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The use of short crack growth model for service-life estimation of pitting-prone surfaces

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SUMMARY

A computational model for determining the service-life of contacting surfaces in regard to surface pitting of mechanical elements is presented. In view of small crack lengths observed in surface pitting, the simulation takes into account the short fatigue crack growth theory. The model considers the material fatigue process leading to pitting, i.e. the conditions required for the development of pits by simulation of the short fatigue crack propagation originating from the initial crack in a single material grain. Due to general lack of required micromechanical fracture parameters, different assumptions are proposed how to bridge the related shortcomings of the used model. The stress field in the contact area and the required functional relationship between the stress intensity factor and the crack length are determined by the finite element method. An equivalent model of two contacting cylinders is used for numerical simulations of crack propagation in the contact area. The probable service life period of contacting surfaces is estimated for surface curvatures and loadings that are most commonly encountered in engineering practice on the basis of numerical results considering some particular material parameters.

Key words*: Short crack growth model, service-life estimation, pitting-prone surfaces, computational model, fatigue, finite element method, crack propagation, material parameters.*

1. INTRODUCTION

Mechanical elements subjected to rolling and sliding contact conditions fail by several mechanisms and the most prominent among these is surface pitting [1, 2]. Pitting is a fatigue phenomenon and is characterised by a gradual deterioration of contacting surfaces, see Figure 1.

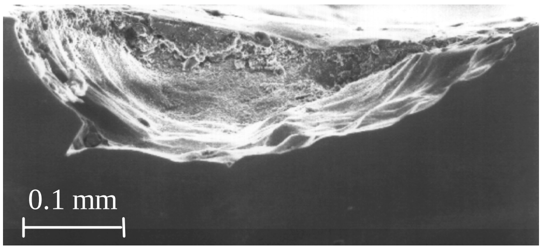

Fig. 1 Typical shape of a pit on the contacting surface

Failure occurs either when the surface has deteriorated so much that the component no longer functions as designed, or when the damage becomes severe enough to lead to failure by another mechanism, such as breakage. The process of surface pitting can be visualised as the formation of small, surface-breaking or subsurface initial cracks which grow under repeated contact loading. Eventually, the crack becomes large enough for unstable growth to occur, which causes the material surface layer to break away. The resulting void is a pit.

The service life of contacting mechanical elements consists of the crack initiation phase and the crack propagation period required for pit formation. The number of the required stress cycles N for the pitting occurrence on contacting surfaces can then be determined as:

$$N = N_o + N_f \tag{1}$$

where N_o is the number of stress cycles required for the initial crack appearance in the material and N_f is the number of stress cycles required for a crack to propagate from the initial to the critical crack length. In polycrystalline metals the crack initiation phase is usually very short if compared to the fatigue life of mechanical elements [3]. There are also many possible stress concentration positions on or under the contact surfaces, grain boundaries, triple points, machining marks, inclusions and large notches, which can immediately initiate a crack either separately or in a variety of combinations. In such applications the service life can be approximately determined as a number of stress cycles N_f required for a crack to propagate from some initial to the critical crack length, which is the approach used in this paper.

The computational model for pitting simulation reported in this paper is restricted to modelling of high precision mechanical components with fine surface finish and good lubrication, i.e. small coefficient of friction. In this case the fatigue crack is usually initiated in the area of the largest contact stresses that appear at a certain depth under the contacting surface [4, 5]. The mechanism of subsurface crack initiation and consequent fatigue growth is here attributed to large shearing stresses at the onset of cracking and mixed mode fracture in the later stages of crack propagation when the crack approaches the surface. The proposed model comprises a two-dimensional fatigue crack growth model based on fracture mechanics where the required materials properties are obtained from common fatigue tests.

2. SHORT FATIGUE CRACK GROWTH UNDER CYCLIC CONTACT LOADING

By considering small crack lengths associated with pitting it is necessary to utilise the short crack growth theory for a proper description of the fatigue crack propagation. The short crack growth is characterised by successive blocking of dislocation motion along persistent slip bands with grain boundaries, which implies the discontinuous character of the crack growth process [8, 9, 10]. The crack growth rate da/dN is assumed to be proportional to the crack tip plastic displacement $\Delta\delta_{pl}$:

$$\frac{da}{dN} = C_o\left(\Delta\delta_{pl}\right)^{m_o} \tag{2}$$

where C_o and m_o are material constants that are determined experimentally. In view of the numerical simulation it is beneficial to express the crack tip plastic displacement $\Delta\delta_{pl}$ in terms of the stress intensity factor K. This relationship has been provided in the form [8, 10]:

