FIZIKA, 1 (1968) 69-80

ATTENUATION OF ELECTROMAGNETIC SURFACE WAVES ON PLASMA COLUMNS

B. A. ANIČIN

Institute »Boris Kidrič«, Beograd

Received 3 June 1968; revised manuscript received 10 July 1968

Abstract: The attenuation coefficient of an electromagnetic surface wave is derived using a perturbation method which takes into account both collisional loss in the plasma and dielectric loss in the surrounding glass. The attenuation coefficient increases as the square of the phase coefficient in the region of low phase velocity. Numerical data are presented for both dipole and axially symmetric modes. The theory is compared with recent experimental observations.

1. Introduction

The inclusion of a small amount of loss in the equations describing the propagation of electromagnetic surface waves on plasma columns was first effected by Trivelpiece and Gould[1] by adding a small imaginary part to the real frequency. The method is therefore inherently limited to plasma losses only. A comprehensive analysis of both propagation characteristics and attenuation of waveguide modes and surface wave modes was published by Clarricoats, Olver and Wong[2]. The attenuation characteristics are derived and numerical data are given for axial annular plasma columns surrounded by glass and a metallic waveguide tube.

Attenuation measurements are reported by Akao, Ida and Oike[3] who measured the Q factor of a length of surface wave guide used as resonator. Anomalous attenuation in the low phase velocity region of the dipolar mode was observed.

The aim of this paper is to provide adequate numerical data on the attenuation of electromagnetic surface waves propagating on open structures i. e., without a metallic wall and give the theoretical background for the attenuation measurements on plasma-glass combinations which have been studied earlier at the Institute »Boris Kidrič«[4, 5] from the standpoint of phase characteristics.

The attenuation coefficient is derived from the characteristic equation describing both the dispersive properties and the attenuation of surface waves using a perturbation technique. The resultant formula is then specialized to the quasi-static dispersion relation in the cold plasma approximation.

2. *Formulation and solution*

We consider a circular column of homogeneous plasma bound by a glass tube with inner and outer radii a and b, respectively. A characteristic equation of the problem is obtained from the requirement that non-zero solutions of Maxwell's equations in the three regions of interest should exist and satisfy the boundary conditions. Irrespective of the simplifications introduced in this procedure, the characteristic equation will depend on the dielectric constant of the plasma ε_p, the dielectric constant of glass and on the propagation constant γ. The dielectric constant ε_p is

$$\varepsilon_p = 1 - \frac{\omega_p^2}{\omega\,(\omega - j\nu)}\,, \tag{1}$$

where ω is the angular signal frequency, ω_p the plasma frequency, and ν the electron-atom collision frequency; the dielectric constant of glass can be written as

$$\varepsilon = \varepsilon' - j\,\varepsilon'', \tag{2}$$

$$\varepsilon'' = \varepsilon' \operatorname{tg} \delta, \tag{3}$$

where tg δ is the loss tangent of glass. For convenience, instead of the propagation constant

$$\gamma = a + j\,\beta \tag{4}$$

we introduce

$$\eta = \beta - j\,a = -j\,\gamma. \tag{5}$$

The characteristic equation then reads

$$F\,(\varepsilon_p,\ \varepsilon,\ \eta) = 0. \tag{6}$$

Under the assumption that the imaginary parts of the arguments of F are much smaller in magnitude than the respective real parts, F can be developed in Taylor series around the point $(\varepsilon_p',\ \varepsilon',\ \beta)$. After the application of some elementary rules on partial derivatives the attenuation coefficient is obtained as

$$a = \frac{\partial\beta}{\partial\omega}\,\frac{\nu}{2} + \frac{\partial\beta}{\partial\varepsilon'}\,\varepsilon''. \tag{7}$$

