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Diagnosis of steady-state characteristics in laminar flow of fluids

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Professional paper

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Abstract

Laminar flow of fluids is one of the most common forms of motion in oilfield practice. In such a flow regime of fluid, the determination of velocity-flow rate performance which takes into account the rheological properties of the fluid is of great importance for the development of hydraulic criteria. On the other hand, from the moment of the beginning of fluid motion in the pipe, a certain time is required to ensure the steady flow of fluid, i.e. independence of its parameters on time. The issues of diagnosing steady-state characteristics in laminar flow of both Newtonian and non-Newtonian fluids are of particular relevance. In this paper, the velocity distribution along the cross-section of a pipe in laminar flow of Newtonian and non-Newtonian fluids is studied while taking into consideration rheological factors, and the change of flow rate is investigated. Determination of the time of transition to the steady-state flow regime and parameters affecting the variation of this time are shown.

Keywords:

viscoplastic fluid; velocity distribution; flow rate; steady flow

1. Introduction

Several issues of managing and improving the efficiency of technological processes in oil and gas production are closely related to the fact that homogeneous and heterogeneous fluids with different rheological and physico-chemical properties have different forms of movement in pipes and hydraulic properties during production, storage and transportation. Pipe calculation in terms of the correct prediction of flow parameters, in turn, requires diagnostics of their steady-state characteristics depending on the rheological properties of fluids (Landau and Lifshits, 1986; Loytsinsky, 1987; Vasiliev et al., 2002; Basniev et al., 2005; Saheed et al., 2012).

Considering the fact that transported hydrocarbon fluids are multicomponent and multiphase, their hydraulic properties are much more complex compared to homogeneous systems (Vasques and Beggs, 1980; Bayron et al., 2002; Robert et al., 2003; Ismayilov et al., 2019; Ismayilova et al., 2023). Energy costs increase during the transportation of such systems, and the presence of various structural forms of motion significantly complicates their hydrodynamic calculations (Sitenkov, 2004). In many cases, it is impossible to diagnose the steady-state modes of motion of these systems. Inevitably, it becomes possible to solve problems based on the establishment of relatively simple mathematical models of these systems and the determination of quasi-steady modes of operation. For the rheological description of these systems, a large number of different rheological models in oil and gas production practice exist and are used (Shuichiro et al. 2015; Tarek and Meftah, 2018; Housiadas, 2020; Wu and Li, 2023).

This paper focuses on the issues of diagnosis of steady-state characteristics for horizontal flow of Newtonian and non-Newtonian (viscoplastic) fluids.

2. Statement and solution of the problem for Newtonian fluids

Suppose that a viscous fluid moves under the action of pressure difference ΔP in a pipe of length l and radius R. It is known that the fluid will initially move unsteadily, and after some time, a steady-state flow will have occurred. Taking into account that the flow is horizontal and the pressure loss is spent on friction and inertia forces in the pipe, we can write following equation (Oliemans and Ooms, 1986; Lurie, 2002; Zehra et al, 2019; Vig and Manikantan, 2023):

$$\Delta P = \frac{8\mu l Q}{\pi R^4} + \frac{m \cdot d\vartheta / dt}{\pi R^2} \quad (1)$$

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Where:

ΔP – pressure loss (Pa),

l – length of the pipe (m),

R – radius of the pipe (m),

Q – flow rate of crude oil flowing through the pipe (m^3/s),

μ – dynamic viscosity of crude oil (Pa s),

m – mass of oil (kg), ($m = \rho \pi R^2 l$),

υ – flow velocity of oil (m/s).

As **Equation 1** shows, the first summand in the right-hand side of the equation is the pressure loss determined by Poiseuille's formula (pressure loss due to friction), and the second summand is the pressure loss to overcome the forces of inertia.

