tsai-Hill strength choice

The Tsai-Wu criterion is based on a complete quadratic expression:

$$G_{TW} = \begin{bmatrix} s_l & s_t & t_{lt} \end{bmatrix}^k \begin{bmatrix} F_{11} & F_{12} & 0 \\ F_{12} & F_{22} & 0 \\ 0 & 0 & F_{66} \end{bmatrix} \begin{bmatrix} s_l \\ s_t \\ t_{lt} \end{bmatrix}^k +$$

$$+ \begin{bmatrix} F_1 & F_2 & 0 \end{bmatrix} \begin{bmatrix} s_l \\ s_t \\ t_{lt} \end{bmatrix}^k = \qquad (2)$$

$$= \left[s_L^k\right]^T \boldsymbol{M}_{TW} s_L^k + \left[V_{TW}\right]^T s_L^k = 1$$

$$(k = 1,...,N_S)$$

where the coefficients of the Tsai-Wu matrix and vector are:

$$F_{11} = \frac{1}{X_T X_C}, \quad F_{22} = \frac{1}{Y_T Y_C}, \quad F_{12} = -\frac{\sqrt{F_{11}F_{22}}}{2},$$
$$F_{66} = \frac{1}{T^2}, \quad F_1 = \frac{1}{X_T} - \frac{1}{X_C}, \quad F_2 = \frac{1}{Y_T} - \frac{1}{Y_C} \qquad (3)$$

The single lamina stresses, which are used in Eqs. (1) and (2), can be estimated considering the constitutive equation [15,17]:

$$\begin{bmatrix} \boldsymbol{D}_{mm} & \boldsymbol{D}_{mf} \\ \boldsymbol{D}_{mf} & \boldsymbol{D}_{ff} \end{bmatrix}_G \begin{bmatrix} e_G \\ c_G \end{bmatrix} = \boldsymbol{D}_G \begin{bmatrix} e_G \\ c_G \end{bmatrix} = \begin{bmatrix} N \\ M \end{bmatrix}_G \qquad (4)$$

where $\boldsymbol{D}_{mm}$ is the membrane laminate stiffness matrix, $\boldsymbol{D}_{ff}$ is the flexural stiffness matrix, while the matrix $\boldsymbol{D}_{mf}$ couples the membrane and flexural effects; e_G is the laminate strain and c_G is the laminate curvature; N and M are respectively the normal and flexural external load referred to the global coordinate system. Calculating e_G and c_G from Eq. (4), the single laminate stress state can be evaluated as:

$$s_L^k = \boldsymbol{D}_L\left(\boldsymbol{T}^k e_G + z^k \boldsymbol{T}^k c_G\right) \quad (k = 1,...,N_S) \qquad (5)$$

where $\boldsymbol{D}_L$ is the lamina stiffness matrix in the local system, $\boldsymbol{T}^k$, the transformation matrix, relates the local stresses to the global ones and z^k is the distance of the lamina k from the neutral surface. Combining the Eq. (5) with Eqs. (1) and (2) the FPF can be calculated. In particular if h is the strength ratio ($h = s_{ultimate}/s_{applied}$) and the applied load is equal to the unity ($s_{applied} = 1$) the FPF can be calculated by the following relations:

$$h^2\left\{\left[s_L^k\right]\boldsymbol{M}_{TH} s_L^k\right\} - 1 = 0 \quad (k = 1,...,N_S) \qquad (6)$$

$$h^2\left\{\left[s_L^k\right]\boldsymbol{M}_{TW} s_L^k\right\} + h\left\{\left[V_{TW}\right]^T s_L^k\right\} - 1 = 0$$
$$(k = 1,...,N_S) \qquad (7)$$

The laminate probabilistic analysis has been done considering the material properties as random variables. Let $\boldsymbol{b}$ the random variables vector:

$$\boldsymbol{b} = \begin{bmatrix} E_l \\ E_t \\ n_{lt} \\ G_{lt} \end{bmatrix} \qquad (8)$$

