C. LI, X. G. LI

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FOURIER SERIES METHOD FOR FUNCTION SYNTHESIS OF SPACE RCCC MECHANISM

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Metallurgical equipment needs to use a large number of linkage mechanism to achieve the given movement requirements, compared with the plane four-bar mechanism, the movement of the spatial mechanism is more stable, this paper through the study of the spatial RCCC mechanism, to establish a new method to realize the function synthesis of the mechanism of the algebraic solution, for the spatial mechanism in the metallurgical equipment in a wide range of applications to lay a theoretical foundation.

Keywords: metallurgical machinery, spatial RCCC mechanism, function synthesis, harmonic analysis

INTRODUCTION

Mechanical equipment is an indispensable tool in the metallurgical industry, which has resulted in a significant increase in the efficiency of metallurgical product production. [1,2,3]Robotic automation has resulted in a fourfold increase in powder metallurgy production, and mechanical equipment such as metallurgical furnace loaders and radial forging machines have also been widely used in the metallurgical industry. This shows the importance of machines and equipment in the metallurgical industry, and as the key to machine movement, the design of the mechanism cannot be ignored. Compared with the plane linkage mechanism, the space linkage mechanism has better performance and more stable movement.The spatial RCCC mechanism can convert the angular and displacement motions of the mechanism to each other, which can meet the motion requirements, in metallurgical equipment. [4,5] Mechanism synthesis can design the mechanism to meet the given requirements, and the functional synthesis methods of space link mechanism mainly include analytical method, optimization method and numerical mapping method.

This paper proposes a new method to establish acomprehensive design equation based on the functional relationship between the design parameters of the mechanism and the harmonic characteristic parameters of the motion output. This method over-comes the shortcomings of the traditional algebraic method which cannot realize multi-point function synthesis due to the limitation of the number of unknown points, realizes the algebraic solution of multi-point function synthesis of spatial connecting rod mechanism, and improves the efficiency and accuracy of function synthesis of spatial RCCC mechanism.

C. Li, X. G. Li (E-mail: hblgyjs@126.com). College of Mechanical Engineering, North China University of Science and Technology, Hebei, Tangshan, China.

FOURIER SCALE DESCRIPTION OF OUTPUT ANGLE AND OUTPUT DISPLACEMENT

As shown in Figure 1 is the institutional sketch of the spatial RCCC mechanism, the connecting frame rods *OAB* and *CD* are the input and output links; the rod lengths of the connecting rods(*CD*, *BC*, *EO*, *OA*, *AB*) are a_1, a_2, a_4, S_4, a_3, the angles between the axes s_1, s_2, s_3, s_4 are α_1, α_2, α_3, α_4; the sliding displacements of the connecting rod *OED*, *CD*, *BC* are S_1, S_2, S_3; the input and output angles of the mechanism are φ and Ψ. The point *O* is located at the origin of the coordinates, the *OE* rod is a rack and coincides with the *x-O-y* axis, and the rotated portion of the AB linkage coincides with the x-axis.

Define the output corner function as $f(t) = e^{i\psi t}$, when the input rod *OAB* of the mechanism moves at constant velocity with angular velocity ω, existsp $\varphi=\omega t$, then $f(t)$ is a periodic function, which can be expanded with Fourier series as:

$$f(t) = \sum_{n=-\infty}^{+\infty} c_n e^{in\omega t} = \sum_{n=-\infty}^{+\infty} c_n e^{in\varphi} \tag{1}$$

where $i=\sqrt{-1}$, c_n is the Fourier level coefficient of the output corner function, n is 0,1,...,(M – 1).

It is difficult to compute c_n by analytical method, therefore, numerical integration method is utilized for the computation and the expression of c_n can be obtained according to the nature of discrete Fourier transform:

$$c_n = \frac{1}{M}\sum_{m=0}^{M-1} e^{i\psi_m}[\cos(nm\Delta) - i\sin(nm\Delta)] \tag{2}$$

where n is 0,1..., (M – 1); m is 0,1..., (M – 1); Δ is $\frac{2\pi}{M}$; M is the number of discrete points.