The first term of this expression also comes from the analysis of Trivelpiece and Gould[1]. It is convenient to rearrange equation (7) for computation purposes. The dispersion relation of surface waves on plasma – glass guides can be expressed as

$$1 - \frac{\omega_p^2}{\omega^2} = \varepsilon' \frac{I_n(A)}{I_n'(A)} \frac{K_n'(B)\,P_n + \varepsilon' K_n(B)\,Q_n}{K_n'(B)\,R_n + \varepsilon' K_n(B)\,S_n}\,, \tag{8}$$

where I_n and K_n are modified Bessel functions, $A = \beta a$, $B = \beta b$ and

$$P_n = I_n'(A)\,K_n(B) - I_n(B)\,K_n'(A)\,,$$

$$Q_n = I_n'(B)\,K_n'(A) - I_n'(A)\,K_n'(B)\,,$$

$$R_n = I_n(A)\,K_n(B) - I_n(B)\,K_n(A)\,,$$

$$S_n = I_n'(B)\,K_n(A) - I_n(A)\,K_n'(B)\,.$$

The relation is valid in the quasistatic case and may be found in a slightly different form in the paper by Akao and Ida[6], where a complete derivation is given. To avoid writing a lengthy programme for the two relevant partial derivatives of (8), the differentiation is performed numerically by varying the two arguments (β and ε'). The derivative $\frac{\partial\beta}{\partial\varepsilon'}$ is avoided using the identity

$$\frac{\partial\beta}{\partial\varepsilon'} = -\frac{\dfrac{\partial\omega}{\partial\varepsilon'}}{\dfrac{\partial\omega}{\partial\beta}}\,. \tag{9}$$

Further, the normalized group velocity v_g is introduced, so from the expression (7) we obtain

$$\alpha a = \frac{1}{v_g}\left(\frac{\nu}{2\omega_p} - \frac{\partial\left(\dfrac{\omega}{\omega_p}\right)}{\partial\varepsilon'}\,\varepsilon''\right), \tag{10}$$

where v_g and $\frac{\partial\left(\frac{\omega}{\omega_p}\right)}{\partial\varepsilon'}$ can be determined by numerical differentiation of the dispersion relation (8) using a computer.

3. *Numerical results*

The attenuation coefficient involves two terms: one associated with plasma loss and the other with glass loss

$$\alpha a = A_p \frac{\nu}{\omega} + A_g \operatorname{tg} \delta . \tag{11}$$

Thus some degree of generality is retained by computing

$$A_p = \frac{\omega}{2\, v_g\, \omega_p} \quad \text{and} \quad A_g = -\frac{\varepsilon}{v_g} \frac{\partial \left(\frac{\omega}{\omega_p} \right)}{\partial \varepsilon} \tag{12}$$

instead of the attenuation coefficient itself.

