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# Decoding Vitruvius <br> - Modulation of a Sequenced Nominal Value of a Fixed Module by Ad Triangulum Geometry 

Dekodiranje Vitruvija<br>- modulacija redane nazivne vrijednosti zadanog modula geometrijom ad triangulum


#### Abstract

The sequencing of circles congruent with a (divided) column diameter - a visualization of the Vitruvian concept of a fixed module - is expanded by the inclusion, in the same scale, of geometric figures intrinsic to the ad triangulum method. The indication of continuous use of the proposed proportional system from Classical Antiquity to the 18th century points to the existence of a single canon for designing intercolumnar plane and fenestration.


KEYWORDS
proportional systems, module, Vitruvius, ad triangulum, intercolumniation, fenestration

## APSTRAKT

Redanje krugova sukladnih (podijeljenom) promjeru stupa - predodžba Vitruvijevog koncepta zadanog modula - prošireno je uvrštenjem, u istoj skali, geometrijskih likova svojstvenih metodi ad triangulum. Naznake neprekidnog korištenja predloženog proporcijskog sustava klasične starine sve do 18. stoljeća upućuju na postojanje jedinog pravila za oblikovanje međustupovne ravnine i fenestracije.

## KLJUČNE RIJEČI

proporcijski sustavi, modul, Vitruvije, ad triangulum, medustupovlje, vrata i prozori

To detect the presence and use of proportions-as-ratios, presumed in historical architecture, are objectives that have long stimulated academical research. Proportional systems are generally divided into geometrical and numerical, i.e., arithmetical. The first category, commonly considered to be medieval, is based on the principle of division of the whole by geometric figures, to which irrational numbers are intrinsic. ${ }^{1}$ On the contrary, in the concept of a module, the principle of addition is applied. ${ }^{2}$ Since the purpose of a modular system is to define a new unit (module) for measuring lengths, it is similar to standard measuring systems - just based upon another unit that allows expression of any magnitude as a multiple or a fraction, leading to an arithmetical sequence of numbers. ${ }^{3}$ Therefore, it is considered to be a rational approach inherent to the revived Classical thought of the Renaissance humanism. ${ }^{4}$
The earliest known mention of the concept is in De Architectura Libri Decem by the Roman architect and military engineer Vitruvius, the only treatise on architecture to survive from antiquity. Since the Renaissance it has been regarded as the first book on architectural theory, which is the "ability to demonstrate and explain the productions of dexterity on the principles of proportion." (Vitruvius, Book I, Chapter 1, 1), as well as a major source on the canon of Classical architecture. ${ }^{5}$ Regardless of the stated importance of geometry and the emphasized necessity of knowing the theory of geometry as well as geometric methods relating to issues of symmetry (I, 1, 3-4), integer multiples or simple fractions based on anthropometry are consistently used to realize the concept of symmetria, provided that a certain part is defined as a standard (III, 1, 9). Accordingly, 5 intercolumnia Vitruvius found appropriate for the Classical orders (except the Doric) - bearing the nominal values (hereafter: NV) of >3, $3,2^{1 / 4}, 2$ and $1^{1 ⁄ 2}$ diameters of the column - classify a temple as araeostylos, diastylos, eustylos, systilos and pycnostylos (III, 3, 2-6). Since the diameter represents a module in the fragmented theory (III, 3, 7) it follows that the NV of intercolumniation inevitably effects the proportions of the whole edifice. Regarding the determination of the total height of a column (III, 3, 10), the column diameter is the fixed parameter in canonized procedure - the multiplications being exclusively of rational numbers. Considering NVs of Vitruvian integers for intercolumnium, the principle of a sequence of the multiplied column diameter with a NV of 1 (according to its shape is represented by a circle) is implied. Therefore, it is necessary to perform a simple logical operation in the case of the non-integer NV, namely
the division of column diameter into 4 or 2 congruent modules of the same NV of 1 (Fig. 1). Sequences of 9 or 3 of that NV result in the specified NV of 2.25 (9/4) or $1.5(3 / 2)$. The congruence in scale, regarding denser colonnades of the Greek Doric order, reinterprets Vitruvius' somewhat strange statement about the division of, logically, the bottom diameter of the Doric column into 2 modules (IV, 3, 4) - although the supposed method for determining proportions of a Greek Doric temple, that also includes the intercolumniation, has been linked to the stylobate and later to the krepis or to the uniform rhythm of triglyphs and metopes with the mutual proportional ratio. ${ }^{6}$ Regarding the Vitruvian NVs - if only the clearly stated application of the concept for determining intercolumniation is considered - that do not correspond to NVs in a large number of extant colonnades, discrepancies could be the result of symmetry-related adaptation to conditions depending on the architect's skill (VI, 2, 1 and 4), but they could also be attributed to an imprecise execution or interventions to the subsequent copies of the text. Therefore, unsurprisingly, many scholars in search of a formula for determining proportions in the Classical architecture apply their own systems in a certain departure from the Vitruvian concept of a module. ${ }^{7}$ Moreover, due to the absence of common mathematics that systematically explains the actual proportions, the existence of proportional systems regarding Roman architecture is legitimately denied. ${ }^{8}$ Considering the discrepancy between ancient theory and practice, as well as the current level of scientific knowledge, this paper aims to present the results of the research that attempts to examine the possibility of canonized determination of actual proportions regarding elements of historical architecture according to Vitruvius' indications about the concept of the module, but with one exception. Divergent from the idea of congruent modules, the concept of systematic application in the same scale of different NVs is proposed, specific to the proportions intrinsic to the geometric figures associated with the elementary ad triangulum method - historically not linked to the proportioning of Classical orders. The exact associated numerical values are derived from the following procedure: since 2 hexagrams joined with pairs of vertices (alternatively, an equilateral triangle mirrored at the base) form a rectangle with the proportional ratio of 1:1.732050807568877 $(1: \sqrt{ } 3)$, the ratio of the height to the width of the hexagram (side to height of an equilateral triangle) is $1.154700538379251: 1$ or $(2 / \sqrt{ } 3): 1$ (Fig. 2). Therefore, when the height of the upright hexagram has a value of 1 - corresponding to

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$\sqrt{ } 3$

### 0.8660250 .866025



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A geometric representation of Vitruvian nominal values for intercolumniation (Duško Čikara)

Geometrijski prikaz Vitruvijevih nazivnih vrijednosti za medustupovlje (Duško Čikara)