$$\Delta\delta_{pl} = \frac{2\kappa}{G\sqrt{\pi}} \cdot \frac{\sqrt{1-n^2}}{n} K\sqrt{a} \tag{3}$$

where G is the shear modulus, and $\kappa=1$ or $1\text{-}\nu$ depending on whether screw or edge dislocations are being considered, with ν being the Poisson's ratio. Parameter n describes the relative position of the crack tip to the grain boundary and is defined as:

$$n = \frac{a}{c} \tag{4}$$

where a is the crack length and c is the crack length together with a dislocation slip band ahead of the crack, which always extends to the grain boundary (see Figure 2). The number of stress cycles required for a crack to propagate through each grain is obtained with the integration of Eq. (2):

$$N_j = \int_{a_{j-1}}^{a_j} \frac{da}{C_o(\Delta\delta_{pl})^{m_o}} \qquad j = 1,2,3,...,z \tag{5}$$

in which z is the number of grains transversed by the crack $(z = a/D)$. The total number of stress cycles N required for a crack to propagate from the initial crack length a_o to any crack length a can then be determined as:

$$N = \sum_{j=1}^{z} N_j \tag{6}$$

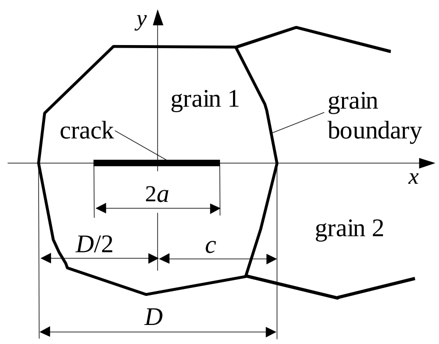

Fig. 2 Initial crack in the first crystal grain

Integration limits a_{j-1} and a_j in Eq. (5) are in each grain determined in relation to the critical value of parameter n, see Figure 3. When n reaches the critical value $n=n_c$, enough strain energy due to dislocation motion is accumulated ahead of the crack to initiate its extension to the next grain. There is no general law to determine n_c precisely and several different expressions may apply. The one most often used by the authors has been provided by Navarro and Rios [8] in the following derived form:

$$n_c = cos\left[\frac{\Delta K}{2\sigma_y} \cdot \sqrt{\frac{\pi}{c \cdot n_c}} \cdot \left(1 - \frac{K_{th}}{\Delta K}\sqrt{n_c}\right)\right] \tag{7}$$

in which σ_y is the yield stress and K_{th} is the threshold stress intensity factor. This equation is solved iteratively by taking $n_c=1$ as the initial seed. Unfortunately, the equation stability is heavily dependent on the used parameters and it is often the case that its roots are impossible to determine. This problem can be overcome, if two limiting cases for possible extensions of the slip band (plastic displacement) ahead of the crack tip are considered. The first case is determined by considering the slip band extending only to the grain boundary containing the crack tip - c_I, and the other when the band extends also through the following grain - c_{II}. For grain j this can be written as:

$$c_I = \frac{i \cdot D}{2} \qquad i = 1,3,5...(2j-1) \tag{8a}$$

$$c_{II} = \frac{(i+2) \cdot D}{2} \tag{8b}$$

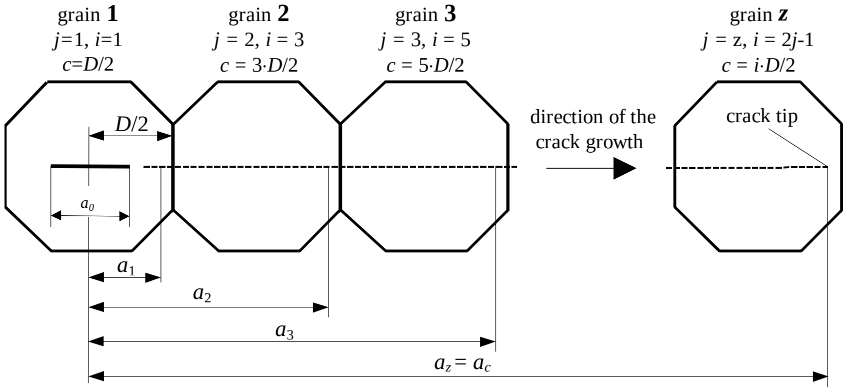

Fig. 3 Schematic representation of the short crack growth

By adopting this assumption it is possible to determine the corresponding integration limits a_{j-1} and a_j for each grain by taking $n=0.99$, which correlates to practical simulation of crack growth between grain boundaries. However, both possible extensions of the plastic zone ahead of the crack are considered when evaluating Eq. (5). By considering these assumptions one obtains the upper band solution by taking $c=c_I$ in Eq. (4), while on the other $c=c_{II}$ returns the lower band solution of the number of loading cycles required for a crack to propagate through the grain. The real number of loading cycles always lays somewhere in between.