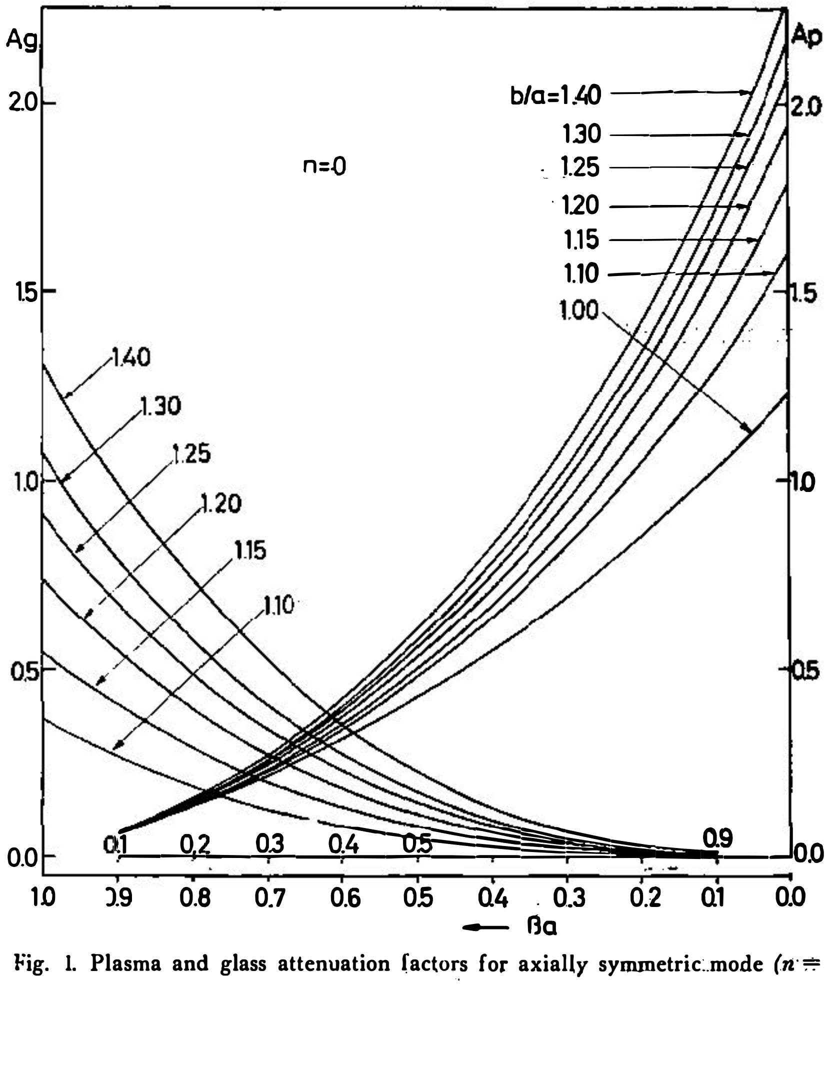

Fig. 1. Plasma and glass attenuation factors for axially symmetric mode ($n = 0$).

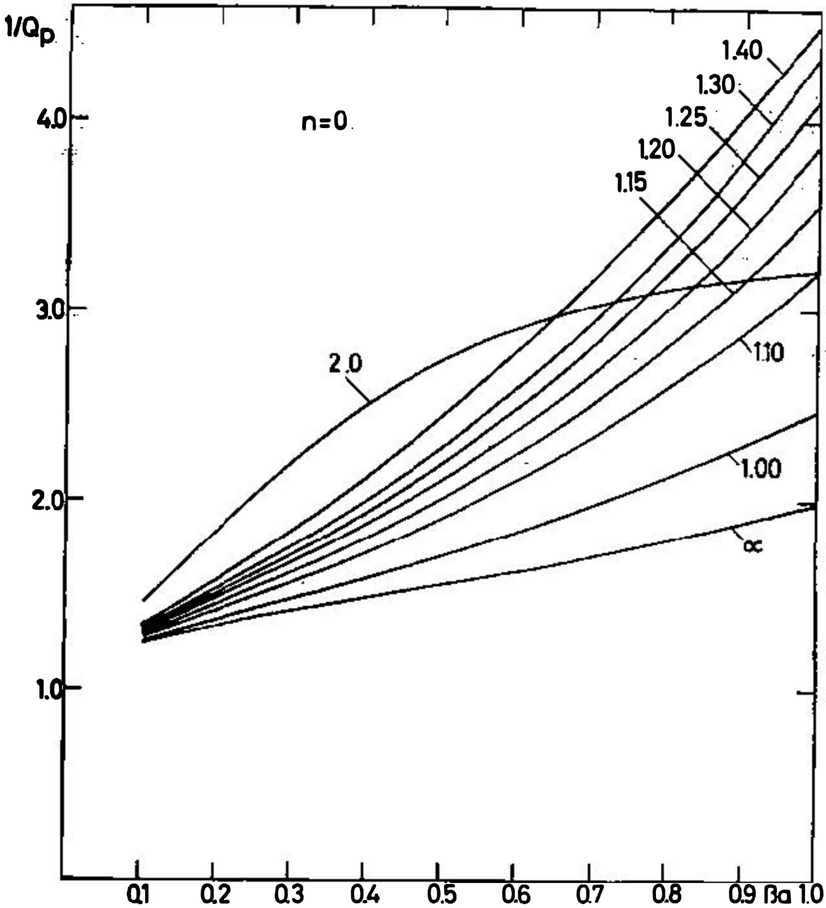

Fig. 2. Plasma Q factor axially symmetric mode

Besides the attenuation coefficient the Q factor of a cavity formed from a length of surface wave guide is also a relevant experimental parameter, related to the attenuation coefficient by $Q = \frac{\beta}{2\alpha}$. The reciprocal Q factor has the additive property and therefore the coefficients $\frac{1}{Q_p} = \frac{2A_p}{\beta_a}$ and $\frac{1}{Q_g} = \frac{2A_g}{\beta_a}$ in the formula

$$\frac{1}{Q} = \frac{1}{Q_p}\frac{\nu}{\omega} + \frac{1}{Q_g}\operatorname{tg}\delta \tag{1.3}$$

have been computed.