2
Calculation of the ratio of 1:1.1547 by geometry ( D . Čikara)

Izračun omjera 1:1,1547 pomoću geometrije (D. Čikara)

## 3

Andrea Palladio, Villa Chiericati at Vancimuglio, floor plan and detail (Ottavio Bertotti Scamozzi, 1781);
A. Palladio, Villa

Valmarana at Lisiera, floor plan and detail (0. Bertotti Scamozzi, 1778); The Temple of Minerva in Assisi, drawings and detail (A. Palladio, 1570) - overlaid with the proportional system by D. Čikara

## Andrea Palladio,

 Villa Chiericati u Vancimugliju, tlocrt i detalj (Ottavio Bertotti Scamozzi, 1781.); A. Palladio, Villa Valmarana u Lisieri, tlocrt i detalj (O. Bertotti Scamozzi, 1778.); Minervin hram u Asižu, crteži i detalj (A. Palladio, 1570.) - preklopi s proporcijskim sustavom D. Čikara
the diameter of the circumcircle that represents the lower/bottom diameter of the column - the width has an exact value of 0.866025 or 1/1.1547 (for brevity, irrational numbers corresponding to proposed NV are initially written with 6 and then with 3 decimal places; the recalculated actual dimensions are the result of calculation operations with 6 decimal places).
The systematic research followed the experimentation on scanned historical plans of relevant edifices, where it had occurred that some intercolumnia correspond to sequences of CAD-generated hexagrams overlaid in the scale of the depicted column diameter. ${ }^{9}$ The inference that the majority of unspecified actual intercolumnia are not simply arbitrary, but the result of the application of a mathematical system, was made after overlays with plans of Palladio's villas. In the etching of Villa Chiericati at Vancimuglio (Fig. 3 above), the central intercolumnium is 3 times a column diameter (hereafter: D) of the standard Vitruvian diastylos, but the lateral prima facie is obtained by intuitive horizontal tripling of upright hexagrams in the same scale. ${ }^{10}$ Concerning the accuracy of the historical drawings it should be noted that, since there


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is no perfect manual drawing, it is always realistic to encounter some deviations. However, the NVs of wider intercolumnia of porticos in Villa Valmarana at Lisiera correspond to iterated 4 identical figures by the same alteration (Fig. 3 centre). ${ }^{11}$ Furthermore, the equally divided portico of Temple of Minerva in Assisi on the woodcut by Palladio might be - except a single perceived and depicted shift - merely proportioned by doubled hexagrams (Fig. 3 below), although the inscriptions (P2@10 or 34 inches and P5¹/4 or 63 inches) suggest 1.852941D, a close approximation of $1+0.866$ or a symmetrical $2 \times 0.933013$. The hypothesis of using ratios of the proposed geometric figures by ancients to determine intercolumniation is further inferred on Palladio's woodcut of temple in Pula. A detail of the column with the inscribed value just above the apophyge inferior, corresponding to the value within the section of the column on the floor plan of the temple ( $\mathrm{P} 2^{1 / 4}$ or 27 inches), points to the standardized position of the lower diameter (Fig. 4 above); it is presumed the sections in previous examples had been determined accordingly. The inscription of the central intercolumnium (P6@1½ or 73.5 inches) suggests 2.722222 D , a

The Temple of Roma and Augustus in Pula, drawings and detail (A. Palladio, 1570); Iktinos and Kallikrates, the Parthenon, floor plan and detail (James Stuart and Nicholas Revett, 1787) overlaid with the proportional system by D. Čikara

## Hram Rome i

 Augusta u Puli, crteži i detalj (A. Palladio, 1570.); Iktin i Kalikrat, Partenon, tlocrt i detalj (James Stuart i Nicholas Revett, 1787.) preklopi s proporcijskim sustavom D. Čikaraclose approximation of $1+(2 \times 0.866)$, although the NV as depicted is $2 \times 1.154$ or 2.3094 D ; within a diagram for the NV of 1.154, a concentric circle - corresponding to the width of the hexagram - subsequently determines a lower/bottom diameter. The inscription for the lateral ( $P 4^{11 / 2}$ or 54 inches) corresponds to the drawing. In descriptions Palladio has laconically rounded NVs according to Vitruvian terms. ${ }^{12}$ The use of the discovered system in determining Greek Doric intercolumniation by the stated division of the diameter into 2 modules is indicated in the overlay with the first accurate plan of the Parthenon (Fig. 4 below). ${ }^{13}$ The NVs of intercolumniation are elegantly determined by the tripling of 0.866 for the main and by the combination of 1 and 0.866 for the extreme intercolumnium in the N part of the W front. The correspondence between the drawing and the inscriptions is evident: the NV of the former could be 1.299038D [( $3 \times 0.866) / 2$ ], accompanied with measures in feet ( $7 \cdot 11 \cdot 5 / 6 \cdot 1 \cdot 8$ or decimally in inches 95.32/73.5=1.29687), and the NV of the latter amounts to approximately $0.933 \mathrm{D}[(1+0.866) / 2]$, the strict intermediate between the 1 and 0.866 - in feet $5 \cdot 8 \cdot 8 / 6 \cdot 1 \cdot 8$ or $68.5 / 73.5=0.931972$.
In the context of the general absence of explicit historical descriptions, for a certain proportional system to be convincing, several of the proposed scientific criteria have to be considered. ${ }^{14}$ To fulfil the criterion of an indication in historical documents beside inference based on the forementioned inscriptions (criteria of simplicity, practicality and accuracy in the paradigmatic overlay of geometrical figures on historical drawings are already decisively fulfilled), the mathematical connection of NVs of Vitruvian intercolumnia and the applied geometrical figures - to the author's knowledge in an authentic way - was found crucial. The first step was to notice that NVs actually represent terms of 2 geometric sequences with a common ratio of $1.333333(4 / 3)$ - the initial terms are 4 and 2 , if the highest NV is in fact 4D. With a theoretical base in Vitruvius' emphasis on the importance of geometry, accompanied by the assumption that in ancient times proportional systems in architecture - as a visual discipline - were derived from primitive regular geometric figures as universal symbols of a higher order, it is logical to infer that the sequences with the specified common ratio were actually selected according to the elegant depiction by the uniform division of the vertical axis of the circumscribed upright hexagram into 4 parts (Fig. 5 above). In order to obtain intercolumnia with NVs that reflect the proportions of geometric figures intrinsic to the ad triangulum method, the detected geometric sequences were simply converted


