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The laterally unsupported steel beams designed by ECCS and AISC specification under snow loads in Croatia

Marta Sulyok-Selimbegovi}
Faculty of Architecture, University of Zagreb, Ka~i}eva 26, HR - 10000 Zagreb, Croatia

SUMMARY

The beam design requirements for limit state lateral torsional buckling are analysed and criteria for two codes (AISC and ECCS) are compared. As the differences in the design requirements are substantial, the comparison of the achieved reliability indeces is made in order to find out the model which is closer to the target reliability level. For this purpose four examples are analyzed concerning various observation data of snow loads on four locations in Croatia. The characteristic values from the actual snow loads on the roof constructions are derived and the laterally unsupported rolled steel beams are designed with various unbraced lengths due to the ultimate moments capacity requirements for the testing models. The statistical parametars of the experimental results are evaluated and FOSM method is used for the procedure of the calibration. As the results vary with the applied snow loads and the slenderness ratios for two examined models of the designed rolled beams, which are compared, it is necessary to achieve target values of reliability by correction of the model and resistance factors.

***Key words**: reliability, rolled beams, welded beams, specifications, FOSM theory, snow loads, distribution density functions.*

1. INTRODUCTION

The laterally unsupported beams, which are examined in the examples of calibration, are subjected to the snow loads with secondary members dividing the main beams into three equal unbraced lengths, with the central critical segment under uniform moment. As the ultimate capacity for lateral torsional buckling varies with the theoretical models used for design of these beams, the statistical parametars of the experimantal results are the same for certain groups of rolled beams in the same slenderness range, but the distances of the beams as well the sections are different in the conjunction with the applied loads and the evaluation models.

It is evident that the differences in the design requirements for various specification, give the different reliability index for the same applied loads calculated by the same global factors, which provides the theoretical models to be compared. The results of the calibrations are compared and they show which models are on the conservative side for certain slenderness range.

2. THEORETICAL MODELS FOR LATERAL TORSIONAL BUCKLING

The differences and similarities are compared for two code-rules which are based on essentially the same theoretical background and the purpose of this section is to compare these methods and to draw conclusions about the impact of the differences between them. These divergences are due to the different perceptions of the effects of initial imperfections. For specifications ECCS and AISC many variabilities arise especially in the inelastic range and for the beams under the moment gradient.

The following general treatments are used for beam design rules:

2.1 The use of the columns formula with the "equivalent" slenderness parameter: $\bar{l}_M = \sqrt{M_p / M_E}$, here M_p is the plastic moment of the cross section and M_E is the elastic lateral torsional buckling moment of the beam with the coefficient $k_M = f\left(\bar{l}_M\right)$ of initial imperfection coefficients a for rolled (0.21) and welled (0.49) beams, such as speciffied in EC3.

2.2 The use of the analytically exact lateral-torsional buckling solutions for the cross section, loading and end condition, empirically modified to account for bucking in the inelastic range, such as in AISC Specification, with the linear interaction in the inelastic range, which can accommodate idealized conservative simplifications.

2.3 The most general equation is the one adopted in Western Europe: $M_u = M_p \left(\frac{1}{1+\bar{l}_M^{\,2n}} \right)^{1/n}$, with the exponent which takes on values 2.5 for rolled and 2.0 for welded beams. The variation of the buckling strength with $\bar{l}_M$ for the various values of *n* and *a* is shown in Figure 1. The advantage of this method is its simplicity and generality. Its disadvantage is that the implied reduction in flexural and torsional stiffness due to the partial plastification of the cross section is the same regardless of the magnitude of the compressive residual stress, and it does not differentiate between cases where the whole segment or most of the unbraced segment is yielded and where only a small part of the length is plastified. This equation is conservative for beams with low residual stress and steep moment gradient.