With $\varepsilon' = 4.8$ (Pyrex glass) the quantities $\frac{\omega}{\omega_p}$, v_g, $\frac{\partial\left(\frac{\omega}{\omega_p}\right)}{\partial\varepsilon'}$, A_p, A_g, $1/Q_p$, $1/Q_g$ have been tabulated for $b/a = 1$, 1.1, 1.15, 1.20, 1.25, 1.30, 1.40, 2.0, ∞ and for the following values of the phase constant: $n = 0$ (axially symmetric mode) $\beta a = 0.1(0.1)1$; 1.2(0.2)3; 4(1)10; n = 1 (dipole mode) $\beta a = 0.2(0.2)3$; 4(1)10*.

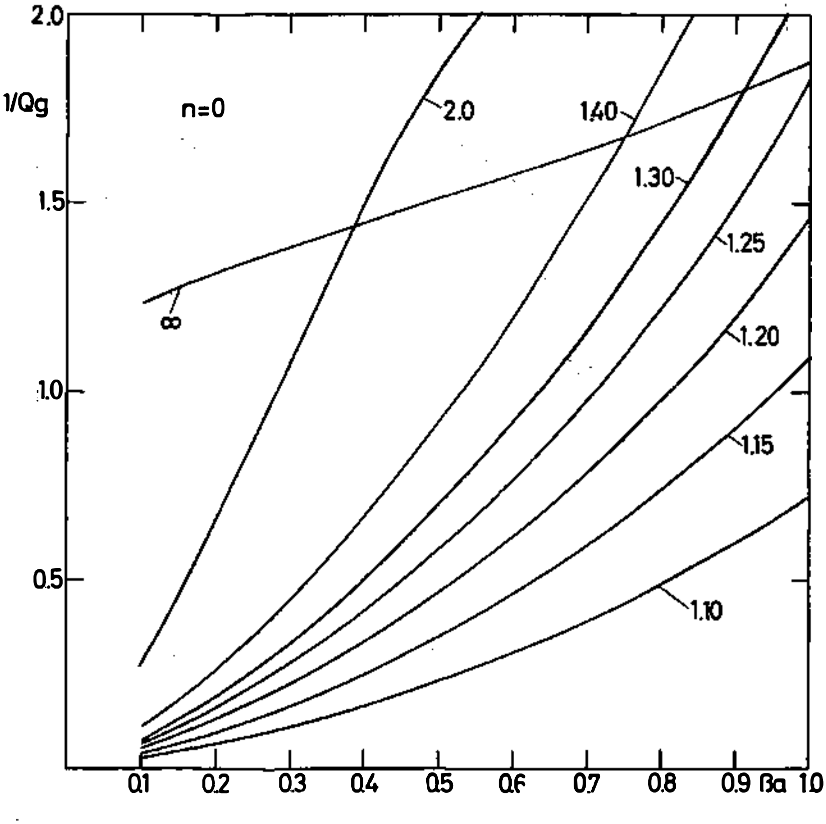

Fig. 3. Glass Q factor for axially symmetric mode.

The resulting attenuation data are shown graphically in figures 1–7. Glass and plasma attenuation factors for the $n = 0$ mode are evident from figure 1. The range of the phase constant in this figure is limited to $\beta a \in (0,1)$. For $\beta a > 1$ the data tend to become less reliable for several reasons: the uniform plasma assumption is no longer correct; warm plasma effects and the Doppler effect of a streaming plasma tilt the asymptote of the dispersion relation; fi

* Five-figure tables as specified above are available from the author on request.