Given that $d\upsilon/dt = (dQ/dt)/(\pi R^2)$, then **Equation 2** can be written as follows:

$$\Delta P = \frac{8\mu l}{\pi R^4} Q + \frac{\rho l}{\pi R^2} \frac{dQ}{dt} \tag{2}$$

If some substitutions are made, then **Equation 3** can be written as follows:

$$\frac{dQ}{dt} + AQ = B \tag{3}$$

Here, $A = 8\mu/\rho R^2$, $B = \Delta P \cdot \pi R^2/\rho l$.

If the differential **Equation 3** is separated into its variables, then **Equation 4** is obtained:

$$\frac{dQ}{1 - \frac{A}{B}Q} = Bt \tag{4}$$

If we integrate **Equation 4**, we obtain the following solution (**Equation 5**):

$$\ln\left(1 - \frac{A}{B}Q\right) + C = -A \cdot t \tag{5}$$

Here, C - is an integer constant. If $t = 0$, then $Q = 0$, $C = 0$.

Then we can write the following expression to determine the flow rate in the pipeline (**Equation 6**):

$$Q = \frac{B}{A}\left(1 - e^{-At}\right) \tag{6}$$

If we consider the expressions A and B in **Equation 6**, we obtain **Equation 7**:

$$Q = \frac{\Delta P \cdot \pi R^4}{8\mu l}\left(1 - e^{-\frac{8\nu}{R^2}t}\right) \tag{7}$$

As **Equation 7** shows, $(\Delta P\ \pi R^4)/(8\mu l) = Q_{st}$ is an expression of steady flow rate established by Poiseuille's formula. Then for the ratio of flow rates we obtain **Equation 8**:

$$\frac{Q}{Q_{st}} = \left(1 - e^{-\frac{8\nu}{R^2}t}\right) \tag{8}$$

According to **Equation 8** which expresses the change of flow rates, it is possible to diagnose the mode flow and determine the time of transition of a viscous fluid to the steady flow, as in **Figure 1**.

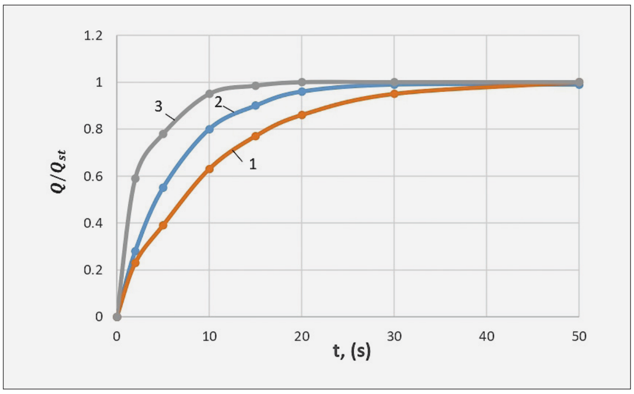

Figure 1: Determination the time of transition to the steady flow for Newtonian fluid (1÷3 – respectively, $(8\nu)/R^2$ = 0.1; 0.16; 3 1/s)

3. Statement and solution of the problem for non-Newtonian (viscoplastic) fluids

Now let us consider the laminar flow of a non-Newtonian, for example, viscoplastic fluid. The variation of velocity in the considered section (1-2) is shown in **Figure 2**.

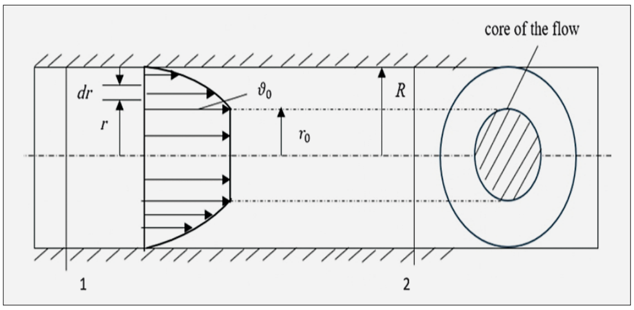

Figure 2: Velocity distribution in viscoplastic fluid flow

As **Figure 2** shows, the velocity is $\upsilon = 0$ at the inner wall of the pipe, where $r = R$.