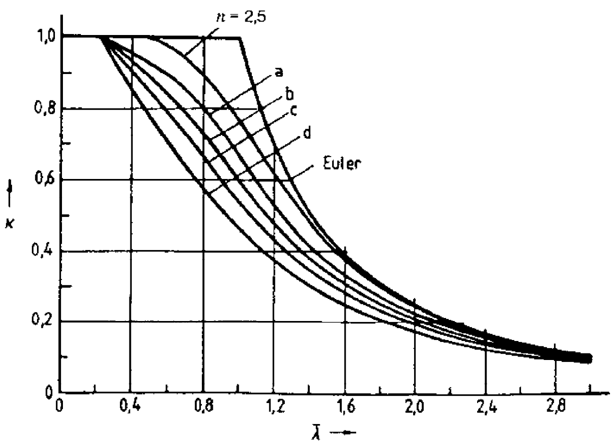

Fig. 1 Buckling curves by ECCS with imperfection coefficients a and sistem factor n=2.5

The generality of the *LRFD* criteria of the *AISC* is comprised in the elastic solution which is expressed in the form: $M_E = C_b M_r$, where M_r is the buckling solution for the case of uniform bending and C_b is the coefficient which accounts for the effect of loading. The inelastic buckling solution is approximated by a straight line (Figure 2). The ultimate moment is determined as follows:

$$M_u = M_p \quad \text{for} \quad L \le L_p$$

$$M_u = C_b \left[M_p - \left(M_p - M_r \right) \frac{\left(L - L_p \right)}{\left(L_r - L_p \right)} \right] \le M_p \qquad (1)$$

$$\text{for} \quad L_p \le L \le L_r$$

$$M_u = M_E \quad \text{for} \quad L \ge L_r \quad \text{where} \quad L_p = 1.76 r_y \sqrt{E/F_y}$$

In these equations are:

E = modulus of elasticity,
F_y = yield stress,
M_r = $S(F_y - F_r)$, yield moment,
M_p = maximum moment capacity,
S = elastic section modulus,
F_r = maximum compresive residual stress,
L_r = unbraced length corresponding to M_r,
L_p = spacing of the braces necessary to reach M_p without rotation capacity.

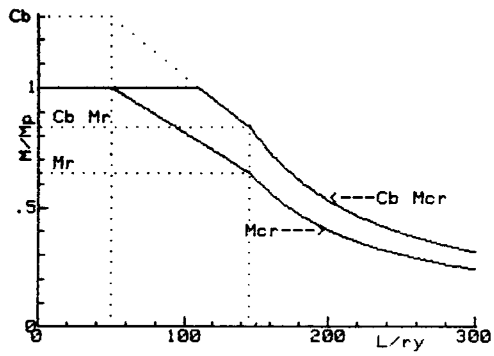

Fig. 2 Variation of M_u with slenderness parameters by AISC Specification

For beams where plastic moments are assumed to develop the distances between lateral support points will be relatively short *(L_p)*, with extreme fiber strain approaching into the strain hardening range. Studies of inelastic lateral buckling have been made by Galambos, Lay, Massey and Pittman, Bansal and Hartmann. The unbraced length corresponding to the elastic limit, L_r, can be obtained by setting $M_E = M_r$, where the corresponding buckling moment M_r, is determined including residual stresses.

As *ECCS* curve for *n=2.50* provides a reasonable mean strength over short and medium slenderness ranges for rolled beams, the curve for *n = 1.50* forms a lower bound for the test points with the *(m-2s)* strength curve shown in Figure 3. In Figure 4 the same is valid for the welded beams with different system factors *n*, which are 2.0 and 1.50 respectively. In order to determine the best fit of the assumed implicit function to the experimental data, such as selected 324 rolled beams and 132 welded beams, the mean values of nondimensional strength coefficients *(d_i)*, as well as 5% fractiles *(m-1.64s)* were evaluated by the method of the least squares in the nonlinear regression analysis:

$$min \sum_{i=1}^{N} \left[\left(\frac{1}{1+\bar{l}^{2n}} \right)^{1/n} - d_i \right]^2, \qquad (2)$$

$$\text{where} \quad d_i = \frac{M_u}{M_p} \quad \text{and} \quad \bar{l} = \sqrt{\frac{M_p}{M_E}}$$

In this problem, as nonlinearity is encountered, the higher-order equations with one independent variable should be tried to fit data with the correlation coefficient near 1.