Based on the mean-centered second-order perturbation technique, the global stiffness matrix, $\boldsymbol{D}_G$ and the global strain vector, e_G is expanded in terms of the random variables b_i:

$$\boldsymbol{D}_G = \boldsymbol{D}_G(b_0) + \sum_{i=1}^{N} \frac{\partial \boldsymbol{D}_G}{\partial b_i}(b_0) Db_i + $$
$$+ \frac{1}{2} \sum_{i,j=1}^{N} \frac{\partial^2 \boldsymbol{D}_G}{\partial b_i \partial b_j}(b_0) Db_i Db_j \qquad (9)$$

$$e_G = e_G(b_0) + \sum_{i=1}^{N} \frac{\partial e_G}{\partial b_i}(b_0) Db_i +$$
$$+ \frac{1}{2} \sum_{i,j=1}^{N} \frac{\partial^2 e_G}{\partial b_i \partial b_j}(b_0) Db_i Db_j \qquad (10)$$

where $Db_i = b_i - b_{i,0}$ and $b_{i,0}$ is the mean value of the random variables b_i.

Substituting the expansions (9) and (10) in the Eq. (4) and equating equal order terms, three order of linear system are determined:

$$\boldsymbol{D}_G(b_0) e_G(b_0) = s_G$$

$$\boldsymbol{D}_G(b_0) \frac{\partial e_G}{\partial b_i}(b_0) = -\frac{\partial \boldsymbol{D}_G}{\partial b_i}(b_0) e_G(b_0)$$

$$(i = 1,...,N)$$

$$\boldsymbol{D}_G(b_0) \frac{\partial^2 e_G}{\partial b_i \partial b_j}(b_0) = -\frac{\partial^2 \boldsymbol{D}_G}{\partial b_i \partial b_j}(b_0) e_G(b_0) + \qquad (11)$$

$$-\left[\frac{\partial \boldsymbol{D}_G}{\partial b_i}(b_0)\frac{\partial e_G}{\partial b_j}(b_0) + \frac{\partial \boldsymbol{D}_G}{\partial b_j}(b_0)\frac{\partial e_G}{\partial b_i}(b_0)\right]$$

$$(i, j = 1,...,N)$$

Using the solutions of the linear Eqs. (11), the strain field is found in the Taylor series expansion form. In order to use the Eq. (5) and the solution just obtained, the stress status for each lamina is calculated in form of Taylor series:

$$s_L^k = s_L^k(b_0) + \sum_{i=1}^{N} \frac{\partial s_L^k}{\partial b_i}(b_0) Db_i + \frac{1}{2} \sum_{i,j=1}^{N} \frac{\partial^2 s_L^k}{\partial b_i \partial b_j}(b_0) Db_i Db_j \qquad (12)$$

$$(k = 1,\dots,N_{lam})$$

where N_{lam} is the number of ply in the laminate and the terms of the expansion are given by:

$$\begin{aligned} s_L^k(b_0) &= \boldsymbol{D}_L(b_0) T^k e_G(b_0) \\ \frac{\partial s_L^k}{\partial b_i}(b_0) &= \boldsymbol{D}_L(b_0) T^k \frac{\partial e_G}{\partial b_j}(b_0) \\ \frac{\partial^2 s_L^k}{\partial b_i \partial b_j}(b_0) &= \boldsymbol{D}_L(b_0) T^k \frac{\partial^2 e_G}{\partial b_i \partial b_j}(b_0) \end{aligned} \qquad (13)$$

$$(k = 1,\dots,N_{lam})$$

Using the Eq. (12) the statistics of the stress for each lamina can be calculated. In particular the mean vector and the covariance matrix of the stress can be calculated using the next relations:

