# COMMAND FILTER ADAPTIVE OUTPUT FEEDBACK CONTROL BASED ON STEEL STRUCTURE ROBOTIC ARM WITH PRESCRIBED PERFORMANCE

Received – Primljeno: 2024-04-24 Accepted – Prihvaćeno: 2024-08-10 Original Scientific Paper – Izvorni znanstveni rad

The paper proposes a command filtering adaptive output feedback control scheme with preset performance for a Robotic Arm Model (RAM) designed specifically for steel structures. Initially, a neural network observer is employed to approximate the nonlinear functions within the model and to estimate the unmeasured states of the system. Subsequently, within the backstepping framework, the integration of preset performance theory and command filtering technology addresses the differential complexity challenges commonly encountered in traditional backstepping methods. This approach ensures rapid convergence of the system's tracking error within predetermined boundaries. The efficacy of this strategy is demonstrated through simulation instances.

Keywords: steel structure, robotic arm, output feedback, command filter, prescribed performance

## **INTRODUCTION**

With the rapid development of the robot industry, steel structure robot arm model (RAM) with series structure has been widely used as an independent robot system in the industrial field [1]. RAM is widely used as an execution unit in service robots, space robots, and other manufacturing sectors, renowned for its high strength and durability acr oss a variety of industrial environments. These robotic arms are particularly suited for tasks such as welding, handling, and quality control, where their rigidity and stability are crucial for operational accuracy and reliability. Additionally, their exceptional adaptability and scalability allow these arms to be customized for specific needs, catering to a diverse range of industrial applications [2].

However, the state of engineering systems such as RAM is often difficult to measure accurately. Therefore, in order to solve the influence of parameter uncertainty and unmeasurable state on the control effect of such non-linear systems, two effective neural network or fuzzy output feedback control strategies are proposed in [3,4].

It should be noted that the backstepping approach in previous studies leads to a problem of differential complexity, which is effectively optimized with the introduction of command filters [5,6]. In addition, most of the previous studies only consider the steady-state performance of the system, which only ensures that the tracking error can eventually converge to a small enough neighborhood near zero, However, the transient

J. H. Zhou (e-mail: zjhzjhan@163.com), D. X. Gao (e-mail: 1074501593@ qq.com), School of Computer Science and Software Engineering, University of Science and Technology Liaoning, Anshan, China. performance of engineering systems is rarely considered. According to the preset performance control schemes proposed for some nonlinear dynamic systems in literature [7,8], it can be seen that improving the transient performance of the system is particularly important in practical applications. In light of this, enhancing the transient performance of RAM is also the key to optimize its overall performance. To address these issues, this paper proposes a command-filtered neural adaptive output feedback control scheme with prescribed performance control.

## STEEL STRUCTURE ROBOTIC ARM MODEL

For the single-link robotic arm made of steel as illustrated in Figure 1, its dynamic behavior can be mathematically modeled with the following equations:

$$M\ddot{q} + \frac{1}{2}mgl\sin(q) = \tau \tag{1}$$

In this model, q corresponds to the angle position,  $\dot{q}$  to the angular velocity,  $\ddot{q}$  to the angular acceleration,  $g = 9.8 m/s^2$  to the acceleration due to gravity, M represents the inertia, l represents the link's length, m represents the link's mass, and  $\tau$  represents the force exerted for control.

In this model,  $x = [x_1,x_1]^T$  denotes state vector, *u* denotes control input, *y* denotes output of the system.

By defining  $x_1 = q$ ,  $x_2 = \dot{q}$  and  $u = \tau$ .

The original equation can be transformed into a state-space form:

$$\begin{cases} \dot{x}_{1} = x_{2} \\ \dot{x}_{2} = \frac{1}{M}u - \frac{mgl\sin(x_{1})}{2M} \\ y = x_{1} \end{cases}$$
(2)

G. F. Cui (e-mail: cuiguofei11@163.com), School of Science, University of Science and Technology Liaoning, Anshan, China.



Figure 1 Schematic Diagram of RAM

Assumption 1: For the above system (2), there are unknown positive constants  $\overline{d}$  and  $\dot{d}$ , such that  $y_d \leq \overline{d}$  and  $\dot{y}_d \leq \dot{d}$ .