The results for the rolled beams from the sellected tests data are *n = 2.64*, which concerns mean values, and *n = 1.425* for 5% fractiles. For the welded beams, on which the same evaluation is performed, the result for mean value is *n = 2.44* and 5% fractile *n = 1.095*.

In order to derive the probabilistic evaluation of lateral-torsional buckling strength of *ECCS* and *AISC* design formula comparing them with test results, the realised indeces of reliability are performed on the following examples.

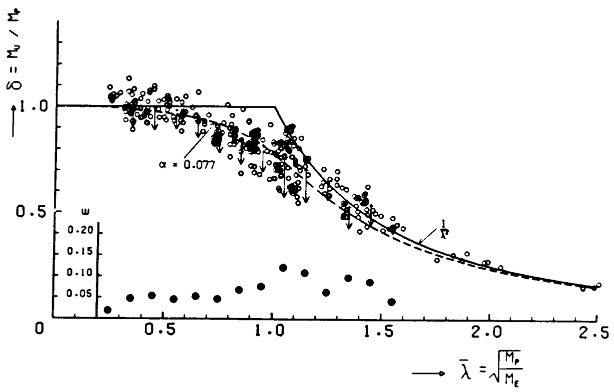

Fig. 3 Experimental results and lateral-torsional buckling curves for rolled beams

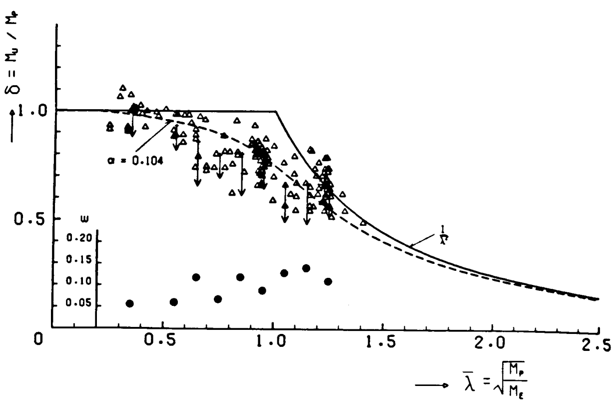

Fig. 4 Experimental results and lateral-torsional buckling curves for welded beams

3. THE EVALUATION OF THE RELIABILITY INDECES FOR THE ROLLED BEAMS UNDER SNOW LOADS

In this example the calibrations of the main roof beams, which are laterally unsupported on the distances between secondary beams, are performed at four different locations for the snow loads in the continental part of Croatia. The snow load is taken as a dominant load during 30 years of measurements of metheorological data with the characteristic values as 95% fractile with the return period of 30 years. The experimental results are selected for the needed slenderness ratios of the rolled beams designed by the theoretical models of *ECCS* and *AISC* Specifications, and the proposed fractile curve with a system factor *n = 1.425*.

3.1 The statistical evaluation of the snow loads data

The snow measurements, which are converted to the snow loads on the flat roofs, are analysed and compared with the extreme probability distribution Gumbel function type *I* as shown in Figure 5 and with the equation:

$$F_0(x) = exp[-exp(-a(x-\dot{x}))] \qquad (3)$$

where:

$\dot{x} = \bar{x} - c/a$ - mod,

$\bar{x}$ - mean value of the distribution,

$a = \dfrac{p}{s_0\sqrt{6}}$ - standard deviation of the distribution.