$$E\left[s_L^k\right] = s_L^k(b_0) + \frac{1}{2} \sum_{i,j=1}^{N} \frac{\partial^2 s_L^k}{\partial b_i \partial b_j}(b_0) Cov\left[b_i, b_j\right] \qquad (14)$$

$$k = 1,\dots,N_{lam}$$

$$Cov\left[s_m^k, s_m^l\right] = \sum_{i,j=1}^{N} \frac{\partial s_m^k}{\partial b_i}(b_0) \frac{\partial s_m^l}{\partial b_i}(b_0) Cov\left[b_i, b_j\right] \qquad (15)$$

$$m,n = 1,2,3 \qquad k,l = 1,\dots,N_{lam}$$

In order to determine the statistics of the FPF, the strength ratio given by the Tsai-Hill and the Tsai-Wu criteria have been expanded via Taylor series, using the second order approximation:

$$h^k = h^k\left(a_0^k\right) + \sum_{i=1}^{N} \frac{\partial h^k}{\partial a_i^k}\left(a_0^k\right) Da_i^k + \frac{1}{2} \sum_{i,j=1}^{N} \frac{\partial^2 h^k}{\partial a_i^k \partial a_j^k}\left(a_0^k\right) Da_i^k Da_j^k \qquad (16)$$

$$(k = 1,\dots,N_{lam})$$

where a_0^k denote the vector of the random variables mean values of the k-th ply used for the FPF calculus. In particular the random vector $\boldsymbol{a}$ contain the stress and strength corresponding to each lamina:

$$a^k = \begin{bmatrix} s_{stress}^k \\ s_{strenght} \end{bmatrix}, \; s_{stress}^k = \begin{bmatrix} s_l^k \\ s_t^k \\ t_{lt}^k \end{bmatrix}, \; s_{strenght} = \begin{bmatrix} X_T \\ X_C \\ Y_T \\ Y_C \\ T \end{bmatrix},$$

$$(k = 1,\dots,N_{lam}) \qquad (17)$$

The mean vector and the covariance matrix of the strength ratio h are defined as:

$$E\left[h^k\right] = h_k\left(a_0^k\right) + \frac{1}{2} \sum_{i,j=1}^{N} \frac{\partial^2 h^k}{\partial a_i^k \partial a_j^k}\left(a_0^k\right) Cov\left[a_i^k, a_j^k\right] \qquad (18)$$

$$(k = 1,\dots,N_{lam})$$

$$Cov\left[h^m, h^n\right] = \sum_{i,j=1}^{N} \frac{\partial h^m}{\partial a_i^m}\left(a_0^m\right) \frac{\partial h^n}{\partial a_j^n}\left(a_0^n\right) Cov\left[a_i^m, a_j^n\right] \qquad (19)$$

$$(m,n = 1,\dots,N_{lam})$$

In this paper the FPF identification is based on the mean value of the lamina strength ratio.

3. NUMERICAL INVESTIGATIONS

As an example of the developed procedure, the numerical calculation has been performed on a symmetric and equilibrate laminate because it is the most common in industrial applications.

To perform the FPF probabilistic calculus a symbolic code has been realized by *Mathematica* package. The code has been organized in two steps: the probabilistic lamina analysis and the FPF probabilistic calculus. The results obtained by this code have been compared with the corresponding Monte Carlo simulations.

The laminate is symmetric-equilibrate, [0°, 90°, 45°, -45°], and the laminate thickness percentage are $(l_1, l_2, l_3, l_4) = (0.4, 0.2, 0.2, 0.2)$. The material lamina is Graphite Epoxy and the property values used for the calculus are summarized in Tables 2 and 4. In the first example the in-plane and positive unit pressure is active in the *x* direction (see Figure 2) and the analysis is based on three different coefficients of variation for the material property values.