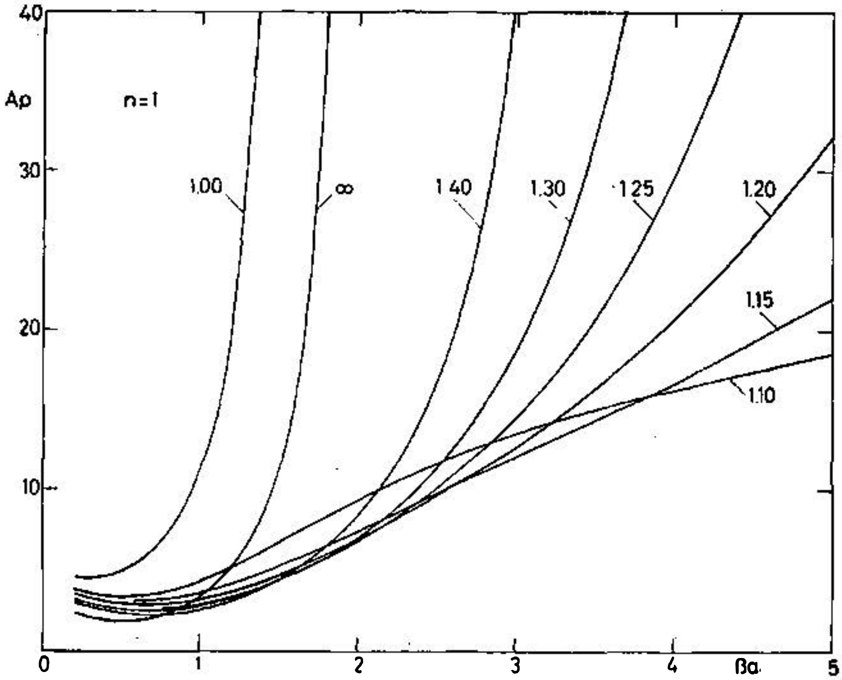

Fig. 4. Plasma attenuation factor for dipole mode ($n = 1$)

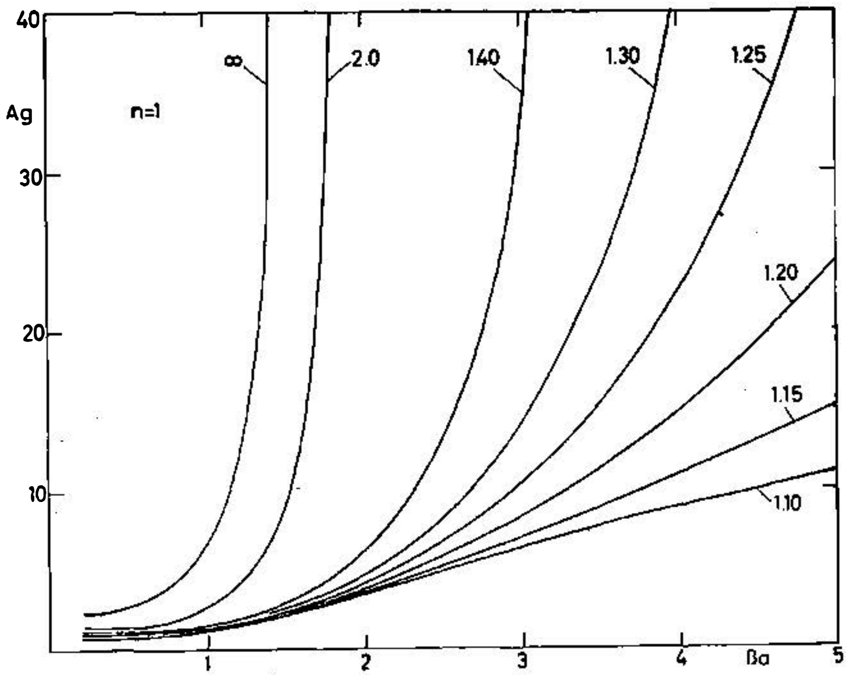

Fig. 5. Dipole mode glass attenuation factor.

nally, near the point $v_g = 0$ the present perturbation theory breaks down. The plasma and glass Q factors for $n = 0$ are given in figures 2 and 3, respectively.

The coefficients A for the $n = 1$ mode are plotted vs. βa in figures 4 and 5 for $\beta a \leqq (0,5)$. The larger interval for the dipole mode is explained by the fact that Akao, Ida and Oike[3] measured Q factors up to $\beta a = 5$ in this mode and by the inherent difference between the $n = 0$ and $n = 1$ dispersion relation. The curves extend only to the first zero of group velocity. The respective data of reciprocals $1/Q$ are given in figures 6 and 7.

It is worth noting that glass and plasma losses will be comparable if $\frac{\nu}{\omega} \simeq \mathrm{tg}\delta$, which is frequently the case with commercial grade glasses, laboratory plasmas and signal frequencies.