Function of the extreme probability distribution during the period of *n* years is:

$$F_n(x) = F_0(x)^n = exp\left[-exp\left(-a\left(x-\dot{x}-\frac{1}{a}ln\,n\right)\right)\right] \qquad (4)$$

where the mean value for the return period of *n* years is:

$$\bar{x}_n = \bar{x}_0\left(1+\frac{\sqrt{6}}{p}V_0\,ln\,n\right) \qquad (5)$$

The characteristic value for the load during the period of *n* years $x_{k,n}$, with the probability *p* that it will not be exceeded is:

$$x_{k,n} = \dot{x} + \frac{1}{a}[ln\,n - ln(-ln\,p)] \qquad (6)$$

$$x_{k,n} = \bar{x}_0\{1-0.4501V_0+0.7801V_0[ln\,n-ln(-ln\,p)]\} \qquad (7)$$

The general expression for the snow loads on the flat roofs with the reduction factor 0.8, concerning the thermal characteristics of the building and the exposure, and with the probability density Gumbel function is defined as:

$$s_{0n} = 0.8\left\{\left(\bar{x}_0 - \frac{\bar{y}_N}{s_N}s_0\right) + \frac{s_0}{s_N}[ln\,n - ln(-ln\,p)]\right\} \qquad (8)$$

where $\bar{y}_N$ and s_N are mean value and standard deviation of the calculated snow loads.

Figure 5 presents the histograms and extreme type *I* distribution function for measuring the data on snow loads in the continental parts of Croatia, which are compared with the theoretical frequencies in Figures 6 and 7.

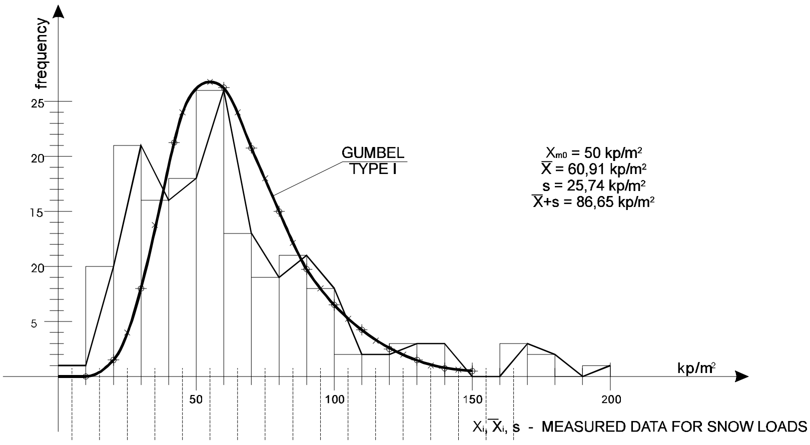

Fig. 5 Histogram and distribution function for the snow load

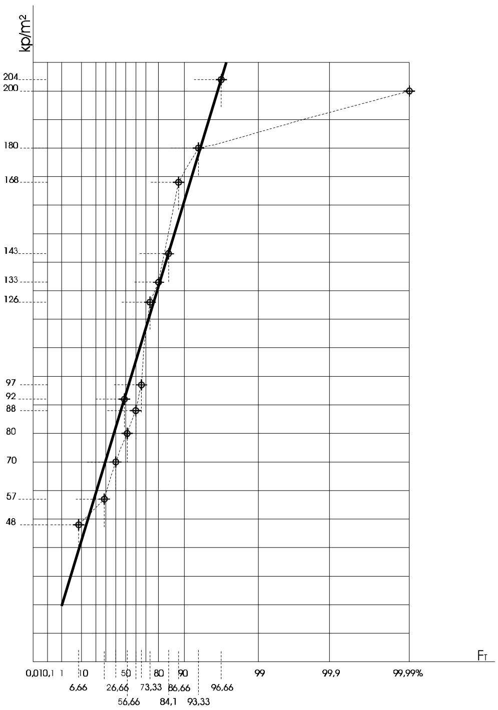

Fig. 6 Comparison of the measured data for snow in Ogulin on the probability paper with Henry-diagram