In Figures 3 and 4 some graphical results and comparisons between the direct solutions and MCM simulations are presented. The Tables 3a and 3b summarize the critical load of each lamina in terms of mean values and coefficient of variation (CV), starting from the first case to the second case. It is interesting to observe increasing of the coefficient of variation. In particular for the critical load of the ply 2, the CV

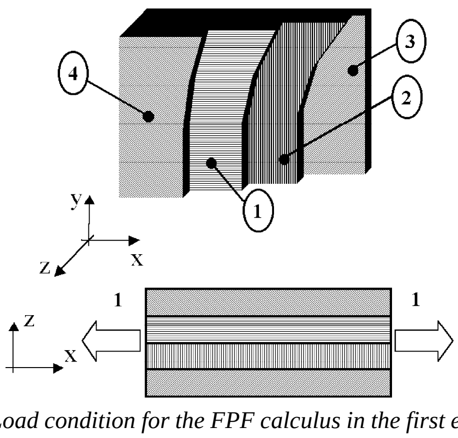

Fig. 2 Load condition for the FPF calculus in the first example

increases from 16% to 30% using the Tsai-Hill failure criterion and, and it increases from the 14% to 28% using the Tsai-Wu failure criterion.

In the second example the FPF load is analyzed in a different load condition, Figure 5. A classical cylindrical load condition is considered: the unit load is active in the x direction and the half unit load is active in the y direction.

The Figures 6 and 7 show the behavior of the probabilistic distribution of the FPF in the second load

Table 2. Material property values used in the three cases of the first example. Three different coefficients of variation have been used to analyze the capability of the Taylor series method.

	Case 1 LogNormal Distributed		Case 2 LogNormal Distributed		Case 3 LogNormal Distributed	
Random Variables	Expected Values	Coefficient of variation	Expected Values	Coefficient of variation	Expected Values	Coefficient of variation
E_l	134000	0.1	134000	0.1	134000	0.2
E_t	7000	0.1	7000	0.1	7000	0.2
ν_{lt}	0.25	0.1	0.25	0.1	0.25	0.2
G_{lt}	4200	0.1	4200	0.1	4200	0.2
X_l	1270	0.1	1270	0.2	1270	0.2
X_2	1130	0.1	1130	0.2	1130	0.2
Y_1	42	0.1	42	0.2	42	0.2
Y_2	141	0.1	141	0.2	141	0.2
T	63	0.1	63	0.2	63	0.2

Table 3a. First example: the strength ratio of the second lamina is the first ply failure load (Tsai-Hill failure criterion)

		Tsai-Hill					
		Case 1		Case 2		Case 3	
		Expected Values	Coeff. of variation	Expected Values	Coeff. of variation	Expected Values	Coeff. of variation
ply 1	Direct Solution	636.284	0.100	636.249	0.200	637.378	0.201
	Montec	636.780	0.101	640.765	0.200	635.700	0.198
ply 2	Direct Solution	**425.014**	**0.156**	**423.096**	**0.226**	**431.475**	**0.297**
	Montec	**426.476**	**0.158**	**426.394**	**0.228**	**435.192**	**0.310**
ply 3	Direct Solution	560.139	0.123	545.859	0.172	574.305	0.232
	Montec	559.142	0.125	546.413	0.168	548.596	0.243
ply 4	Direct Solution	560.139	0.123	545.859	0.072	574.305	0.232
	Montec	559.142	0.125	546.413	0.168	548.596	0.243

Table 3b. First example: the strength ratio of the second lamina is the first ply failure load. (Tsai-Wu failure criterion)

		Tsai-Wu					
		Case 1		Case 2		Case 3	
		Expected Values	Coeff. of variation	Expected Values	Coeff. of variation	Expected Values	Coeff. of variation
ply 1	Direct Solution	639.638	0.099	639.478	0.198	640.367	0.199
	Montec	639.213	0.100	643.171	0.198	637.947	0.196
ply 2	Direct Solution	**385.724**	**0.144**	**383.375**	**0.210**	**389.119**	**0.275**
	Montec	**385.441**	**0.143**	**384.508**	**0.210**	**390.624**	**0.282**
ply 3	Direct Solution	551.050	0.136	542.240	0.198	545.967	0.265
	Montec	552.072	0.141	544.458	0.198	548.550	0.276
ply 4	Direct Solution	551.050	0.136	542.240	0.198	545.967	0.265
	Montec	552.072	0.141	544.458	0.198	548.550	0.276