4. *On the anomalous attenuation of e. m. surface waves*

The Q factors of cavities formed from sections of plasma surface wave guides operated in the dipolar mode have been recently measured by Akao, Ida and Oike[3]. The main result of their investigation is the constancy of Q in

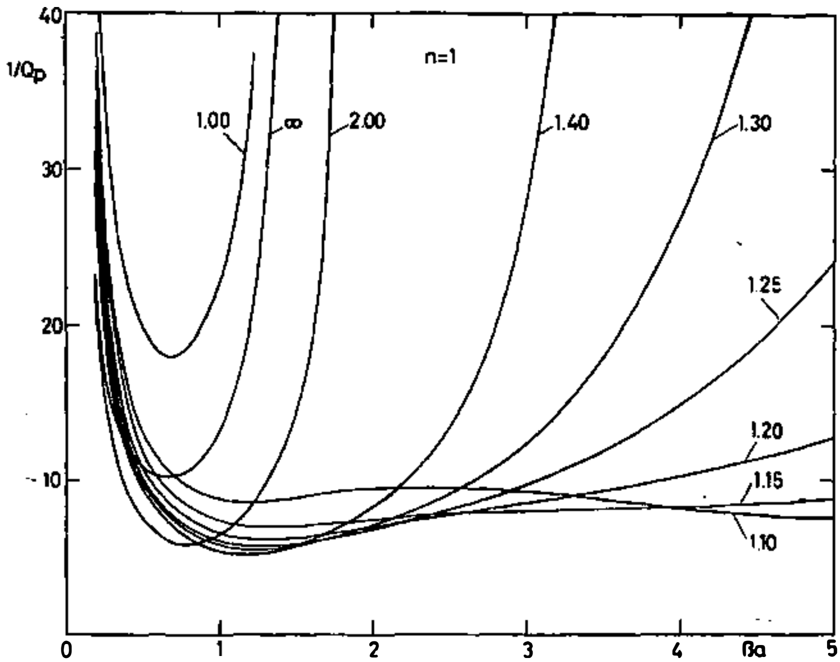

Fig. 6. Dipole mode plasma Q factor.

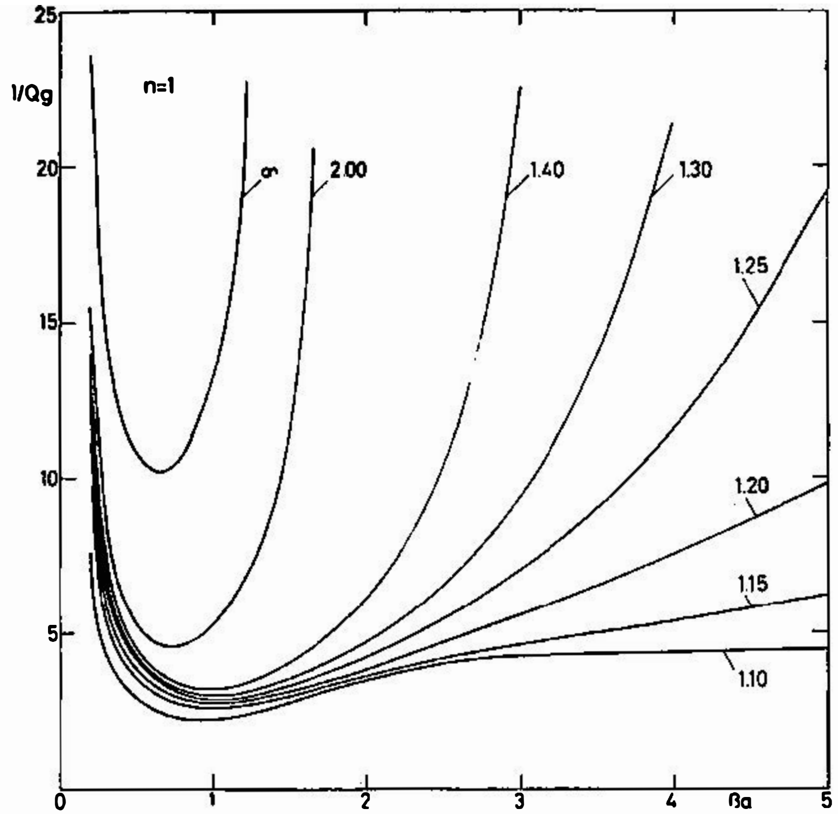

Fig. 7. Dipole mode glass Q factor.

a wide range of phase velocity, which is in accordance with a theory predicting $Q = \frac{\omega}{\nu} = \frac{\text{angular frequency}}{\text{collision frequency}}$, and an increase of attenuation for small phase velocities, which appears to be anomalous in the light of the above theory. The Landau damping was considered as a possible mechanism to account for the decrease in Q near the asymptote of the dispersion relation, but numerical agreement did not prove to be rewarding.