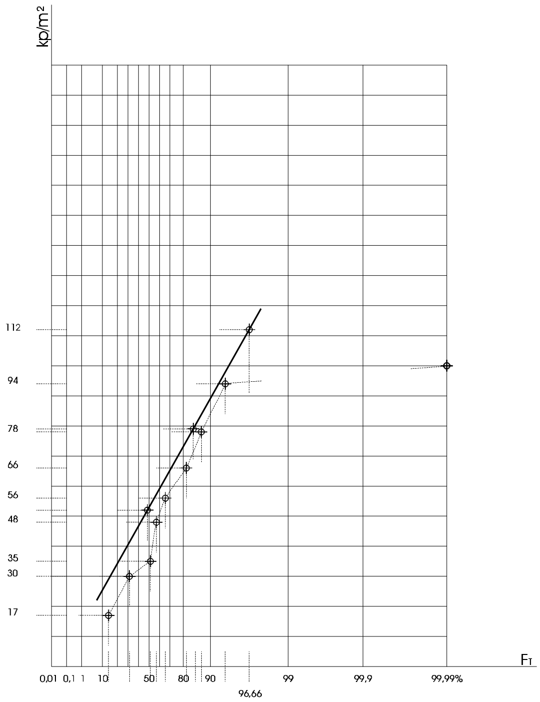

Fig. 7 Comparison of the measured data for snow in Vara`din on the probability paper with Henry-diagram

3.2 The statistical parameters for the load and the resistance variable

The examples are derived for the ultimate lateral-torsional buckling resistances of the rolled beams with sections I 200×100×8×5.5, which are designed by the above stated models, under the characteristic values of snow loads with the different length of lateral supports and distances between the beams.

The first example is for a location in Zagreb with the statistical parametar for the snow loads:

$\overline{q}_0 = 0.35\ kN/m^2\,;\ s_0 = 0.21\ kN/m^2;\ V_0 = 0.60$

$q_{p,n} = \overline{q}_0\ (1+4.52V_0) = 1.30\ kN/m^2$... characteristic values of the loads

$\overline{q}_n = \overline{q}_0\ (1+2.652V_0) = 0.907\ kN/m^2$... mean values for *n* years

The evaluated girder is from the group of the tested beams with the variable strength shown in Figure 5, designed by the values of the loads $q_{p,n}$, the distances between lateral supports l_y, and with ultimate strength M_u.

The second example is for a location in Vara`din with the statistical parameters for the snow loads:

$\overline{q}_0 = 0.51\ kN/m^2;\ s_0 = 0.28\ kN/m^2;\ V_0 = 0.55$

$q_{np} = 0.51(1+4.52V_0) = 1.78\ kN/m^2$

$\overline{q}_n = 0.51(1+2.652V_0) = 1.254\ kN/m^2;\ V_n = 0.223$

The calculated values of the ultimate strength M_u and the distances between the girders are specificed in the Tables 1 - 4.

The third example is for a location in Ogulin with the parameters of snow loads:

$\overline{q}_0 = 0.929\ kN/m^2;\ s_0 = 0.51\ kN/m^2;\ V_0 = 0.55$

$q_{np} = 3.20$ kN/m^2; $\overline{q}_n = 2.26\ kN/m^2;\ V_n = 0.225$

The fourth example is for a location in Slavonski Brod with the characteristics of snow loads:

$\overline{q}_0 = 0.490\ kN/m^2;\ s_0 = 0.244\ kN/m^2;\ V_0 = 0.490$

$q_{np} = 2.960$ kN/m^2; $\overline{q}_n = 2.120\ kN/m^2;\ V_n = 0.113$

3.3 The evaluation of the reliability indeces for the calibration of the rolled beams

The reliability index is derived from the equation of the ultimate limit state with two basic variables, which are statistically independent, $g(x) = R - Q$, with the probability of the failure:

$$p_f = P(R < Q) = \int F_R(x)\, f_Q(x)\, dx \qquad (9)$$

where is:
F_R - cumulative distribution function of resistance R,
f_Q - probability density function of load Q.