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A segment of Stonehenge at Amesbury (The United Kingdom), the primordial construction system of post-and-lintel (D. Čikara after Diego Delso, delso. photo, License CC BY-SA, 2014); the relevant zones for proportioning intercolumnar plane and clear opening of fenestration by the proposed system regarding articulation of supports (D. Čikara)

Dio Stonehengea u Amesburyju (Ujedinjeno Kraljevstvo), praiskonski gredni konstrukcijski sustav (D. Čikara prema: Diego Delso, foto: CC BY-SA, 2014); bitne zone za proporcioniranje medustupovne ravnine i svijetlog otvora fenestracije glede raščlambe nosača (D. Čikara)

concept supposedly based on arithmetics within an invaluable historic source for the theory of architecture, might represent a significant theoretical contribution to the study of proportional systems in historical architecture. ${ }^{16}$ Since Modulation is also indicated in the above prominent oeuvre from Cinquecento, the possibility of the continuous application from Antiquity arose - regardless of Vitruvius, especially if presumed that the parts of the treatise relevant to the concept of the module are integrally copied until the dissemination in the Renaissance. It was therefore considered appropriate to examine the persistent application within an extended period, including residential architecture, although the focus had been primarily on Classical temples, largely existing in the Vitruvian epoch. Accordingly, the selected sample of applied Modulation is presented, from the beginning of late Antiquity and the departure from the canon, to the 11th-15th century and the 16th-18th century periods of contrasting attitudes towards Classical Antiquity. To meet the criterion of comparability, in order to assess the extent of chronological and geographical application of the Modulation within the limits of the article, sufficiently accurate numerical data and architectural documentation from independent modern surveys proved crucial beside archival data. The hypothesis is, for the sake of greater accuracy, tested by overlaying CAD geometric diagrams with relevant CAD drawings. ${ }^{17}$ Regarding high technology, significant deviations from the actual state are not expected. Irregularities should be considered as an inevitable consequence of imprecision in the diverse stages of design and construction: from the drafting, translation of the unit of measurement, poor manufacturing, to the installation. Factors such as erosion or destruction followed by anastylosis should also be considered. The particular selection of precise surveys, produced mainly for conservation/restoration purposes, is deliberately partially inconsistent from the aspect of art history to minimize idiosyncrasy and coincidence. The metric values of all dimensions in question are
thoroughly recalculated, since the mensuration (metrological analysis) is essential for the research - primarily in order to confirm the hypothesis of the exclusive derivation of specific proportional ratios (obtained by addition of rational and irrational numbers intrinsic to Modulation) of clear distance from a dimension of material component (diameter or front), presumably precisely executed in the finest practical contemporary unit of measurement. Secondly, to prove that the actual clear distance in question - determined by Modulation - is conversely rounded in a convenient manner. Therefore, the relevant dimensions, including those stated in feet (piedi, P) and spans (palmi maiores, $3 / 4$ of a foot), are converted into digits (dita, 1/16 of a foot). The non-anthropomorphic minuto, obtained by the division of an inch (oncia, $1 / 12$ of a foot) in quarters, is presumed to be the finest proper unit. ${ }^{18}$ According to the mutual ratio of 12:16 between inches and digits, a minuto enables setting i.e., carrying out in practice of measurements in thirds of a digit ( 0.25 oncie equals 0.33 dita, $0.50=0.66,0.75=1,1=1.33$, $1.25=1.66,1.5=2,1.75=2.33,2=2.66$, etc). Quadripartite and the derived tripartite structure of a hexagram optimally suits the anthropometric, duodecimal system of measurement due to the elegantly divisible longer axis in equal parts. ${ }^{19}$ It has to be considered that metrological values differ chronologically and geopolitically; e.g. the values of Doric and Roman feet are not reliable - the latter varies considerably, from 0.293 m to $0.2975 \mathrm{~m} .{ }^{20}$ Therefore from the earlier periods in the sample, only the NV of Modulation is reported. More accurately determined is the metric value of a Venetian piede; 0.34672 m from the 15th-17th century and 0.34776 m in the 18 th century. ${ }^{21}$ Dito accordingly amounts to 0.02167 m and 0.021735 m .
The concept of dividing the bottom diameter into modules by Ancient Greeks, inferred in the plan of the Parthenon - for the exclusive purpose of determining clear distances by the NVs of Modulation on condition of the denser disposition of columns - is evident by clear distances calculated with the compiled data
(interaxial distances related to the presumed bottom diameters) from independent surveys of a dozen Doric temples (Table 1). ${ }^{22}$ The relation of simpler division into modules to the temples of Athena, Zeus and Hera is remarkable. The systematic use of Modulation in Doric order is indicated by the geometrically determined proportion of the column shaft calculated upon the measurement of the bottom diameter ( 1.555 m ) and the reconstructed height ( 8.885 m ) of the 6-drum shaft from the Temple of Athena Alea in Tegea. The height of the trachelion ( 0.138 m ), added to the top in regard to fluting, results in the total height of 9.023 m. ${ }^{23}$ According to the data, the actual NV is 5.802572 D ( $\approx 5.799038$ or $4+0.933+0.866$ ).
For further examination, compiled numerical data relating to the values of the lower diameters and intercolumnia of Capitoline temples are considered as credible (Table 2). ${ }^{24}$ Again, the values are first converted from interaxial to the values of clear distances. Since most of the intercolumnia in question are too narrow to be expressed by the smallest possible combination of NV of Modulation, a solution is again found in the proposed division of the lower diameter into modules that resulted in unexpectedly acceptable deviations from measured/reconstructed dimensions. The most evident sign of the actual application of the proposed system are NVs of the slightly different frontal and flank intercolumnia of the Temple of Venus Genetrix, obtained by inversion ( $0.866+1+0.866$ and $1+0.866+1$ ).
To examine the systematic use in Roman provincial architecture from the turn of the millennium, Modulation is overlaid with a drawing based on a 3D scan of the Temple of Roma and Augustus in Pula (Fig. 7). It is to be noted that the portico was bombed during WW II and was soon after reassembled. ${ }^{25}$ Therefore, deviations in intercolumnia and in the dimensions of the shafts can also be attributed to the damage and extensive reconstruction, and not only to the usual conditions. Of the 6 shafts, 4 are 0.800 m in diameter and the remaining are 0.780 m . Since the apophyge inferior (together with the fillet) is lost on all columns, the accurate lower diameter is calculated by measuring the upper diameter of the shaft, thanks to the 4 preserved apophyge superior. The Vitruvian proportional system of diminution is applied; for the category of columns tall between 20 and 30 feet, the ratio between the upper and lower diameters is 6:7 (1:1.166666). ${ }^{26}$ Considering that the frontal columns have a total height of approximately 8.150 m and that the frontal 3-drum shafts are $6.979 \mathrm{~m}, 6.980 \mathrm{~m}$ and two 6.965 m tall and that the best preserved upper