**Lemma 1** [4]: The neural networks (NNs) can uniformly approximate the continuous nonlinear function f(x) with arbitrary precision on the compact set  $\Omega_x$  as follows

$$\sup_{x \to 0} |f(x) - W^T \varphi(x)| \le \varepsilon(x)$$
(3)

where  $\varepsilon_x$  is approximate error, and satisfy  $|\varepsilon(x)| < \varepsilon$ .  $\varphi(x)$  is basis function. Due to this approximation capability, we can assume that the nonlinear terms in (2) can be approximated as

$$f(x \mid \hat{W}) = \hat{W}^T \varphi(x) \tag{4}$$

where  $\tilde{W} = W - \hat{W}$ ,  $\hat{W}$  is an estimate of W, and W is a given ideal parametric regression vector, can be expressed as

$$W = \underset{W \in \Omega_{W}}{\operatorname{arg\,min}} \{ \underset{x \in \Omega_{x}}{\sup} | f(x | \hat{W}) - f(x) | \}$$
(5)

where  $\Omega_{W}$  is a compact set.

Lemma 2 [5]: Define the command filter as follows:

$$\begin{cases} \dot{\psi}_1 = \omega_2 \psi_2 \\ \dot{\psi}_2 = -2\zeta \omega_2 \psi_2 - \omega_2 (\psi_1 - \alpha_1) \end{cases}$$
(6)

If the input signal  $\alpha_1$  satisfies that for all  $t \ge 0$ , there are  $|\dot{\alpha}_1| \le \delta_1$  and  $|\ddot{\alpha}_1| \le \delta_2$ , where  $\delta_1$  and  $\delta_2$  are positive constants with  $\Psi_1(0) = \alpha_1(0)$ ,  $\Psi_2(0) = 0$ . Then for any  $\gamma > 0$ , there is  $\omega_2 > 0$ ,  $0 < \zeta < 1$ , such that  $|\Psi_1 - \alpha_1| \le \gamma$ ,  $|\dot{\Psi}_1|$ ,  $|\ddot{\Psi}_1|$  and  $|\ddot{\Psi}_1|$  are bounded.

# NEURAL NETWORK (NN) OBSERVER AND CONTROLLER DESIGN

In this section establishes a NN observer. Subsequently, theory with an adaptive mechanism is designed utilizing the Lyapunov stability analysis method.

### **Neural Network Observer Design**

Selecting an appropriate K ensures that the matrix A is a strictly Hurwitz matrix. It is assumed that the state  $x_2$  of system (2) are not available for feedback. In this situation, to estimate the state of the system, a NN observer is designed for (2) as

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Ky + B\left(\frac{1}{M}u + f(\hat{x} \mid \hat{W})\right) \\ y = C\hat{x} \end{cases}$$
(7)

where 
$$A = \begin{bmatrix} -k_1 & 1 \\ -k_2 & 0 \end{bmatrix}$$
,  $K = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$  and  $C = [1,0]$ .

Moreover,  $\hat{x}$  is the estimate of state vector x,  $f(\hat{x} | \hat{W}) = \hat{W}^T \varphi(\hat{x})$ , and the observer error is defined as  $e = x - \hat{x} = [e_1, e_2]^T$ . Therefore, there exist positive definite matrices P and Q, such that  $A^T P + P^T A = -2Q$ .

From (7), we can obtain

$$\dot{e} = Ae + B\left(\tilde{W}^{\mathsf{T}}\varphi(\hat{x}) + \varepsilon\right) \tag{8}$$

where  $\mathcal{E} = f(x) - f(x | W)$ .

To ensure the boundedness of the (8), choose Lyapunov function as  $V_0 = e^T P e$ .