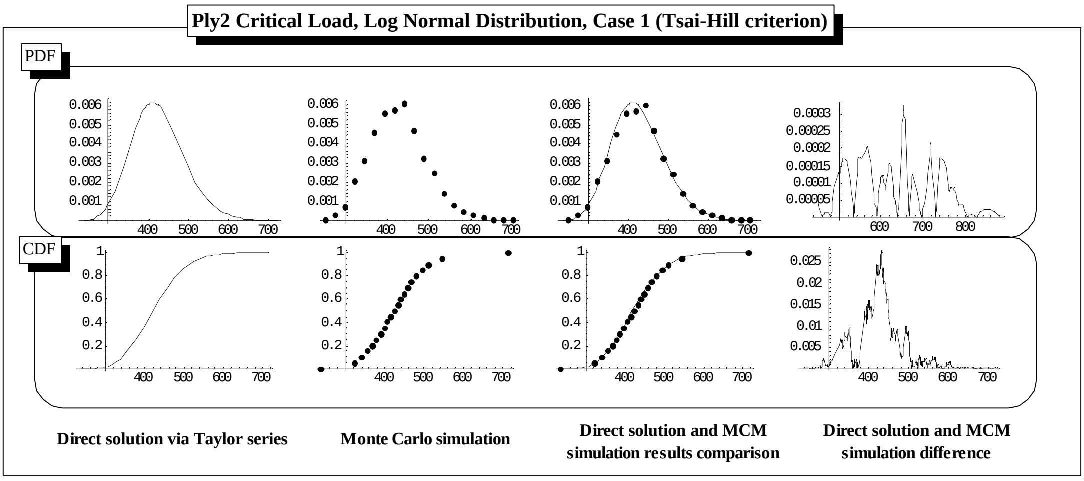

Fig. 3 FPF load corresponding to the second lamina strength ratio using the Tsai-Hill criteria, Case 1

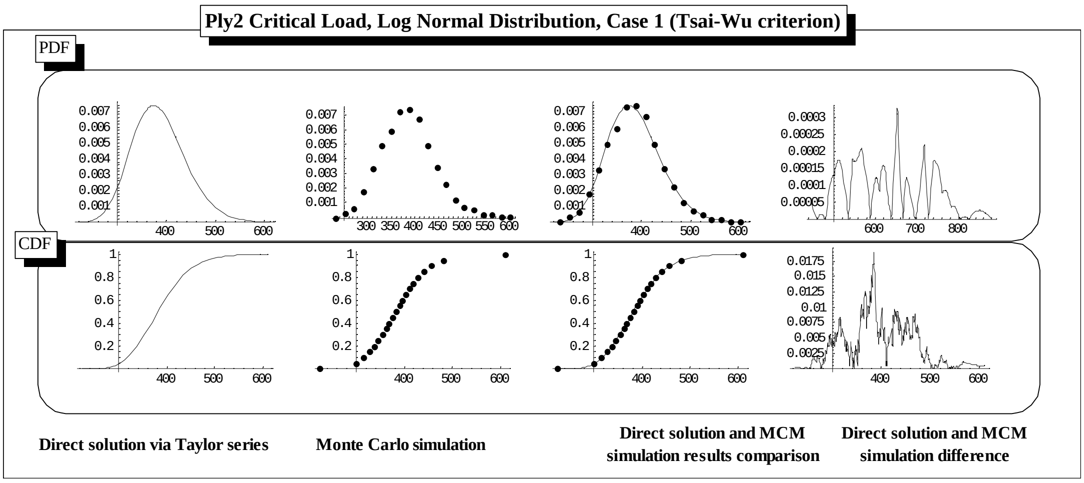

Fig. 4 FPF load corresponding to the second lamina strength ratio using the Tsai-Wu criteria, Case 1

case. The first case (Table 4) and the second case (Table 2) results comparison show that the coefficient of variation is similar in spite to different load condition. It is also interesting to notice that in both examples the error between the CDF obtained via Direct solution and the MCM simulation is restricted on the second decimal digit.