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Front colonnade of the Temple of Roma and Augustus in Pula (Croatia) overlaid with the proportional system (D. Čikara, architectural drawing: VEKTRA d.o.o. Varaždin)

Čeono stupovlje hrama Rome i Augusta u Puli preklopljeno s proporcijskim sustavom
(D. Čikara, arhitektonski crtež: VEKTRA d.o.o. Varaždin)
diameter (on the left shaft of the central intercolumnium) is 0.687 m , then the ideal lower diameter should be 0.801 m - the virtually insignificant difference confirms the implementation of the system. Since the closest NV from Modulation to the actual state is $10 \times 0.866=8.66025 \mathrm{D}(0.801 \times 8.66=6.936 \mathrm{~m})$, the lengths of the shafts relative to the established lower diameter could have been determined excluding the 0.059 m thick astragal, although they are carved from the same block (completely destroyed on the corner shafts). ${ }^{27}$ Therefore, the relevant total heights of the shafts, including the upper fillet, are $6.920 \mathrm{~m}, 6.921$ m and 6.906 m . It can be assumed that the difference is due to rounding down to a convenient unit of measurement or the anastylosis. The mutual difference of 0.014 m points to the latter. The central intercolumnium is 1.700 m and the lateral are 1.600 m . If the NVs of Modulation are scaled to the diameter of 0.8009 m , actual central intercolumnium closely approximates 2.1547 D or $2 \times 1.077 \mathrm{D}(1.726 \mathrm{~m})$ and lateral 2D (1.602 m). ${ }^{28}$
An overlay with the drawing of the view from the W of the northernmost part of the eastern arcade of the


Peristyle of Diocletian's Palace in Split built around 300 AD is performed to detect geometry in the proportions of the likely re-used monolithic Classical shafts, but also the possibility of a wider use of Modulation - since it is not a temple. The drawing is the result of combining precise methods of survey (Fig. 8).29 Starting from the $N$, shafts (two from Greek marble and one from a red granite) of $0.618 \mathrm{~m}, 0.634 \mathrm{~m}$ and 0.621 m in diameter are $5.315 \mathrm{~m}, 5.196 \mathrm{~m}$ and 5.258 m tall - both fillets included and astragals excluded. The NVs of $8.600323,8.195583$ and 8.466989 correspond to 8.598075 D that would amount to 5.314 m ( $8.598 \times 0.618$ ), then to 8.19615 D that would amount to $5.196 \mathrm{~m}(8.196 \times 0.634)$ and finally to 8.4641 D that would amount to $5.256 \mathrm{~m}(8.464 \times 0.621)$. Besides the strong indication of Modulation, it is worth noting that the inversion of the NVs 1 and 0.866 in determining the lengths of the marble shafts points to a certain system of gradation, presumably within the same original edifice. The intercolumniation, in the same order, is $2.661 \mathrm{~m}, 2.665 \mathrm{~m}$ and 2.683 m wide. Since the lower diameters differ, it is difficult to determine the intended NV. The closest NV from Modulation is 4.330 D , if a lower diameter of 0.618 m was selected as referential. The product is a clear span of 2.678 m . The deviation in relation to the measured distances - a possible consequence of imprecise placement of


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Northernmost part of Eastern colonnade of the Peristyle of Diocletian's Palace in Split (Croatia), view from the W overlaid with the proportional system (D. Čikara, architectural drawing: Croatian Conservation Institute)

Krajnji sjeverni dio istočnog stupovlja Peristila Dioklecijanove Palače u Splitu, pogled sa zapada preklopljen proporcijskim sustavom (D. Čikara, arhitektonski crtež: Hrvatski restauratorski zavod)

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Northern portal inside the Vestibule of Diocletian's Palace in Split (Croatia) overlaid with the proportional system (D. Čikara, architectural drawing: GEODATA d.o.o. Split)

Sjeverni portal unutar Vestibula Dioklecijanove Palače u Splitu preklopljen proporcijskim sustavom (D. Čikara, arhitektonski crtež: GEODATA d.o.o. Split)
columns, with different diameters being an aggravating circumstance - is $0.02 \mathrm{~m}, 0.01 \mathrm{~m}$ and 0.01 m (less than 1\%).
To examine the fenestration, Modulation is overlaid with the late Roman portal inside the Vestibule of Diocletian's Palace in Split. According to the drawing based on computer-generated orthogonal photography referenced by total station (Fig. 9), non-articulated jambs of unequal fronts (the left is 0.506 m and the right is 0.461 m wide) are aligned with the outer edges of the lintel (a somewhat more massive, 0.562 $\mathrm{m})$. Therefore, as a relevant detail for establishing their intended width, the distance of the astragal strip on the lintel - in relation to the upper and side edges of the clear opening - is selected starting from the logical assumption that it should be identical. Accordingly, the right jamb occurs to be more relevant. The clear span, after the correction amounting to 2.577 m, might have been obtained by rounding down the NV of $5.598075 \mathrm{~F}(0.461 \times 5.598=2.580 \mathrm{~m})$ and the height of the opening $(4.003 \mathrm{~m})$ from the rounding down of the NV $8.73205 \mathrm{~F}(0.461 \times 8.732=4.026 \mathrm{~m})$.
The survival of the ancient method is suggested by the overlay with the drawing based on a survey with total station of the highly carved proto-Romanesque monophore from 11th century on the E and N facades of St. Michael's, a court chapel at Ston. ${ }^{30}$ Accordingly, only the wider right jambs ( $0.104 \times 0.909 \mathrm{~m}$ and $0.099 \times 0.901 \mathrm{~m})$, inserted in the partial recesses in the sills, determine the proportions of the 0.539 m wide openings expressed by Modulation (Fig. 10). Lengths to the sill of the right jambs closely correspond to the NVs of $8.732 \mathrm{~F}(0.104 \times 8.732=0.908 \mathrm{~m})$ and $9.129163 \mathrm{~F}(0.099 \times 9.129=0.903 \mathrm{~m})$, and clear spans to the NVs of $5.19615 \mathrm{~F}(0.104 \times 5.196=0.540$ $\mathrm{m})$ and $5.4641 \mathrm{~F}(0.099 \times 5.464=0.541 \mathrm{~m})$ in the same order. The chipped sill of the N window exposes total lengths of the jambs ( 0.953 m ) that indicate a NV of $7+0.933+1.732=9.665063 \mathrm{D}\left(\approx 9^{2 / 3}\right)$. Geometrically, the difference between the total and once visible length is a matter of replacing 4 circles with 4 hexagrams. Another medieval sign of Modulation occurs in a contract from 1324, far before the renaissance of Vitruvius, wherein a stonemason and builder Vlaho Tomadov undertook the production of columns for a house in Dubrovnik. Since similar contracts usually confirm separate carving of capitals, the above probably referred to typical cylindrical shafts. ${ }^{31}$ The lesser elongated is 9 palmi tall and $\mathrm{P}_{1} 1 / 2$ in diameter and three are 10 palmi tall and P1 plus 2 dita in diameter. Although dimensions converted to digits (108/24 and $120 / 18$ ) reveal simple fractions $4^{1 / 2}$