Then, we have

$$V_0 = e^{\mathrm{T}} (A^{\mathrm{T}} P + P A^{\mathrm{T}}) e + 2e^{\mathrm{T}} P B (\tilde{W}^{\mathrm{T}} \varphi(\hat{x}) + \varepsilon) \quad (9)$$

By using Young's inequality, can calculate

$$2e^{\mathrm{T}}PB\tilde{W}^{\mathrm{T}}\varphi(\hat{x}) \le \|P\|^2 \|e\|^2 + \tilde{W}^{\mathrm{T}}\tilde{W}$$
(10)

$$2e^{\mathsf{T}}PB\varepsilon \le \left\|P\right\|^2 \left\|\varepsilon\right\|^2 + \varepsilon^2 \tag{11}$$

Substituting (10), (11) into (9) yields

$$\dot{V}_0 \le -\beta \|e\|^2 + \tilde{W}^{\mathrm{T}} \tilde{W} + \varepsilon^2$$
(12)

where 
$$\beta = \lambda_{min}(Q) - 2 \|P\|^2$$
.

### **Controller Design**

In order to ensure that the RAM tracking error can converge to a preset set, before designing the controller, let's define a smooth, bounded, and strictly positive performance function  $\mu$  as follows:

$$\mu = (\mu_0 - \mu_\infty)e^{-at} + \mu_\infty$$
(13)

where a,  $\mu_0$  and  $\mu_{\infty}$  are positive design parameter. Then, introduce a smooth and strictly monotonically increasing function  $\Psi(\chi)$  that satisfies  $-\underline{\kappa} < \Psi^{-1} < \overline{\kappa}$ , is defined as follows.

$$\Psi(\chi) = \frac{\overline{\kappa}e^{\chi} - \underline{\kappa}e^{-\chi}}{e^{\chi} + e^{-\chi}}, \ \eta = \mu\Psi(\chi)$$
(14)

where  $\overline{\kappa} > 0$ ,  $\underline{\kappa} > 0$ , tracking error  $\eta = x_1 - y_d$ , according to the strictly monotonically increasing property of function  $\Psi(\chi)$ , its inverse function  $\chi$  can be expressed as

$$\chi = \Psi^{-1}(\frac{\eta}{\mu}) = \frac{1}{2} \ln \frac{\Psi + \underline{\kappa}}{\overline{\kappa} - \Psi}$$
(15)

$$\dot{\chi} = \Pi(\dot{\eta} - \frac{\mu}{\mu}\eta) \tag{16}$$

where  $\chi$  is the error conversion function, and  $\Pi = \frac{1}{2\mu} \left( \frac{1}{\Psi + \underline{\kappa}} - \frac{1}{\Psi - \overline{\kappa}} \right).$  If appropriate parameters are selected, the tracking error satisfies the following conditions:

$$-\underline{\kappa}\mu < \eta < \overline{\kappa}\mu \tag{17}$$

Defining error variables  $z_1 = \chi - \frac{1}{2} \ln \frac{\Delta}{\overline{\kappa}}$ ,  $z_2 = \hat{x}_2 - \gamma$ , where  $\gamma$  is the output of the command filter. In  $z_2$ , the command filter (6) is used to replace the derivative of the virtual control signal in the traditional backstepping design, which can effectively avoid the differential complexity problem. However, research shows that the introduction of command filter will bring errors, so it is necessary to introduce the following compensation mechanism:

$$\dot{\xi}_{1} = -c_{1}\xi_{1} + \Pi(\xi_{2} + \gamma - \alpha_{1}) 
\dot{\xi}_{2} = -\Pi\xi_{1}$$
(18)

where  $\xi_i(i = 1,2)$  is the compensation signal, and  $\xi_i(0) = 0$ . Subsequently, the compensated error is defined as

$$v_i = z_i - \xi_i \ (i = 1, 2)$$
 (19)

**Step 1:** Selecting the Lyapunov function as  $V_1 = V_0 + \frac{1}{2}v_1^2$ .

Then, the time derivative of  $V_1$  yields

$$\dot{V}_{1} = \dot{V}_{0} + v_{1}\dot{v}_{1} 
= \dot{V}_{0} + v_{1}(\dot{z}_{1} - \dot{\xi}_{1}) 
\leq \dot{V}_{0} + \Pi v_{1}(\hat{x}_{2} - \dot{y}_{d} - \frac{\dot{\mu}}{\mu}\eta - \xi_{2} 
-\gamma + \alpha_{1}) + c_{1}v_{1}\xi_{1} 
\leq \dot{V}_{0} + \Pi v_{1}(v_{2} + \alpha_{1} - \dot{y}_{d} - \frac{\dot{\mu}}{\mu}\eta) + c_{1}v_{1}\xi_{1}$$
(20)

Choose the virtual control function  $\alpha_1$  as

$$\alpha_{1} = -\frac{c_{1}}{\Pi} z_{1} + \dot{y}_{d} + \frac{\dot{\mu}}{\mu} \eta$$
 (21)

where  $c_1 > 0$  is design parameter.