Since the gravity force is not taken into account, the tangential stress arising in the pipe cross-section can be determined from the equality of friction and pressure forces acting on the fluid due to the equilibrium condition (**Equation 9**):

$$\Delta P \cdot \pi r^2 = \tau \cdot 2\pi r l \tag{9}$$

Consequently, the distribution of tangential stresses across the section will follow the following law (**Equation 10**):

$$\tau = \frac{\Delta P \cdot r}{2l} \tag{10}$$

As **Equation 10** shows, when $r = 0$, tangential stress gets the value of $\tau = 0$, and when $r = R$ it gets the value of $\tau = \tau_{max} = (\Delta Pr)/2l$.

In the section of $0 < r < r_0$ the flow rate remains constant, and since $\tau \leq \tau_0$, the cylindrical part of the fluid moves as a solid and is considered to be the core of the flow. Considering the abovementioned expression for the tangential stress and assuming $\tau = \tau_0$, the radius of the core (r_0) can be found as following (**Equation 11**):

$$r_0 = \frac{2l \cdot \tau_0}{\Delta P} \tag{11}$$

Taking into account the last statement when $r = R$ then $\tau = \tau_0$, i.e. the value of pressure drop ΔP_0 (initial pressure drop) corresponding to the steady state of viscoplastic fluid, is found from the following (**Equation 12**):

$$\Delta P_0 = \frac{2l \cdot \tau_0}{R} \tag{12}$$

Thus, for fluid motion in a horizontal cylindrical pipe, it is important to fulfil the condition of $\Delta P > \Delta P_0$.

It is known that the rheological model of a viscoplastic fluid is written as follows according to the Shvedov-Bingham equation expressing the state of motion $\tau > \tau_0$ (**Equation 13**):

$$\tau = \tau_0 - \mu \frac{d\vartheta}{dr} \tag{13}$$

For the case under consideration, the equilibrium condition of tangential stresses and pressure forces can be written as follows (**Equation 14**):

$$2\pi r l \tau - 2\pi r_0 l \tau_0 = \pi(r^2 - r_0^2) \cdot \Delta P \tag{14}$$

If we consider the value of τ_0 according to **Equation 11** in **Equation 14**, we obtain:

$$\tau = \frac{\Delta P}{2l} \cdot r \tag{15}$$

Thus, as can be seen, the law of distribution of tangential stresses does not depend on the rheological properties of the fluid and is expressed by the same equation. If we consider **Equation 11** in **Equation 15** and divide it by variables, we obtain the following (**Equation 16**):

$$d\vartheta = -\frac{\Delta P}{2\mu l} r dr + \frac{\tau_0}{\mu} dr \tag{16}$$

By integrating **Equation 16**, we obtain **Equation 17** for the velocity distribution:

$$\vartheta = -\frac{\Delta P}{4\mu l} r^2 + \frac{\tau_0}{\mu} r + C \tag{17}$$

The integer constant (C) can be found from the following boundary condition – $r = R$, $\vartheta = 0$, then $C = (\Delta PR^2)/(4\mu l) - (\tau_0 R)/\mu$

If we take into account the value of C in **Equation 17**, we obtain the last expression characterising the velocity distribution over the cross-section (**Equation 18**):

$$\vartheta = \frac{\Delta P}{4\mu l}\left(R^2 - r^2\right) - \frac{\tau_0}{\mu}(R - r) \tag{18}$$

The velocity distribution in the cross-section is shown in **Figure 2**. **Equation 18** is valid in the interval $r_0 < r < R$ of the current radius.