As the present theory does not predict a constant Q, independent of the mode number and phase coefficient, it is of interest to establish whether it can provide an explanation of the high attenuation of the dipolar surface wave for low phase velocity. For illustration in figure 8 the computed data are compared with the results of measurements[3)] performed at 770 MHz, $T_e = 2.6 \times 10^4$ ^{0}K, assuming $\nu = 25$ MHz[8)], $\varepsilon' = 4.8$, tg $\delta = 0$, 2.4×10^{-3} for glas and, 8.9×10^{-3} for Pyrex glass[7)]. Figure 8 has to be regarded as an illustration of the fact that the decrease in Q is to be expected within the framework of cold plasma theory and not as an actual comparison of theory and experiment. The properties of glass, for instance, are not quoted in ref.[3]. The increase in attenuation is associated with the decrease in group velocity near the asymptote of the dispersion relation.

The fact that the ratio of phase velocity and thermal velocity v_p/v_T appears to be a normalizing parameter[3] also finds some justification in cold plasma theory. Assuming very thick glass walls, we can prove (Appendix I) that the attenuation coefficient near the asymptote of the dispersion relation is proportional to the square of the phase constant

$$\alpha a \sim \frac{1+\varepsilon'}{\varepsilon'} \frac{\nu}{\omega} (\beta a)^2 . \tag{14}$$

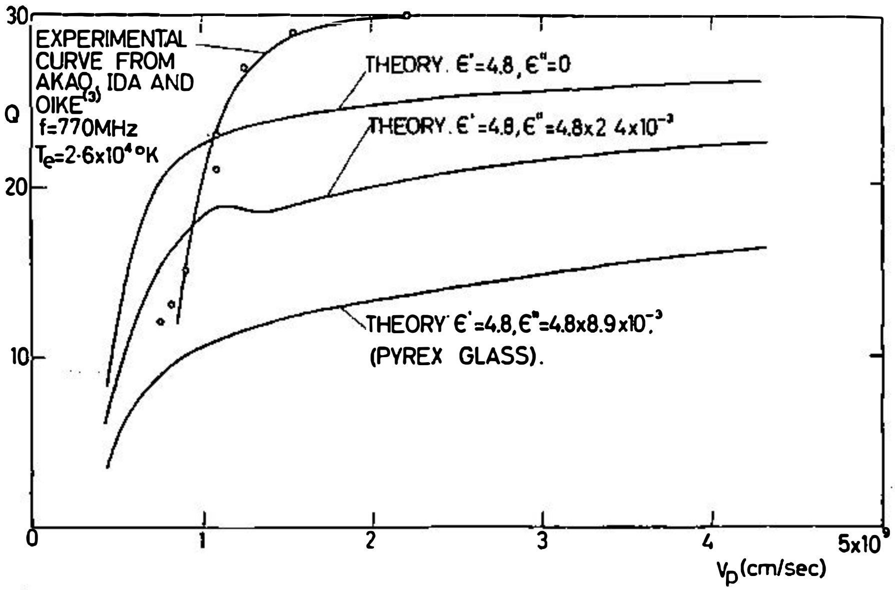

Fig. 8. Theoretical and experimental Q factor *vs.* phase velocity.

The collision frequency is given by $\nu = pP_c v_T$, where p is the gas pressure, P_c the collision probalility, v_T the electron thermal velocity, and the phase velocity is given by $v_p = \frac{\omega}{\beta}$. This gives

$$Q \sim \frac{\varepsilon'}{1+\varepsilon'} \frac{1}{2\, a_p P_c} \frac{v_p}{v_T} . \tag{15}$$

In conclusion, cold plasma theory seems to be capable of explaining the increased attenuation of surface waves in the region of low phase velocity.