By using (21), can be obtain

$$\dot{V}_{1} \leq -\beta \|e\|^{2} + \tilde{W}^{\mathrm{T}}\tilde{W} - c_{1}v_{1}^{2} + \Pi v_{1}v_{2} + \varepsilon^{2}$$
(22)

Step 2: Choose the following Lyapunov function

$$V_{2} = V_{1} + \frac{1}{2}v_{2}^{2} + \frac{1}{2\Gamma}\tilde{W}^{T}\tilde{W}$$
(23)

where  $\Gamma > 0$  is design parameter. The time differentiation of (23) can be represented as

$$\dot{V}_{2} = \dot{V}_{1} + v_{2}\dot{v}_{2} - \frac{1}{\Gamma}\tilde{W}\tilde{W}$$

$$\leq \dot{V}_{1} + v_{2}\left(\frac{1}{M}u + \hat{W}^{\mathsf{T}}\varphi(\hat{x}) + \tilde{W}^{\mathsf{T}}\varphi(\hat{x}) - (24)\right)$$

$$+\varepsilon + c_{2}\xi_{2} + \Pi\xi_{1} - \frac{1}{\Gamma}\tilde{W}\tilde{W}$$

By utilizing the Young's inequality, have

$$v_2 \varepsilon \le \frac{1}{2} v_2^2 + \frac{1}{2} \varepsilon^2 \tag{25}$$

Design the following controller u and adaptive law  $\hat{W}$ :

$$u = M\left(-c_2 z_2 - \frac{1}{2}v_2 - \hat{W}^{\mathsf{T}}\varphi(\hat{x}) - \Pi z_1\right)$$
(26)

$$\hat{\hat{W}} = \Gamma v_2 \varphi(\hat{x}) - \sigma \hat{W}$$
(27)

where  $c_2$  and  $\sigma$  are positive design parameters.

#### STABILITY ANALYSIS

**Theorem 1**: Consider the RAM system (2), under Assumption 1, Lemmas 1-2, if choosing the NN observer (7), the command filter (6), the virtual control function (21), the controller (26), and the adaptive law (27), such that all signals of the closed-loop system are bounded, and the prescribed performance control is realized.

**Proof**: Substituting (22) and (25)-(27) into (24) and carrying out some computations, one has

$$\dot{V}_{2} \leq -\beta \|\boldsymbol{e}\|^{2} + \tilde{\boldsymbol{W}}^{\mathrm{T}} \tilde{\boldsymbol{W}} - \sum_{i=1}^{2} c_{i} v_{i}^{2} + \frac{\sigma}{\Gamma} \tilde{\boldsymbol{W}} \hat{\boldsymbol{W}} + \frac{3}{2} \boldsymbol{\varepsilon}^{2} \quad (28)$$

Utilizing the Young's inequality yields

$$\tilde{W}\hat{W} = \tilde{W}(W - \tilde{W}) \le -\frac{1}{2}\tilde{W}^{\mathsf{T}}\tilde{W} + \frac{1}{2}W^2 \qquad (29)$$

It follows from (29)

$$\dot{V}_{2} \leq -\beta \|e\|^{2} - (\sigma - 2\Gamma) \frac{1}{2\Gamma} \tilde{W}^{\mathsf{T}} \tilde{W} - \sum_{i=1}^{2} c_{i} v_{i}^{2} + B \quad (30)$$

where  $B = \frac{3}{2}\varepsilon^2 + \frac{1}{2}W^2$  From (30), we can see that we only need to let  $\sigma - 2\Gamma > 0$  to conclude that both  $v_i$ and  $\tilde{W}$  are bounded. Then (30) can be rewritten as

$$\dot{V}_2 \le -AV_2 + B \tag{31}$$

where  $A = min\{\beta, 2c_i, \sigma - 2\Gamma, i = 1, 2\},\$ 

According to (19), we know that  $z_1$  is bounded. The prescribed performance of  $\eta$  in the sense of (17) is satisfied for all  $t \ge 0$ .