3. COMPUTATIONAL DETERMINATION OF THE STRESS INTENSITY FACTOR

To find the solution to Eq. (3) and consequently Eqs. (5) and (6) it is necessary to determine the relationship between the stress intensity factor and the crack length $K=f(a)$. The Virtual Crack Extension method (VCE) in the framework of the finite element analysis was used for this purpose. The VCE method is based on the criteria of released elastic energy, which serves as a basis for the determination of the stress intensity factor [11, 12]. Using the VCE method the stress intensity factor is determined in several different possible crack extension directions and the crack is actually extended in the direction of the maximum stress intensity factor. Crack extensions are incremental, where the size of the crack increment is prescribed in advance. The incremental procedure is stopped when the stress intensity factor reaches the critical value K_{Ic}, i.e. when uncontrolled crack growth occurs and full fracture is expected. Thus one can numerically determine the functional relationship $K = f(a)$ and the critical crack length a_c.

For computational simulations of surface pitting it is advantageous to use an equivalent model of two contacting cylinders instead of simulating the actual contact of mechanical elements [12]. The stress field in the contact area can then be much more easily determined by using the standard Hertzian contact theory, along with general-purpose finite element codes. This avoids the need to use special algorithms to determine the actual contact state which are not generally available. Various rolling and sliding contact conditions between contacting mechanical elements can then be simply simulated by analysing the equivalent cylinder of radius R^* and Young's modulus E^* with applied Hertzian contact conditions along the contact area. Interested readers should refer to reference [12] for more details.

This paper deals only with the contact between mechanical elements with smooth surfaces and good lubrication. For such cases the friction coefficient is in average equal to $\mu=0.04$ [7], which is also the value used to prescribe the tangential frictional contact forces in all computations reported in this paper.

The equivalent contact model is first used to determine the position of the maximum contact stress, where it is assumed that the fatigue crack will be most likely initiated. The maximum equivalent stress $\sigma_{E_{max}}$ and its position H under the surface are estimated for different combinations of the maximum contact pressure p_o and equivalent curvature radius R^* by analysing the equivalent finite element model. In all computations Young's modulus was equal to $E_1=E_2=2.06\times10^5$ *MPa* and Poisson's ratio to $\nu=0.3$. The results of such numerical computations performed with parameters that are typical for gears and bearings are summarised in [12]. The computations show that equivalent stress $\sigma_{E_{max}}$ and its depth H under the contact surface clearly depend on the contact geometry and loading. They both increase with the increase of p_o and R^*.

The initial crack of the size equal to the grain diameter $a_o=D=0.05$ mm is then positioned at the point of the maximum equivalent stress $\sigma_{E_{max}}$, i.e. at depth H under the contact surface, depending on geometry and contact loading. The VCE computational procedure is then applied to simulate the fatigue crack growth for a particular data set. The results of these computations and corresponding relationships $K=f(a)$ are given in detail in reference [12].

Numerical analyses have shown, that upon initial crack appearance, the stress intensity factor is much higher at the crack tip, which coincides with the direction of the applied friction force. Therefore, it is assumed that the crack will start to propagate in the direction of acting frictional forces, which is incrementally computed with the VCE method.

Analyses have shown that when the crack breaks through to the surface the stress intensity factor at the other crack tip exceeds K_{Ic}, which implies that the corresponding crack length can be taken as the critical crack length a_c. Critical crack lengths for different combinations of p_o and R^* are summarised in Table 1.

Table 1. Critical crack lengths a_c for different combinations of p_o and R^*

Maximum contact pressure p_o [MPa]	Equivalent curvature radius R^* [mm]				
	6	8	10	14	20
1000	0.18	0.27	0.32	0.42	0.57
1200	0.22	0.27	0.37	0.52	0.72
1400	0.27	0.32	0.42	0.57	0.77
1550	0.32	0.37	0.47	0.62	0.87
1700	0.32	0.37	0.52	0.67	0.97

4. ESTIMATION OF THE SERVICE LIFE

The service life of contacting surfaces in regard to pitting is in this study estimated for mechanical elements made of flame hardened steel AISI 4130 with a yield stress $\sigma_y=900$ *MPa* and the threshold stress intensity factor $K_{th}=269\ MPa\sqrt{mm}$. For this material the average grain diameter was measured to be equal to $D=0.05$ *mm* and the micromechanical material constants in Eq. (2) have been previously determined as $C_o=120.57$ and $m_o=3.069$ [10].