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The widest Northern nave arcade of Šibenik cathedral (Croatia), view from the South overlaid with the proportional system (D. Čikara, architectural drawing: VEKTRA d.o.o. Varaždin)

Najširi sjeverni luk u ladi šibenske katedrale, pogled s juga preklopljen proporcijskim sustavom (D. Čikara, arhitektonski crtež: VEKTRA d.o.o. Varaždin)
and $62 / 3$ (determined by congruent multiples, with a prerequisite division of the diameter into 2 and 3 modules), the latter could have been determined alternatively by $4+0.933+1.732=6.665063 \mathrm{D}$ since $18 \times 6.665063=119.97$ (Fig. 11). The impossibility of the same elegant alternative, regarding the former, points to the proposed division of a diameter by reason of a greater variety among proportionally shorter shafts, like in narrower intercolumnia.
The late medieval use of Modulation is indicated by the overlay with the 3D scan-based drawings of Šibenik Cathedral. The nave is divided by columns from the Gothic phase of construction (1431-1441). ${ }^{32}$ According to the drawing of the N arcade, the diameters of 5 columns vary between 0.500 and 0.489 m , small deviations from 23 dita ( 0.498 m ). ${ }^{33}$ The cylindrical shafts, excluding tori and astragals, starting from the W are $3.434 \mathrm{~m}, 3.429 \mathrm{~m}$ (3 columns) and 3.41 m tall. Applying Modulation, the hypothetical NV could be determined by $8 \times 0.866 \mathrm{D}(0.498 \times 6.9282=3.450 \mathrm{~m})$. The height corresponds to 159 dita or 3.446 m . The intercolumnia, in the same order, are $3.182 \mathrm{~m}, 3.179 \mathrm{~m}$, $3.298 \mathrm{~m}, 3.179 \mathrm{~m}$ and 4.216 m , while the easternmost is 3.159 m . It can be concluded, after selecting a lower
diameter of 0.498 m as relevant, that the largest intercolumnium should be $8.4641 \mathrm{D}(0.498 \times 8.464=4.215$ m , rounded to $194^{2} / 3$ dita (Fig. 12). The Modulation is also overlaid with the contemporary W (Fig. 13 left) and $N$ (Fig. 13 right) portals regarding the innermost frames. The evenly cut frames of pointed lunettes ( 0.406 m and 0.340 m ) are selected as relevant since they exclusively determine the proportions by Modulation. ${ }^{34}$ It can be considered that the intended fronts were $18^{2 / 3}$ dita and $15^{2 / 3}$ dita ( 0.405 m and 0.339 m ). The clear spans of 2.212 m and 1.989 m almost have the exact NVs of 5.464 F and 5.866025 F , probably rounded to $102^{1 / 3}$ dita ( 2.217 m ) and $91^{2 / 3}$ dita ( 1.986 m). Shafts are 2.991 m (perhaps $137^{2 / 3}$ dita plus joints) and $2.792 \mathrm{~m}\left(128^{1 / 3}\right.$ dita) tall, close approximations of the NVs of $6.464+0.933=7.397114 \mathrm{~F}(2.300 \mathrm{~m})$ and 8.196F ( 2.787 m ).

The use of Modulation in fenestration is again indicated by the overlay with the drawing of the W monofora from the piano nobile of the Moise Palace in Cres, attributed to the workshop Marangonich from the first half of the Cinquecento. ${ }^{35}$ From the drawing based on digital photography referenced by total station, the jambs are not uniform, nor constant in width (Fig. 14 left). In addition to the deviation from the perpendiculars, they are unevenly installed in relation to the axis of symmetry, since below slightly protruding capitals jambs were usually to be positioned in the continuous plane regarding springing points of an arch. Therefore, the width of the frame at the springing points ( 0.161 $\mathrm{m} \approx 7^{1 / 3}$ dita) is selected as relevant. The diameter of the intrados ( $0.879 \mathrm{~m} \approx 40^{2 / 3}$ dita) corresponds to the NV of $5.464 \mathrm{~F}(5.464 \times 0.161=0.879 \mathrm{~m})$. The shafts are 1.196 m and 1.204 m tall ( $\approx 1.199 \mathrm{~m}$ or $55^{1 / 3}$ dita), which resembles the NV of $7.4641 \mathrm{~F}(7.464 \times 0.161=1.202 \mathrm{~m})$. The Modulation is again indicated on the basis of a recalculation, to the finest practical unit of measurement, of dimensions from the archived contract; in 1599 the stonemason Stjepan Bokanić undertook the cutting of monolithic columns $\mathrm{P}_{13} 1 / 2$ tall and $\mathrm{P}_{2}$ minus 4 dita wide for the church of St. Simon in Zadar. ${ }^{36}$ The stated certainly refers to shafts, for carving a complete column - especially of such dimensions in a single cuboid of stone would be an irrational endeavour. The proportional ratio converted into digits is 7.714285 (216/28), a rounded-down NV of 7.73205D or 216.4974 digits (Fig. 14 right).
The persistence in proportioning of fenestration by Modulation is illustrated in the fortified Marinčević-Gligo complex at Bobovišće Luka on the island of Brač; the outline around 1740 is attributed to protomagister Ivan III. Macanović from Trogir. ${ }^{37}$ The evenly cut