The basic variable is not distributed by the normal probability distribution function, so *FOSM* method is not applicable and *Rackwitz & Fiessler-method* is used with the transformation of the basic variable into the equivalent of the normal distribution and the parameters $\bar{x}^N$, s_x^N under the circumstances, so that the cumulative distribution and probability density functions are the same, as for the basic and aproximated variables, in the reper points on the ultimate limit state plane $g(x_1^*, x_2^*, \ldots, x_n^*) = 0$.

The equivalent mean value and standard deviation of the basic variable is:

$$\bar{x}^N = x_i^* - F^{-1}\left[F_i\left(x_i^*\right)\right] s_{x_i}^N$$
$$s_{x_i}^N = \frac{f\left\{F^{-1}\left[F_i\left(x_i^*\right)\right]\right\}}{f_i\left(x_i^*\right)} \qquad (10)$$

where is:
F, f - distribution and density function of the basic variable x_i,
F, f - cumulative distribution and the density function of the standard normal variable.

The iterative procedure for approaching the minimum value of b is obtained by the equation system:

$$a_i = \frac{\left(\frac{\P g}{\P x_i}\right) s_{x_i}^N}{\sqrt{\sum_{i=1}^{m}\left(\frac{\P g}{\P x_i}\right) s_{x_i}^{N\,2}}} \qquad (11)$$

$$x_i^* = \bar{x}_i - a_i\, b\, s_{x_i}^N$$
$$g\left(x_1^*, x_2^*, \ldots, x_n^*\right) = 0 \qquad (12)$$

The partial derivations $\P g / \P x_i$ are evaluated for x_i^*, and a_i of the basic variables x_i. After the convergence of this algorythm, the reliability index b^* is evaluated, and the approximate value of the probability of failure $p_f @ 1 - F(b^*)$.

4. THE RESULTS OF THE CALIBRATION FOR THE ROLLED BEAMS

From the statistical parametars of the buckling test results for rolled beams and the predicted model values, the evaluation of the reliability indices is performed and tabulated (Tables 1 - 4) for the stated slenderness ratios and the snow loads at four locations.

Table 1. Ultimate strength and calibration for the snow load data in Zagreb

$\bar{l} = 0.605$	Theoretical results M_{teo} [kNm] for models:			M_{exp} [kNm]
	ECCS	AISC	(m-2s) curve	
$\bar{M}_{max}$	21.63	21.22	19.22	57.45
s_{Mmax}	5.01	4.91	4.45	1.58
b	3.8	3.9	4.20	

Table 2. Ultimate strength and calibration for the snow load data in Vara`din

$\bar{l} = 0.77$	Theoretical results M_{teo} [kNm] for models:			M_{exp} [kNm]
	ECCS	AISC	(m-2s) curve	
$\bar{M}_{max}$	20.51	19.75	17.17	46.30
s_{Mmax}	4.51	4.40	3.83	2.77
b	3.20	3.50	4.20	

Table 3. Ultimate strength and calibration for the snow load data in Ogulin

$\bar{l} = 0.918$	Theoretical results M_{teo} [kNm] for models:			M_{exp} [kNm]
	ECCS	AISC	(m-2s) curve	
$\bar{M}_{max}$	18.40	18.10	15.07	47.27
s_{Mmax}	4.14	4.07	3.40	5.69
b	3.21	3.10	3.81	

Table 4. Ultimate strength and calibration for the snow load data in Slavonski Brod

$\bar{l} = 1.17$	Theoretical results M_{teo} [kNm] for models:			M_{exp} [kNm]
	ECCS	AISC	(m-2s) curve	
$\bar{M}_{max}$	14.51	14.88	11.93	40.53
s_{Mmax}	1.64	1.69	1.35	3.40
b	5.18	4.80	6.20	