For the laminar flow of fluid, the indicated velocity distribution can also be obtained by describing its variation in the form of the following trinomial (**Equation 19**):

$$\vartheta = Ar^2 + Br + C \tag{19}$$

Lets use the following conditions to find the coefficients A, B, C included in **Equation 19**: when $r = R$, that is, on the pipe wall $\upsilon = 0$, if $r = r_0$, then $d\upsilon/dr = 0$, which gives us **Equation 20**.

$$\tau_{\max} = -\mu \frac{d\vartheta}{dr}\int_{r=R} + \tau_0;\ \tau_{\max} = \frac{\Delta P \cdot R}{2l};$$

$$\tau_0 = \frac{\Delta P \cdot r_0}{2l} \tag{20}$$

Then we obtain the following expressions (the system of equations) for finding the coefficients A, B, C (**Equation 21**):

$$\left.\begin{aligned} Ar^2 + Br + C &= 0 \\ 2Ar_0 + B &= 0 \\ -\mu\left(2AR + B\right) &= \frac{\Delta P}{2l}(R - r_0) \end{aligned}\right\} \tag{21}$$

Solving the system of **Equation 15**, we obtain the following expressions for the coefficients A, B, C (**Equation 22**):

$$\left.\begin{aligned} A &= -\frac{\Delta P}{4\mu \cdot l} \\ B &= \frac{\Delta P \cdot r_0}{2\mu \cdot l} \\ C &= \frac{\Delta P \cdot R^2}{4\mu l} - \frac{\Delta P r_0}{2\mu l} \end{aligned}\right\} \tag{22}$$

If we consider **Equation 22** in **Equation 19**, we get the following **Equation 23**, which shows the velocity distribution along the pipe cross-section and corresponds to **Equation 18**:

$$\vartheta = \frac{\Delta P}{4\mu l}\left(R^2 - r^2\right) - \frac{\tau_0}{\mu}(R - r) \tag{23}$$

In laminar flows, the flow characteristics can also be determined from the velocity distribution. It is clear that the fluid flow rate in such flows should be defined as the total flow rate of fluids – $Q = Q_0 + Q_1$. Here, Q_0-flow rate of fluid flowing in the core of the pipe and, Q_1-flow rate of fluid flowing in the cross-section of $r_0 \le r \le R$. Then the following expressions can be written for the flow rates Q_0 and Q_1, respectively (**Equation 24**):

$$Q_0 = \vartheta_0 \cdot \pi r_0^2$$

$$Q_1 = \int_{r_0}^{R} \vartheta 2\pi r dr \tag{24}$$

If we consider the values of υ_0 and υ, integrate them in the last expressions and consider expressions $r_0 = (2l\tau_0)/\Delta P$; $\tau_0 = (\Delta PR)/2l$, we obtain the following **Equation 25** for the steady-state flow rate of a non-Newtonian fluid in a pipe:

$$Q = \frac{\pi R^4 \cdot \Delta P}{8\mu l}(1 - \frac{4}{3}\frac{\Delta P_0}{\Delta P} + \frac{1}{3}\frac{\Delta P_0^4}{\Delta P^4}) \tag{25}$$

if we take into account $\Delta P_0/\Delta P = r_0/R$ in **Equation 25**, we get **Equation 26**:

$$Q = \frac{\pi R^4 \cdot \Delta P}{8\mu l}(1 - \frac{4}{3}\frac{r_0}{R} + \frac{1}{3}\frac{r_0^4}{R^4}) \tag{26}$$

Considering that $r_0^4/(3R^4)$ is ~ 0, we obtain **Equation 27** for determining the flow rate of laminar flow of viscoplastic fluid.

$$Q = \frac{\pi R^4 \cdot \Delta P}{8\mu l}(1 - \frac{4}{3}\frac{r_0}{R}) \tag{27}$$

As can be seen from **Equation 27**, the difference between the flow rate of viscoplastic laminar flows and flow rate of Newtonian fluids determined by the Poiseuille formula can be characterised by the following flow relation (**Equation 28**):

$$\frac{Q}{Q_0} = 1 - \frac{4}{3}\frac{r_0}{R}; \; Q_0 = \frac{\pi R^4 \cdot \Delta P}{8\mu l} \tag{28}$$

The dependence of the Q/Q_0 on the r_0/R, is described in **Figure 3**.