$\longrightarrow \quad 5 \mathrm{~m}$

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ancient theory might have indeed been founded if the actual proportions of the elements in question are elegantly determined by multiples in the same scale of the value of 1 (represented by a circle) and the values of 0.866025 and 1.1547 (1:0.866025), intrinsic to a hexagram constructed by the ad triangulum method. Beside different combinations of the above values, if appropriate, intermediate values 0.933013 and 1.07735 may also be included. Due to the irrational ratios of geometric origin, the design system discovered and proposed here is not strictly modular, but a specific modulation of the value of the fixed module.
It can be argued that the results of this research exceptionally met the criteria set for proportional systems in historical architecture. In addition to the almost perfect match of the proposed system with the lower diameter-based proportions of several precisely surveyed Classical Antique shafts (astragal excluded) and intercolumniation, the numerical values obtained from independent surveys strongly point to a particular strategy: the intercolumniation of prominent Doric and Roman temples is determined by uniform division of the bottom/lower diameter in 2,3 or 4 modules. It may be concluded that the unexplained

Western and Northern portal of Šibenik cathedral (Croatia) overlaid with the proportional system (D. Čikara, architectural drawing: VEKTRA d.o.o. Varaždin, 2013)

Zapadni i sjeverni portal šibenske katedrale preklopljeni proporcijskim sustavom (D. Čikara, arhitektonski crtež: VEKTRA d.o.o. Varaždin, 2013.)



## 14

Monofora on the Northern façade of the Moise Palace in Cres (Croatia) overlaid with the proportional system; geometric representation of the column shaft dimensions in digits from a contract in 1599 (Zadar, Croatia) (D. Čikara, architectural drawing: University of Zagreb, Faculty of Architecture, Institute for Build Heritage)

Monofora na sjevernom pročelju palače Moise u Cresu preklopljena proporcijskim sustavom; geometrijski prikaz dimenzija tijela stupova izraženih u prstima iz ugovora sklopljenog u Zadru 1599. godine (D. Cikara) (D. Čikara, arhitektonski crtež: Zavod za graditeljsko naslijede Arhitektonskog fakulteta Sveučilišta u Zagrebu)


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Portal of the fortified complex Marinčević-Gligo at Bobovišće Luka (island of Brač, Croatia) overlaid with the proportional system (D. Čikara, architectural drawing: Anka Ćurić)

Portal utvrđenog
sklopa Marinčević-
Gligo u Bobovišću Luci na otoku Braču preklopljen proporcijskim sustavom (D. Čikara, arhitektonski crtež: Anka Ćurić)
division of the diameter of the Doric column into 2 modules made sense as a part of the solution to the issue of composing a large number of different narrow intercolumnia from the discovered system, currently the only logical reason for dividing the diameter into modules. As it is not probable that division of any length into a systematic sequence of irrational values follows the prior division of the same length by integers, a persistent presence of values derived from proposed system strongly indicates a methodological inversion regarding the established theory of designing a Greek Doric temple. Since the research so far suggests that Vitruvius should be taken cum grano salis, the strategy - not contradictory to the source, but logically derived from the specific postulate of dividing the diameter into 2 modules - warrants scientific validation. Furthermore, the proportions of a complete fluted zone of a Greek Doric column might have been determined by the proposed system. The architectural elements in question, derived from the primitive trabeated system, supports the hypothesis of the ancient canonical application of a rudimentary ad triangulum method. Therefore, the division of a column diameter into modules should be considered as a subsequent adjustment to denser colonnades, implying a non-Doric origin of the Modulation.
A more important discovery is the correspondence, given the high degree of accuracy regarding the samples, between the nominal values of the proposed

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system that match architectural elements of Classical Antiquity and the fixed module-based proportions from the late Roman Empire to the 18th century. Accordingly, it can be concluded that the proposed system is not limited to supposedly distinct Graeco-Roman principles of architectural design - it might persist as canon in later periods. Conversely, the units of Gre-co-Roman metrology-in-practice might be inferred from the determination of the relevant dimensions in digits, documented in the 14th and 16th centuries The hypothesis of extended use should be verified on a larger sample. Future research should also try to define a chronological and geographical framework in which the proposed system can be established by the same methodology, including the detected patterns. Different combinations of geometrically generated components derived from a fixed module to a great extent correspond to the actual proportions. Therefore, along with dimensions in the finest practical units of measurement, the proposed system explains perceived anomalies in construction and thus largely refutes the equally radical view of the nonexistence of proportional systems. ${ }^{38}$ Considering the nominal values (resulting from specific repetition of hexagrams) that virtually correspond to fractions, a single mathematical method of encoding dimensions that were empirically recognized as suitable from the aspect of firmitas and utilitas is nevertheless the elegant way of obtaining final proportions by avoiding the division of the fixed module. Although, the observed inclusion of the single nominal value of 0.933 points to the ability of the proportion within a specified vertical support to be, if required, also expressed in simple fractions (Fig. 10 extreme right, 11 and 13 left). Although the proposed system does not seem to reflect philosophical ideas or religious symbolism of Antiquity, it can be speculated upon whether it - used with great discipline - fulfils a certain notion of venustas or even belief in a higher order since the process appears to always result in clear distances determined by symmetric horizontal sequencing.
The proposed system allows for a new paradigm, that neither changes of style nor the evolution of a particular style necessitates a change in proportional system. Yet, the issue of widespread emulation of proportions or recalculation of the numerical values from extant buildings should be considered, given the proportions determined by the proposed system imposes itself as a prerequisite for such a reproduction. Therefore, it is logical to posit a rather limited repertoire of mathematical tools and strategies, although the existence of a "universal" system of proportions is a paradox in

Western thought since it calls into question the axiom of individual creative activity and continuous change/ progress based upon it.
Since none of the largely anonymous medieval protomagistri nor any famous individuals that would be expected to participate in humanist discourse left a usable description or drawing, the selectively restricted knowledge of a single common proportional system is presumed. The results show that architecture could have been systematically designed despite the general lack of descriptions, although Vitruvius' treatise cannot be judged without pause because it is not known if its integrity was preserved until the 15th century. But, regarding the theoretical achievements of early modern period treatises as manuals - the results of this research clearly invite a grounded re-evaluation - it can be said that they are primarily a display of intellectual power emulating the ancient literary model, not serving to elaborate essential theoretical knowledge of the profession. It should be examined closer, whether the proportions in architecture by trattatisti resemble an idiosyncratic design or the proposed system.