5. CONCLUSION

The analysis of laterally unsupported steel beams for various design models is obtained by which the ultimate limit state of the lateral torsional buckling strength is evaluated for the purpose of the calibration of the rolled beams under the snow loads from the measured data in Croatia. The results of the calibration vary with the applied snow loads and the slenderness ratio for three examined designed models, while for *ECCS* criteria they are in the range from the realised reliability indices $b = 3.20$ to 5.19, for *AISC* Specifications indices are lower, such as $b = 3.10$ to 4.80, and for the model of the proposed system factor n is quite on the target safety side with $b = 4.20$ and 6.20. As the calibration is performed with the designed model by the global and constant safety factor, the differences are the result of the basic formulations of the buckling curves. It is evident that there it is not necessary to change the system factor n of the buckling curves but to correct the evaluation model by the model and resistance factors with the target reliability level in order to achieve uniform reliability with the proposed loads factors concerning the applied loads in certain cases.

6. REFERENCES

[1] American Institute of Steel Construction, *Load and Resistance Factor Design*, AISC, 1986.

[2] Eurocode 3. Design of Steel Structures-Part 1-1: General rules and rules for buildings, 1987.

[3] R.E. Melchers, *Structural Reliability - Analysis and Prediction*, John Wiley & Sons, 1987.

[4] M. Sulyok-Selimbegovi} and T.V. Galambos, The evaluation of the resistance factors for the laterally unsupported rolled and welded beams and plate girders designed by LRFD of the AISC, *Steel Structures: Advances, Design and Construction*, Elsevier Applied Science, London and New York, 1987.

[5] T.V. Galambos, *Guide to Stability Design Criteria for Metal Structures*, 4th edition, John Wiley&Sons, 1988.

[6] J. Lindner, Design aplications for beams restrained by adjecent members, In: P.J. Doweling, J.E.Harding, R. Bjorhovde and E. Martinez-Romero, *Constructional Steel Design*, Elsevier Applied Sc., London and New York, 1992.

BO^NO NEPODUPRTE ^ELI^NE GREDE PROJEKTIRANE PREMA ECCS I AISC PROPISIMA POD OPTERE] ENJEM SNIJEGOM U HRVATSKOJ

SA@ETAK

U radu se analiziraju zahtjevi u projektiranju greda za grani~no stanje bo~nog torzijskog izvijanja te se uspore|uju kriteriji za dva propisa (AISC i ECCS). Budu}i da postoje znatne razlike u projektiranju uspore|uju se dobiveni indeksi pouzdanosti kako bi se pronašao model koji je bli`i zahtjevanoj razini pouzdanosti. Stoga se analiziraju ~etiri primjera koja se odnose na razli~ite podatke promatranja optere}enja snijegom na ~etiri razli~ita mjesta u Hrvatskoj. Izvode se karakteristi~ne vrijednosti stvarnih optere}enja snijegom na krovnim konstrukcijama i projektiraju se grede od valjanog ~elika, bo~no nepoduprte, razli~itih du`ina do kojih dolazi zbog zahtjeva kapaciteta krajnjih momenata za testiranje modela. U radu se statisti~ki procjenjuju parametri eksperimentalnih rezultata, dok se za postupak kalibracije koristi FOSM metoda. Budu}i da rezultati variraju ovisno o primijenjenom optere}enju snijegom i o omjerima vitkosti kod dva analizirana modela projektiranih valjanih greda koje se uspore|uju, potrebno je posti}i zahtjevane vrijednosti pouzdanosti, korigiraju}i model i faktore otpora.

***Klju~ne rije~i**: pouzdanost, grede od valjanog ~elika, zavarene grede, propisi, FOSM teorija, optere}enje snijegom, funkcija gusto}e razdiobe.*