By determining the functional relationship $K=f(a)$ and the critical crack length a_c for each combination of p_o and R^* in the previous Section, it is now possible to estimate the number of loading cycles required for a crack to propagate from the initial crack length of $a_o=D=0.05$ mm to the critical crack length a_c, which in turn defines the contacting surface service life. This can be done by following the procedure outlined in Section 2.

The number of grains transversed by the crack is first estimated to be equal $z=a_c/D$. Then the crack lengths together with the plastic zone ahead of the crack c_I and c_{II} are determined for each grain $j=1,2,...,z$ according to Eqs. (8a) and (8b). Corresponding integration limits of Eq. (5) are for each grain determined by considering that $a_j=0.99\cdot c_I$. Integration of Eq. (5) is for each grain performed twice, first by considering that $n=a/c_I$ and secondly that $n=a/c_{II}$. Thus, the upper bound and lower bound solution can be obtained, see Section 3. The total service life can be obtained by summing the respective number of loading cycles for all grains transversed by the crack, see Eq. (6). The results of such service life estimation are given in Table 2 for the combination $p_o=1400$ *MPa* and $R^*=8$ *mm*. The propagation rate of the fatigue crack growth can be estimated for any chosen crack length according to Eq. (7). The results of such computations are combined in Figure 4, where the successive blocking of crack growth by grain boundaries is clearly illustrated.

Table 2. Fatigue crack growth data for $p_o=1400$ MPa, $R^*=8$ mm

Grain j		1	2	3	4	5	6
Length of damage zone [mm]	c^*	0,075	0,125	0,175	0,225	0,275	0,325
	c^{**}	0,125	0,175	0,225	0,275	0,325	0,375
Integration limits [mm]	a_{j-1}	$a_o/2=0,025$	$a_1=0,0743$	$a_2=0,1238$	$a_3=0,1733$	$a_4=0,2228$	$a_5=0,2723$
	a_j	$a_1=0,0743$	$a_2=0,1238$	$a_3=0,1733$	$a_4=0,2228$	$a_5=0,2723$	$a_6=a_c=0,32$
Number of loading cycles	N_j^*	4838330	1253250	591812	247849	146685	65931
	N_j^{**}	139354	67003	34457	18626	11375	6852
$N_f=N_1+N_2+N_3+N_4+N_5+N_6$ => $N_f^*=7{,}144\cdot10^6$ and $N_f^{**}=2{,}777\cdot10^5$							