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|  | Reconstructed clear distance (interspacing minus lower diameter) in mm | Recalculated D after reconstruction (clear distance ) lower diameter) | Proposed D (proposed value) modules in diameter) | Geometric representation of proposed intercolumnium | Theoretical clear distance (error) (proposed D× lower diameter) in mm |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Divus Iulius | $\begin{gathered} 1760 \\ (2920-1160) \end{gathered}$ | $\begin{gathered} 1.517241 \\ (1760 / 1160 \end{gathered}$ | $\begin{gathered} 1.515543 \\ (6.062175 / 4) \end{gathered}$ |  | $\begin{gathered} 1758(-2) \\ (1.515543 \times 1160 \end{gathered}$ |
| Venus <br> Genetrix <br> front | $\begin{gathered} 1700 \\ (2950-1250 \end{gathered}$ | $\begin{gathered} 1.36 \\ (1700 / 1250 \end{gathered}$ | $\begin{gathered} 1.366025 \\ (1+1.73205 / 2) \end{gathered}$ |  | $\begin{gathered} 1708(+8) \\ (1.366025 \times 1259 \end{gathered}$ |
| Venus <br> Genetrix <br> flank | $\begin{gathered} 1790 \\ (3040-1250 \end{gathered}$ | $\begin{gathered} 1.432 \\ (1790 / 1250 \end{gathered}$ | $\begin{gathered} 1.433012 \\ (2+0.866025 / 2) \end{gathered}$ |  | $\begin{gathered} 1791(+1) \\ (1.433012 \times 1250 \end{gathered}$ |
| Apollo <br> Sosianus | $\begin{gathered} 2160 \\ (3630-1470) \end{gathered}$ | $\begin{gathered} 1.469387 \\ (2160 / 1470) \end{gathered}$ | $\begin{gathered} 1.466506 \\ (4+1.866025 / 4) \end{gathered}$ |  | $\begin{gathered} 2156(-4) \\ (1.466506 \times 1479) \end{gathered}$ |
| Saturn | $\begin{gathered} 3150 \\ (4500-1350) \end{gathered}$ | $\begin{gathered} 2.333333 \\ (3150 / 1350 \end{gathered}$ | $\begin{gathered} 2.333333 \\ (7 / 3) \end{gathered}$ |  | $\begin{gathered} 3150 \\ (2.333333 \times 1359) \end{gathered}$ |
| Jupiter <br> Optimus <br> Maximus | $\begin{gathered} 3530 \\ (5000-1470) \end{gathered}$ | $\begin{gathered} 2.40136 \\ (3530 / 1470) \end{gathered}$ | $\begin{gathered} 2.398716 \\ (2+5.19615 / 3) \end{gathered}$ |  | $\begin{gathered} 3526(-4) \\ (2.398716 \times 1470) \end{gathered}$ |
| Jupiter <br> Opt. Max. <br> central | $\begin{gathered} 5930 \\ (7400-1470 \end{gathered}$ | $\begin{gathered} 4.034013 \\ (5930 / 1470) \end{gathered}$ | $\begin{gathered} 4.031087 \\ (2+6.062175 / 2) \end{gathered}$ |  | $\begin{gathered} 5926(-4) \\ (4.031087 \times 1470 \end{gathered}$ |

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## NOTES

1 Matthew A. Cohen, "Two Kinds of Proportion," in Proportional Systems in the History of Architecture. A Critical Reconsideration, ed. Matthew A. Cohen and Maarten Delbeke (Leiden: Leiden University Press, 2018), 19.
2 Mark Wilson Jones, Principles of Roman Architecture (New Haven and London: Thames \& Hudson, 2003), 120.
3 Marie-Pascale Corcuff, "Modularity and Proportions in Architecture and their Relevance to a Generative Approach to Architectural Design," Nexus Network Journal 14 no. 1 (2012): 55-56.
4 Matthew A. Cohen, "Two Kinds," 19.
5 Vitruvius, The Ten Books of Architecture, tr. by Morris Hicky Morgan (Cambridge: Harvard University Press, 1914).
6 Gene Wadell, "The Principal Design Methods for Greek Doric Temples and Their Modification for the Parthenon," Architectural History 45 (2002): 1-8.
7 Within the limits of the paper, it is impractical to objectively select relevant literature concerning proportional systems of Roman architecture. The reader is referred to the book in note 2 , which contains an extensive bibliography, and the book in note 1 , considering relevant recent researches. The following article serves as a general introduction to proportional systems: Ralf Weber and Sharon Larner, "The Concept of Proportion in Architecture: An Introductory Bibliographic Essay," Art Documentation: Journal of the Art Libraries Society of North America 12 no. 4 (winter 1993): 147-154.
8 Mark Wilson Jones, Principles, 6.
9 To visualise the relevant NV, a hexagram is preferred over an equilateral triangle due to its biaxial symmetry.
10 Ottavio Bertotti Scamozzi, Le fabbriche e i disegni di Andrea Palladio. Tomo terzo (Vicenza: Francesco Modena, 1781.), T. XLVII.

11 Ottavio Bertotti Scamozzi, Le fabbriche e i disegni di Andrea Palladio. Tomo secondo (Vicenza: Francesco Modena, 1778.), T. XVIII.

12 Andrea Palladio, I quattro libri dell'architettura. Libro quarto (Venezia: Appresso Dominico de’Franceschi, 1570.), 103, 107.
13 James Stuart and Nicholas Revett, The Antiquities of Athens. Volume the Second (London: John Nichols, 1787), Ch.I.PI.II.
14 J. J. Coulton, "Towards Understanding Doric Design: The Stylobate and Intercolumniations," The Annual of the British School at Athens 69 (1974): 61; Mark Wilson Jones, "Ancient Architecture and Mathematics: Methodology and the Doric Temple," in Nexus VI: Architecture and Mathematics, ed. Sylvie Duvernoy and Orietta Pedemonte (Torino: Kim Williams Books, 2006), 150-157.
15 Francisco Roldán, "Method of Modulation and Sizing of Historic Architecture," Nexus Network Journal 14 no. 3 (2012): 539-553.
16 Duško Čikara, "A Proposal for the Canonical Design of an Aisleless Floor Plan by Hexagram - a Dozen Vernacular Churches from the Island of Brač," in Neglected Topics in Art and Humanities. Book of Essays in Honour of Professor Emeritus Vladimir P. Goss, ed. Danko Dujmović and Palma Karković Takalić (Rijeka: University of Rijeka, Faculty of Humanities and Social Sciences: 2022), 73-74.
17 Grateful acknowledgment is given to architects Ivana Gobec and Ema Babić.
18 Andrea Palladio, Libro secondo, 4.
19 Duško Čikara, "A Proposal," 69.
20 Mark Wilson Jones, "Doric Measure and Architectural Design 1: The Evidence of the Relief from Salamis," American Journal of Archaeology 104 no. 1 (Jan. 2000): 74-75, 77.
21 Sanja Buble, "Istraživanje povijesnih mjera u Trogiru - mjere za dužine u arhitekturi rane renesanse," in Zbornik Tomislava Marasovića, ed. Ivo Babić, Ante Milošević and Željko Rapanić (Split: Sveučilište u Splitu and Muzej hrvatskih arheoloških spomenika u Splitu: 2002), 461.