For the spatial RCCC mechanism, the mechanism output displacement varies periodically with the change of the mechanism input angle, which can be expressed as the sum of Fourier series for the thought variables:

$$S_1(t) = \sum_{n=-\infty}^{\infty} c_n' e^{in\varphi} \tag{3}$$

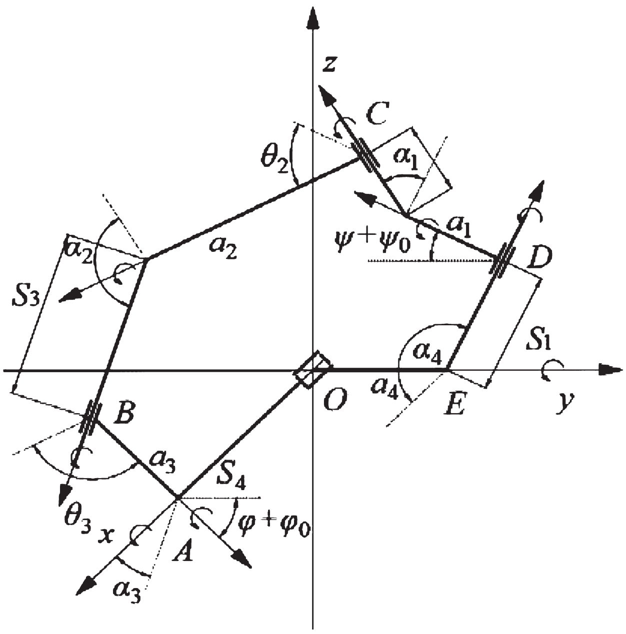

Figure 1 General position diagram of spatial RCCC mechanism

where $i=\sqrt{-1}$, c_n' is the Fourier level coefficient of the output displacement function, n is 0,1,...,(M – 1).

According to the properties of the discrete Fourier transform the expression for c_n' can be obtained as:

$$c_n' = \frac{1}{M}\sum_{m=0}^{M-1} S_1[\cos(nm\Delta) - \mathrm{i}\sin(nm\Delta)] \qquad (4)$$

where n is 0,1..., (M – 1); m is 0,1..., (M – 1); Δ is $\frac{2\pi}{M}$; M is the number of discrete points.

ESTABLISHMENT OF INTEGRATED DESIGN EQUATIONS

In order to facilitate the establishment of the integrated design equations, the clamp angle and rod length parameters of the mechanism are treated separately. For the axis angle α_1, α_2, α_3, α_4 and the starting position φ_0 of the input angle and the starting position $\varPsi_0$ of the output angle, the following relation exists:

$$\begin{aligned}&\sin\alpha_1\sin\alpha_4\cos\alpha_3\cos(\varphi+\varphi_0) - \sin\alpha_1\cos\alpha_4\sin\alpha_3\\&\cos(\varphi+\varphi_0)\cos(\psi+\psi_0)+\sin\alpha_1\sin\alpha_3\sin(\varphi+\varphi_0)\\&\sin(\psi+\psi_0) + \cos\alpha_1\sin\alpha_4\sin\alpha_3\cos(\psi+\psi_0)+\\&\cos\alpha_1\cos\alpha_4\cos\alpha_3 - \cos\alpha_2=0\end{aligned} \qquad (5)$$