Table 4. Material property values used in the three cases of the second example. Three different coefficients of variation have been used to analyze the capability of the Taylor series method.

	Case 1 LogNormal Distributed		*Case 2 LogNormal Distributed*	
Random Variables	*Expected Values*	*Coefficient of variation*	*Expected Values*	*Coefficient of variation*
E_l	134000	0.1	134000	0.2
E_t	7000	0.1	7000	0.2
ν_{lt}	0.25	0.1	0.25	0.2
G_{lt}	4200	0.1	4200	0.2
X_1	1270	0.2	1270	0.4
X_2	1130	0.2	1130	0.4
Y_1	42	0.2	42	0.4
Y_2	141	0.2	141	0.4
T	63	0.2	63	0.4

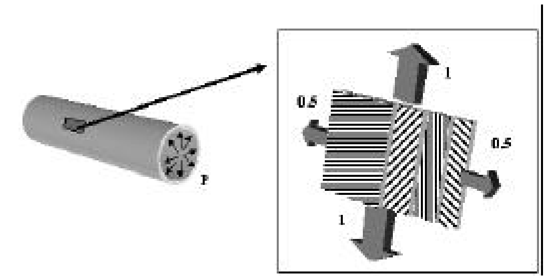

Fig. 5 Load condition for the FPF calculus in the second example

4. CONCLUSIONS

A procedure for probabilistic analysis of a composite laminates with one failure mode has been studied on the basis of the second order Taylor series method. The procedure developed considers the lamina material properties and the lamina strength as random variables and the lognormal distribution has been used to describe their behavior. Two load cases have been studied and in both the MCM simulations to validate the accuracy of the direct solution results. In particular it has been shown that the difference between the FPF cumulative distribution function (CDF), log normally distributed, obtained by Taylor method and the one obtained by the MCM is restricted on second decimal digit.