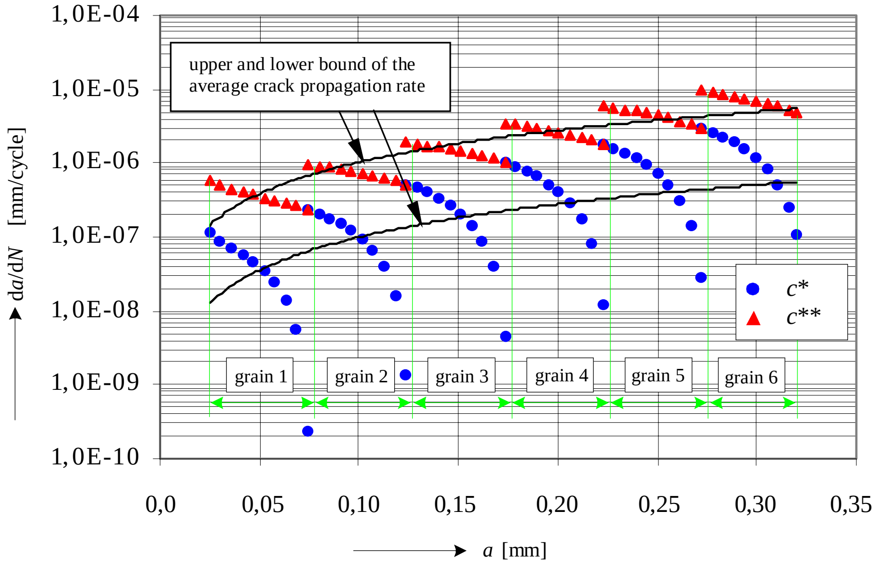

Fig. 4 Diagram of crack growth propagation rate for p_o = 1400 N/mm^2, R^*=8 mm

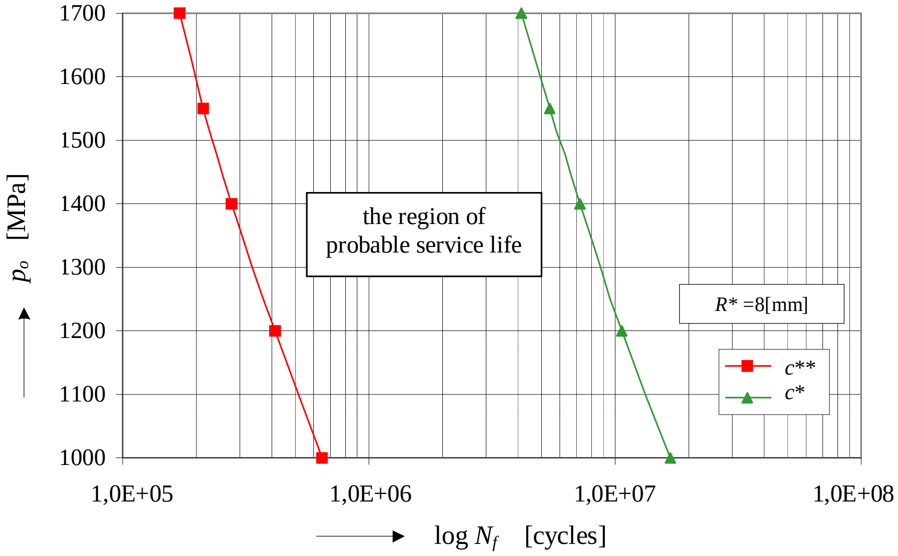

Fig. 5 Diagram of probable service life for equivalent curvature radius R=8 mm*

Performing such computational operations for each combination of p_o and R^* it is possible to combine the results in diagrams, where the region of probable service life for each equivalent curvature radius in regard to different maximum contact pressure can be clearly illustrated. Figure 5 shows the combined results for equivalent curvature radius $R^*=8$ *mm*. Such diagrams can be devised for each curvature radius or even for each maximum loading pressure and can serve the designer as a quick estimate of a probable service life of contacting mechanical elements with regard to pitting.

5. CONCLUSIONS

This paper presents a new model for determining the service life of contacting mechanical elements with regard to pitting. The model simulates the fatigue process of contacting surfaces, i.e. it includes the simulation of the short fatigue crack propagation from some initial crack length to the critical crack length. The presented numerical model appropriately simulates the influence of the microstructure on the fatigue crack growth and is easy enough to be used by designers in industry. Numerical computations have shown that the service life with regard to pitting is reduced by increasing the maximum contact pressure and by reducing the curvature radius of the contact surface.

The advantage of the developed model is that it combines the micromechanics of the fatigue crack growth with common material parameters. The results can also be presented in a clear and concise manner in diagrams. The model can be applied to any engineering problem where the pitting of contacting surfaces may at some stage influence the operation of a given device and where accurate service life estimation is required.

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PRIMJENA MODELA RASTA MALIH PUKOTINA ZA PROCJENU VIJEKA TRAJANJA O[TE]ENIH POVR[INA

SA@ETAK

Predstavljen je novi numeri~ki model za odre|ivanje `ivotne dobi kontaktnih površina strojnih elemenata, u zavisnosti od opasnosti o{te}enja povr{ina. O{te}enje kontaktnih površina povezano je s pukotinama vrlo kratkih du`ina i zbog toga se razvijeni model temelji na teoriji rasta kratkih pukotina uslijed zamora. Model u cjelini obuhva}a proces zamora materijala koji vodi do o{te}enja te potrebne uvjete za nastanak po~etne zamorne pukotine u jednom kristalnom zrnu i proces njenog širenja do nastanka površinskog o{te}enja. Zbog nepoznavanja potrebnih mikromehani~kih lomnih parametara, u ~lanku je prikazan na~in prevladavanja povezanih nedostataka upotrijebljenog modela. Naponsko polje u kontaktnom predjelu i potrebna funkcijska zavisnost faktora intenziteta napetosti od du`ine pukotine, odre|ene su prora~unima po metodi kona~nih elemenata. Za numeri~ke simulacije širenja pukotine u kontaktnom predjelu upotrijebljen je ekvivalentni model dvaju kontaktnih cilindara. Vjerojatna `ivotna dob kontaktnih površina ustanovljena je na osnovu numeri~kih rezultata, upotrebom odre|enih parametara materijala za razli~ite zakrivljenosti kontaktnih površina i razli~ita optere}enja, koja se mogu naj~eš}e na}i u in`injerskoj praksi.

***Klju~ne rije~i**: model rasta malih pukotina, procjena trajanja, o{te}enja povr{ina, prora~unski model, zamor, metoda kona~nih elemenata, razvoj pukotina, parametri materijala.*