The proof for Theorem 1 is now complete.

#### SIMULATION EXAMPLE

In this section, a simulation example will be provided for the RAM system. The corresponding system parameters are chosen as  $m = 10 \ kg$ ,  $M = 0.5 \ kg/m^2$ ,  $l = 1 \ m$ , and the reference signal  $y_d$  is selected as  $y_d = \sin(t)$ . In addition, the NN basis functions are chosen to be Gaussian functions uniformly distributed in the range [-5,5].

The control parameters are selected as  $c_1 = 15$ ,  $c_2 = 30$ ,  $k_1 = 15$ ,  $k_2 = 300$ ,  $\Gamma = 2$ ,  $\sigma = 10$ ,  $\zeta = 0.4$ ,  $\omega_2 = 50$ , a = 1,  $\mu_0 = 2.8$ ,  $\mu_{\infty} = 0.1$ , and the initial values are selected as  $x_1(0) = x_2(0) = 0.2$ ,  $\hat{x}_1(0) = \hat{x}_2(0) = -0.2$ ,  $\hat{W}(0) = 0.5$ .

The simulation results are shown in Figures 2 and 3.



**Figure 2** Trajectories of  $x_1 \hat{x}_2, \eta$ 

## CONCLUSIONS

This paper presents a NN adaptive command filter output feedback control scheme for a steel structure RAM with prescribed performance. This scheme can effectively estimate the observed state on the premise that the system can realize the preset performance control. The simulation results show that the control strategy can guarantee both steady state and transient performance of the system. Therefore, this strategy is of great significance for improving the overall performance of RAM and has the potential of practical application.



**Figure 3** Trajectories of  $\hat{W}$  and u

#### REFERENCES

 Yin H, Huang S., He M., et al. An overall structure optimization for a light-weight robotic arm[C]//2016 IEEE 11<sup>th</sup> Conference on Industrial Electronics and Applications (ICIEA). IEEE, (2016), 1765-1770.

- [2] Jahnavi K., Sivraj P. Teaching and learning robotic arm model[C]//2017 International Conference on Intelligent Computing, Instrumentation and Control Technologies (ICICICT). IEEE, (2017), 1570-1575.
- [3] Liu Y. J., Gong M. Z., Tong S. C., et al. Adaptive fuzzy output feedback control for a class of nonlinear systems with full state constraints[J]. IEEE Transactions on Fuzzy Systems, 5 (2018) 26, 2607-2617.
- [4] Liu L., Cui Y., Liu Y. J., et al. Observer-based adaptive neural output feedback constraint controller design for switched systems under average dwell time[J]. IEEE Transactions on Circuits and Systems I: Regular Papers, 9 (2021) 68, 3901-3912.
- [5] X. L. Zheng, X. B. Yang. Command filter and universal approximator based backstepping control design for strictfeedback nonlinear systems with uncertainty[J]. IEEE Transactions on Automatic Control, 3 (2019) 65, 1310-1317.
- [6] Y. X. Li. Finite time command filtered adaptive fault tolerant control for a class of uncertain nonlinear systems[J]. Automatica, (2019) 106, 117-123.
- [7] Q. Liang, Q. M. Yang, W. C. Meng, et al. Adaptive finitetime control for turbo-generator of power systems with prescribed performance[J]. Asian Journal of Control, 4 (2022) 24, 1597-1608.
- [8] H. D. Shan, H. Xue, S. H. Hu, et al. Finite-time dynamic surface control for multi-agent systems with prescribed performance and unknown control directions[J]. International Journal of Systems Science, 2 (2022) 53, 325-336.
- Note: The responsible translators for English language is S. M. Pan-University of Science and Technology Liaoning, China