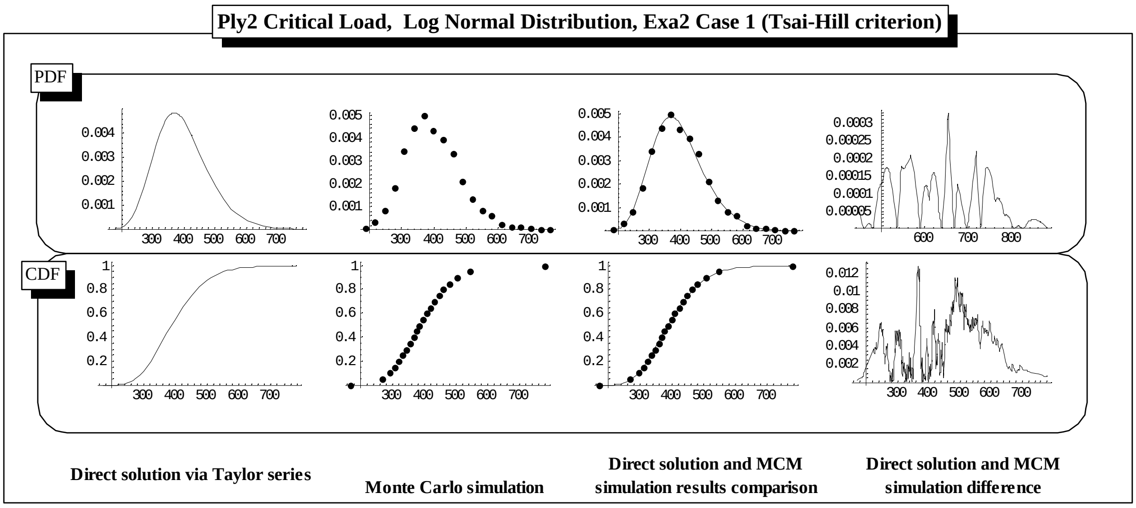

Fig. 6 FPF load corresponding to the second lamina strength ratio using the Tsai-Hill criteria, Case 2

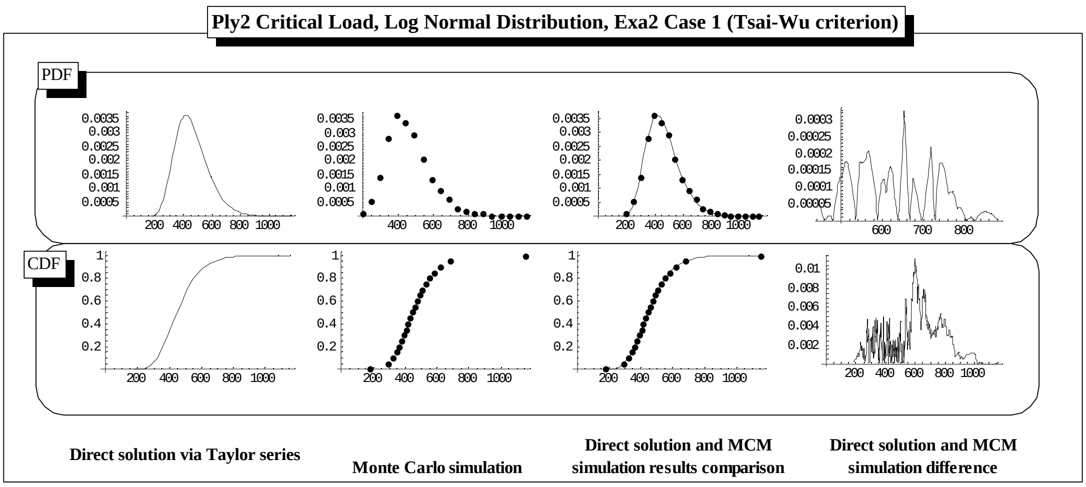

Fig. 7 FPF load corresponding to the second lamina strength ratio using the Tsai-Wu criteria, Case 2

Table 5. Second example results

		Tsai-Hill				*Tsai-Wu*			
		Case 1 (3000 Simulations)		*Case 2 (10000 Simulations)*		*Case 1 (3000 Simulations)*		*Case 2 (10000 Simulations)*	
		Expected Values	*Coefficient of variation*	*Expected Values*	*Coefficient of variation*	*Expected Values*	*Coefficient of variation*	*Expected Values*	*Coefficient of variation*
ply 1	*Direct Solution*	*449.700*	*0.170*	*419.976*	*0.361*	*527.116*	*0.225*	*510.501*	*0.461*
	Montec	*448.425*	*0.164*	*419.960*	*0.333*	*530.361*	*0.222*	*518.429*	*0.462*
ply 2	*Direct Solution*	***396.249***	***0.219***	***392.659***	***0.431***	***460.302***	***0.258***	***475.112***	***0.484***
	Montec	***395.130***	***0.214***	***387.410***	***0.408***	***462.225***	***0.264***	***474.141***	***0.542***
ply 3	*Direct Solution*	*421.347*	*0.194*	*398.956*	*0.402*	*493.999*	*0.238*	*485.078*	*0.476*
	Montec	*419.752*	*0.184*	*397.294*	*0.354*	*496.291*	*0.235*	*485.812*	*0.474*
ply 4	*Direct Solution*	*421.347*	*0.194*	*398.956*	*0.402*	*493.999*	*0.238*	*485.078*	*0.476*
	Montec	*419.752*	*0.184*	*397.294*	*0.354*	*496.291*	*0.235*	*485.812*	*0.474*

5. REFERENCES

[1] D. Gay, *Matériaux Composites*, Hermes, 1997.