Appendix I

When the glass thickness is large ($b/a \to \infty$) the dispersion relation (15) is simplified to the expression

$$\varepsilon'_p = \varepsilon' \frac{I_n(\beta a)\, K_n'(\beta a)}{I_n'(\beta a)\, K_n(\beta a)} . \tag{16}$$

We choose the function F in equation (6) as

$$F = -\log \varepsilon' + \log I_n(\beta a) - \log I_n'(\beta a) + \log K_n'(\beta a) - \log K_n(\beta a) ,$$

$F = 0$ obviously yields the same information as (16). The relevant partial derivatives are

$$\frac{\partial F}{\partial \varepsilon'_p} = -\frac{1}{\varepsilon'_p} , \qquad \frac{\partial F}{\partial \varepsilon'} = \frac{1}{\varepsilon'} ,$$

$$\frac{\partial F}{\partial (\beta a)} = \frac{I_n'(\beta a)}{I_n(\beta a)} - \frac{I_n''(\beta a)}{I_n'(\beta a)} + \frac{K_n''(\beta a)}{K_n'(\beta a)} - \frac{K_n'(\beta a)}{K_n(\beta a)} .$$

After some manipulation with Wronskians we obtain

$$\frac{\alpha a}{\beta a}\left[\frac{1}{I_n K_n} + \frac{1}{I_n' K_n'}\left(1 + \frac{n^2}{(\beta a)^2}\right)\right] = \frac{\varepsilon_p''}{\varepsilon_p'} - \frac{\varepsilon''}{\varepsilon'} \tag{17}$$

Using the first three terms of the asymptotic developments of the modified Bessel functions we have

$$\alpha a \sim \left(-\frac{\varepsilon_p''}{\varepsilon_p'} + \frac{\varepsilon''}{\varepsilon'}\right) (\beta a)^2 , \tag{18}$$

irrespective of the mode number n. Finally, after inserting the asymptotic value of the plasma dielectric constant and plasma loss we obtain

$$\alpha a \sim \left(\frac{1+\varepsilon'}{\varepsilon'} \frac{\nu}{\omega} + \mathrm{tg}\delta\right) (\beta a)^2 . \tag{19}$$

Appendix II

A simplified formula expressing the attenuation of surface waves on thin glass tubes can be derived from equation (10). First, the equation is modified to

$$\alpha a = \frac{\omega/\omega_p}{2\, v_g} \left[\frac{\nu}{\omega} + \left(1 - \frac{\omega^2}{\omega_p^2}\right) \frac{\partial \log \varepsilon'_p}{\partial \varepsilon'} \varepsilon''\right] . \tag{20}$$

Then the expressions P_n, Q_n, R_n and S_n entering the dispersion relation (8) are developed in powers of b/a around the point $\frac{b}{a} = 1$. The resulting attenuation formula is

$$a a \simeq \frac{\omega/\omega_p}{2\, v_g} \left\{ \frac{\nu}{\omega} - \beta a \left(\frac{b}{a} - 1 \right) \left(1 - \frac{\omega^2}{\omega_p^2} \right) \frac{K_n'}{\varepsilon' K_n} + \frac{\varepsilon' K_n}{K'_n} \left(1 + \frac{n^2}{\beta^2 a^2} \right) \right\}, \quad (21)$$

where the argument of K_n and K_n' is βa.

References

1) A. W. Trivelpiece and R. W. Gould, J. Appl. Phys., 30 (1959) 1784;
2) P. J. B. Clarricoats, A. D. Olver and J. S. L. Wong, Proc. I. E. E. 113 (1966) 755;
3) Y. Akao, Y. Ida and T. Oike, Proc. VIII Int. Conf. on Phen. in Ionized Gases, Vienna, 1967, p. 344;
4) B. A. Aničin, Bull. of the Boris Kidrič Institute of Nuclear Sciences, 17 (1966) 89;
5) D. Ilić, Bull. of the Boris Kidrič Institute of Nuclear Sciences Physics, 18 (1967) 37;
6) Y. Akao and Y. Ida, J. Appl. Phys., 35 (1964) 2565;
7) Th. Moreno, Microwave Transmission Design Data, p. 202, McGraw-Hill 1948;
8) F. W. Crawford and S. A. Self, Int. J. of Electronics, 18 (1965) 569.

SLABLJENJE ELEKTROMAGNETNIH POVRŠINSKIH TALASA NA STUBU PLAZME

B. A. ANIČIN

Institut »Boris Kidrič«, Beograd

Koeficijent slabljenja površinskih elektromagnetnih talasa izveden je jednom perturbacionom metodom koja omogućuje da se uzme u obzir gubitak u plazmi i gubitak u okružujućem staklu. Analiza je pokazala da koeficijent slabljenja raste sa kvadratom fazne konstante u oblasti malih faznih brzina. Izneseni su numerički podaci za dipolni i aksijalno simetrični tip talasa. Izvršeno je upoređenje teorije sa eksperimentalnim podacima drugih autora.