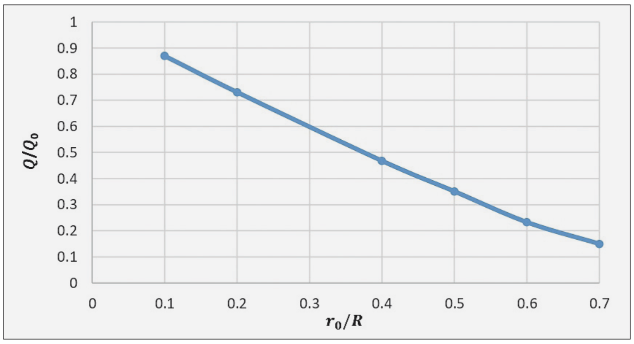

Figure 3: The dependency of $Q/Q_o = f(r_o/R)$

According to **Figure 3**, as the ratio (r_0/R) increases, the flow rate ratio Q/Q_0 decreases significantly. In other words, when the flow core is reduced and the pipe diameter is large, the flow rate of the fluid determined by **Equation 27** will be less different from Q_0. This means that the accuracy will be high in determining the flow rate. Taking into account the abovementioned and pressure losses due to inertia forces, the following **Equation 29** can be obtained in order to determine the flow rate for laminar flow of viscoplastic fluids:

$$Q = \frac{B}{A_1}\left(1 - e^{-A_1 t}\right) \tag{29}$$

Where:

$$A_1 = \frac{8\mu}{\rho R^2 (1 - \frac{4}{3}\frac{r_0}{R})}; \; B = \frac{\Delta P \pi R^2}{\rho l};$$

$$\frac{B}{A_1} = \frac{\Delta P \pi R^4}{8\mu l}\left(1 - \frac{4}{3}\frac{r_0}{R}\right) \tag{30}$$

Taking into account the coefficients A_1, B, B/A_1 in **Equation 29**, the following mathematical expression for determining the flow rate in laminar- viscoplastic flows is obtained (**Equation 31**):

$$Q = \frac{\Delta P \cdot \pi R^4 \left(1 - \frac{4}{3}\frac{r_0}{R}\right)}{8\mu l}(1 - e^{-\frac{8\nu t}{R^2\left(1 - \frac{4}{3}\frac{r_0}{R}\right)}}) \tag{31}$$

Considering that the steady flow rate for viscoplastic fluid is:

$$Q_{st}^{'} = \frac{\Delta P \cdot \pi R^4 \left(1 - \frac{4}{3}\frac{r_0}{R}\right)}{8\mu l} \tag{32}$$

Then, for the flow rate ratio we can write:

$$Q / Q_{st} = 1 - e^{-\frac{8\nu t}{R^2\left(1 - \frac{4}{3}\frac{r_0}{R}\right)}} \tag{33}$$

At $(8\nu)/R^2 = 0.16$ 1/s and different values of the ratio (r_0/R), dependences reflecting the change in the flow rate ratio coefficient as a function of time are shown in **Figure 4**.

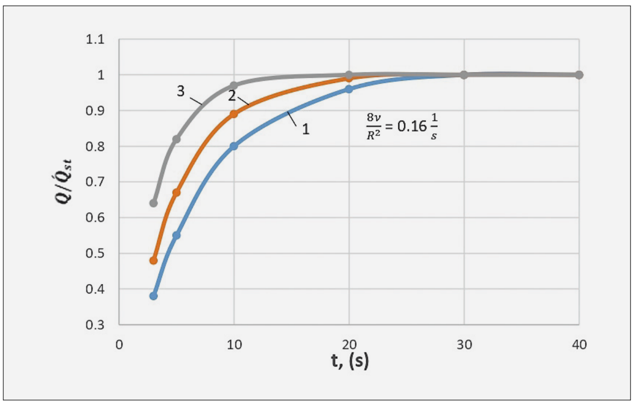

Figure 4: Determination of the time of transition to a steady flow for viscoplastic fluid (1÷3 – respectively, if $(r_o/R) = 0; 0{,}2; 0{,}4$)