22 Naomi J. Norman, "The Temple of Athena Alea at Tegea," American Journal of Archaeology 88 no. 2 (Apr. 1984): 172; Bonna Daix Wescoat, The Temple of Athena at Assos (Oxford: Oxford University Press, 2012), 38, 41; Mark Wilson Jones, "Doric Measure and Architectural Design 2: A Modular Reading of the Classical Temple," American Journal of Archaeology 105 no. 4 (2001): 702-710.
23 Jari Pakkanen, The Temple of Athena Alea at Tegea. A Reconstruction of the Peristyle Column (Helsinki: Department of Art History at the University of Helsinki and Finnish Institute at Athens, 1998), 13, 15, 17, 34.
24 John W. Stamper, The Architecture of Roman Temples. The Republic to the Middle Empire (Cambridge: Cambridge university Press, 2005), 126.
25 Ines Vanjak and Marko Špikić, "Principi restauriranja Augustova hrama u Puli 1946. i 1947. godine," Godišnjak zaštite spomenika kulture Republike Hrvatske 37/38 (2013): 8.
26 David A. Villa Domini, "The Diminution of the Classical Column: Visual Sensibility in Antiquity and the Renaissance," Nexus Network Journal 5 no. 1 (2003): 103.
27 Ines Vanjak and Marko Špikić, „Principi," 16.
28 A small difference from the actual lower diameter confirms that Palladio, while measuring in 16th C piedi Vicentini $\left(2^{1 / 4 \times 0.347 m}=0.78075 \mathrm{~m}\right)$, applied traditional methodology of determining the module. Regarding his higher NV of 2.309D for central intercolumnium, a recurrent clerical error is first to be assumed; since the lateral intercolumnia maintained the stated NV of 2D, it would have meant a significant contraction $(0.1 \mathrm{~m})$ of the whole front as a result of anastylosis, which is less likely.
29 Petar Puhmajer and Ivana Gobec, "Dokumentiranje stanja na sjeveroistočnom dijelu Peristila 2004. godine," Godišnjak zaštite spomenika kulture Hrvatske 31/32 (2007/2008): 45-46.
30 Ivana Tomas, "Nova promišljanja o crkvi Sv. Mihajla u Stonu," Ars Adriatica 6 (2016): 41-42, 45, 47-48.
31 Cvito Fisković, Prvi poznati dubrovački graditelji (Dubrovnik: Historijski institut JAZU u Dubrovniku, 1955), 80-81.
32 Predrag Marković, Katedrala sv. Jakova u Šibeniku (Zagreb: Naklada Ljevak, 2010), 160-161.
33 With the importance of the edifice and the involvement of foreigners in the construction, it is assumed that the measures were thoroughly determined in Venetian and not in local units.
34 The fronts of the monolithic jamb shafts of N portal amounts to 0.3502 m . On the W portal differ ( 0.417 m and 0.411 ), which is usual, since numerous examples have unequally wide jamb shafts of irregular outer edges.
35 Tea Sušanj Protić, "Renesansna kuća Moise u Cresu - rezultati konzervatorskih istraživanja 2011. godine," Ars Adriatica 4 (2014): 284-285.
36 The State Archives in Zadar; HR-DAZD-31, Bilježnici Zadra, Francesco Primizio (1590.-1608.), kut. 189, b. II, fasc. V, p. 43v-44r.
37 Duško Čikara, "Kasnobarokne reprezentativne bračke kuće tripartitnog tlocrta," in Razmjena umjetničkih iskustava u jadranskome bazenu, ed. Jasenka Gudelj and Predrag Marković (Zag'reb: FF Press, 2016), 90, 97.
38 Rudolf Wittkower, "The Changing Concept of Proportion," Dædalus 89 no. 1 (winter 1960): 209.

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SAžETAK

# Dekodiranje Vitruvija <br> - modulacija redane nazivne vrijednosti zadanog modula geometrijom ad triangulum 

Najstariji spomen sustava razmjera za oblikovanje arhitekture zamišljenog na načelu dodavanja nalazimo u Vitruvijevom traktatu i to za određivanje nazivnih vrijednosti medustupovlja. Međutim, redanje krugova sukladnih (ravnomjerno podijeljenom) promjeru stupa - predodžba Vitruvijevog koncepta zadanog modula - daje iznose u cijelim brojevima ili razlomcima, koji ne odgovaraju znanstveno ustanovljenim nazivnim vrijednostima postojećeg ili rekonstruiranog svijetlog razmaka u stupovlju klasične starine. Svrhovitost se pokazala tek pokusnim uvrštenjem, u istoj skali, geometrijskih likova svojstvenih metodi ad triangulum. Potom je na dovoljno velikom uzorku postignut visok stupanj poklapanja zrcalnih nizova s točnim skorašnjim nacrtima te brojčanim vrijednostima neovisnih izmjera. Ovakav prošireni koncept primijenjen je za određivanje međustupovnog razmaka, uključivo dorski red, te odnosa promjera i visine tijela stupa. Sustav je, s istovjetnim rezultatima, odgovarajuće primijenjen na fenestraciju s obzirom na to da se stupovlje i uokvireni otvori mogu smatrati izvedenicama praiskonske gredne gradnje. Naznake korištenja otkrivenog sustava klasične starine na hrvatskom tlu do u 18 . stoljeće - poput ugovorenih mjera za stupove koje su podudarne odgonetnutim nazivnim vrijednostima - dovode u pitanje zamisao o kreativnoj ulozi pojedinca glede odredivanja razmjera jer upućuju na postojanje jedinog, nepisanog pravila za oblikovanje navedenih elemenata arhitekture zapadnog svijeta od grčko-rimskog doba - neovisno o tiskanoj inačici Vitruvijevog rukopisa koji je možda odavno prepravljen. Stoga će se i teorijski dosezi modernih traktatista, u slučaju poklapanja predloženog sustava s točnim nacrtima njihovih ostvarenja, ispostaviti tek kao puki iskazi moći uma prema drevnom predlošku.

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