[2] K.K. Chawla, *Composite Materials, Science and Engineering*, Springer, 1998.

[3] E.J. Barbero, *Introduction to Composite Materials Design*, Taylor&Francis, 1998.

[4] F. Cesari, Introduction to Composite Materials, University of Bologna, Engineering Faculty, Internal notes, DIEM, 1999.

[5] S.W. Tsai and E.M. Wu, A general theory of strength for anisotropic materials, *J. of Composite Materials*, Vol. 5, pp. 58-80, 1971.

[6] The Composites Materials Handbook - MIL - 17, Volume 3 - Materials Usage, Design and Analysis, 1999.

[7] L.S. Sutherland and C. Guedes Soares, Review of probabilistic models of strength of composite materials, *Reliability Engineering and System Safety*, Vol. 56, pp. 183-196, 1997.

[8] W.F. Wu, H.C. Cheng and C.K. Kang, Random field formulation of composite laminates, *Composite Structures*, Vol. 49, pp. 87-93, 2000.

[9] M.R. Gurvich and R.B. Pipes, Probabilistic strength analysis of four-directional laminated composites, *Composite Science and Technology*, Vol. 56, pp. 649-656, 1996.

[10] H. Nakayasu and Z. Maekawa, A Comparative study of failure criteria in probabilistic fields and stochastic failure envelopes of composite materials, *Reliability Engineering and System Safety*, Vol. 56, pp. 209-220, 1997.

[11] E.M. Wu and C.S. Robinson, Computational micro-mechanics for probabilistic failure of fiber composite in tension, *Composite Science and Technology*, Vol. 58, pp. 1421-1432, 1998.

[12] T.Y. Kam, S.C. Lin and K.M. Hsiao, Reliability-analysis of nonlinear laminated composite plate structures, *Composite Structures*, Vol. 25, No. 1-4, pp. 503-510, 1993.

[13] S.C. Lin, T.Y. Kam and K.H. Chu, Evaluation of Buckling and First-Ply Failure Probabilities of Composite Laminates, *Int. J. Solids Structures*, Vol. 35, No. 13, pp. 1395-1410, 1998.

[14] T.Y. Kam and E.S. Chang, Reliability formulation for composite laminates subjected to first-ply failure, *Composite Structures*, Vol. 38, No. 1-4, pp. 447-452, 1997.

[15] A. Zucchelli and F. Cesari, Reliability analysis of laminates in composite materials, University of Bologna, Engineering Faculty, Internal notes, DIEM, 2000.

[16] W.K. Liu, T. Belytschko and A. Mani, Probabilistic finite elements for nonlinear structural dynamics, *Comp. Meth. in Applied Mech. and Eng.*, Vol. 56, pp. 61-81, 1986.

[17] P.B. Thanedar and C.C. Chamis, Reliability considerations in composite laminate tailoring, *Computers and Structures*, Vol. 54, pp. 131-139, 1995.

PROBABILISTI^KA ANALIZA SLOMA PRVE RAZINE SLOJA LAMINATA U KOMPOZITNOM MATERIJALU

SA@ETAK

Ovaj rad govori o op}oj metodi pomo}u koje se mo`e napraviti probabilisti~ka analiza kompozitnih materijala. Drugi red Taylor-ovog niza koristi se za odre|ivanje statistike sloma prve razine sloja simetri~ki uravnote`enog kompozita. Monte Carlo-vom metodom potvr|uje se to~nost opisane metode.

Klju~ne rije~i: *slom prve razine kompozita, probabilisti~ka analiza, Taylor-ov niz.*