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As can be seen from **Figure 4**, in laminar flow of viscoplastic fluids the time of transition to a steady mode of motion increases exponentially depending on the parameter (r_0/R). It was studied and the variation of the time of transition to a steady flow was achieved with $(8v)/R^2$. According to **Figure 5**, the time of transition to a steady mode of motion for Newtonian and viscoplastic fluids, decreases with an increase in parameter $(8v)/R^2$ (see **Figure 5**).

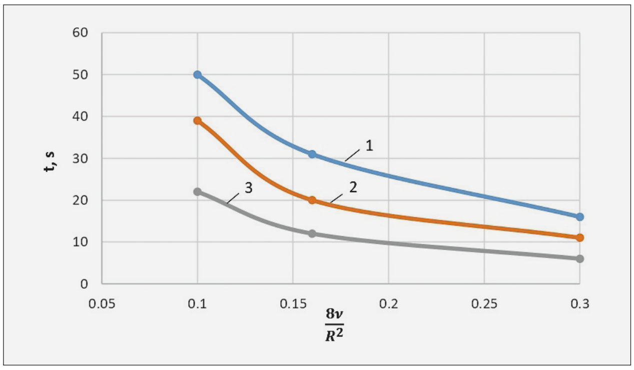

Figure 5: The dependence of the time of transition to a steady flow on $(8v)/R^2$ (1 – Newtonian fluid; 2-Pseudoplastic fluid (r_0/R = 0,2); 3 - Pseudoplastic fluid (r_0/R = 0,4))

4. Conclusions

1. Based on the flow characteristics of Newtonian and viscoplastic fluids and taking into account the forces of inertia in laminar flow, a methodology for determining the time of transition to a steady flow has been developed.
2. It is found that the time of transition to a steady flow for viscous fluids mainly depends on the parameter $(8v)/R^2$, and for viscoplastic flows it also varies depending on how the flow core is formed, which is determined by the initial pushing stress (r_0/R).
3. In order to eliminate the difference and improve accuracy when determining the flow rate using the Poiseuille formula for viscoplastic laminar flows, the necessity of reducing the (r_0/R) (or $\Delta P_0/P$) ratio is shown.

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SAŽETAK

Utvrđivanje karakteristika stacionarnoga stanja u laminarnome protjecanju fluida

Laminarni protok fluida najčešći je oblik protjecanja fluida na naftnim poljima. Kod takva režima protoka fluida od iznimne je važnosti određivanje odnosa brzine i protoka fluida uzimajući u obzir reološka svojstva fluida. S druge strane, od trenutka početka gibanja fluida u cijevi potrebno je određeno vrijeme da se postigne stacionarni protok fluida, odnosno neovisnost njegovih parametara o vremenu. I za njutnovske i za nenjutnovske fluide od posebne je važnosti odrediti karakteristike stacionarnoga stanja u laminarnome protoku. U ovome su radu istraživane raspodjela brzine njutnovskih i nenjutnovskih fluida u poprečnome presjeku cijevi tijekom laminarnoga protjecanja te promjena protoka uzimajući u obzir reološke čimbenike. U radu je prikazano utvrđivanje vremena prijelaza u stacionarni režim protjecanja te čimbenici koji utječu na promjene toga vremena.

Ključne riječi:
viskoplastični fluid, raspodjela brzine, protok, stacionarni protok

Author’s contribution

Gafar Ismayilov (1) (Doctor of technical sciences, Professor, oil and gas engineering) proposed the idea and supervised the research. **Fidan Ismayilova (2)** (PhD, Associated Professor, transportation and storage of oil and natural gas) examined the results and edited the manuscript. **Gulnara Zeynalova (3)** (PhD student of transportation and storage of oil and natural gas) performed tests and provided the report.