Expand $\sin(\varphi+\varphi_0)$, $\cos(\varphi+\varphi_0)$, $\sin(\psi+\psi_0)$, $\cos(\psi+\psi_0)$ using Euler's formula, the following equation can be obtained:

$$\begin{aligned}&l_{-4}\mathrm{e}^{-\mathrm{i}(\varphi-\psi)} + l_{-3}\mathrm{e}^{-\mathrm{i}(\varphi+\psi)} + l_{-2}\mathrm{e}^{-\mathrm{i}\varphi} + l_{-1}\mathrm{e}^{-\mathrm{i}\psi} + l_0 +\\&l_1\mathrm{e}^{\mathrm{i}\psi} + l_2\mathrm{e}^{\mathrm{i}\varphi} + l_3\mathrm{e}^{\mathrm{i}(\varphi+\psi)} + l_4\mathrm{e}^{\mathrm{i}(\varphi-\psi)}=0\end{aligned} \qquad (6)$$

where

$l_{-4} = \frac{1}{4}(1-\cos\alpha_4)\mathrm{e}^{-\mathrm{i}(\varphi_0-\psi_0)}; l_{-3} = -\frac{1}{4}(1+\cos\alpha_4)\mathrm{e}^{-\mathrm{i}(\varphi_0-\psi_0)};$

$l_{-2} = \frac{1}{2}\mathrm{e}^{-\mathrm{i}\varphi_0}\cot\alpha_1\sin\alpha_4; l_{-1} = -\frac{1}{2}\mathrm{e}^{-\mathrm{i}\psi_0}\cot\alpha_3\sin\alpha_4;$

$l_0 = \cos\alpha_4\cot\alpha_1\cot\alpha_3 - \cos\alpha_2\csc\alpha_1\csc\alpha_3;$

$l_1 = -\frac{1}{2}\mathrm{e}^{\mathrm{i}\psi_0}\cot\alpha_3\sin\alpha_4; l_2 = -\frac{1}{2}\mathrm{e}^{\mathrm{i}\varphi_0}\cot\alpha_1\sin\alpha_4;$

$l_3 = -\frac{1}{4}(1+\cos\alpha_4)\mathrm{e}^{\mathrm{i}(\varphi_0+\psi_0)}; l_4 = \frac{1}{4}(1-\cos\alpha_4)\mathrm{e}^{\mathrm{i}(\varphi_0-\psi_0)}.$

[6]From the analysis in the literature, it can be seen that the original function can be better described by using Fourier series to represent the original function by taking a finite number of harmonics, and in the paper, we take the 4th harmonic, $\mathrm{e}^{\mathrm{i}\psi}$ and $\mathrm{e}^{\mathrm{i}\psi}$ as a sum of Fourier series with φ as the variable:

$$\begin{aligned}\mathrm{e}^{\mathrm{i}\psi} &= c_{-4}\mathrm{e}^{-4\mathrm{i}\varphi}+c_{-3}\mathrm{e}^{-3\mathrm{i}\varphi}+c_{-2}\mathrm{e}^{-2\mathrm{i}\varphi}+c_{-1}\mathrm{e}^{-\mathrm{i}\varphi}+c_0\\&+c_1\mathrm{e}^{\mathrm{i}\varphi}+c_2\mathrm{e}^{2\mathrm{i}\varphi}+c_3\mathrm{e}^{3\mathrm{i}\varphi}+c_4\mathrm{e}^{4\mathrm{i}\varphi}\end{aligned} \qquad (7)$$

$$\begin{aligned}\mathrm{e}^{-\mathrm{i}\psi} &= \overline{c}_{-4}\mathrm{e}^{4\mathrm{i}\varphi}+\overline{c}_{-3}\mathrm{e}^{3\mathrm{i}\varphi}+\overline{c}_{-2}\mathrm{e}^{2\mathrm{i}\varphi}+\overline{c}_{-1}\mathrm{e}^{\mathrm{i}\varphi}+\overline{c}_0\\&+\overline{c}_1\mathrm{e}^{-\mathrm{i}\varphi}+\overline{c}_2\mathrm{e}^{-2\mathrm{i}\varphi}+\overline{c}_3\mathrm{e}^{-3\mathrm{i}\varphi}+\overline{c}_4\mathrm{e}^{-4\mathrm{i}\varphi}\end{aligned} \qquad (8)$$

Substituting Equations (7) and (8) into Equation (6) and combining and simplifying them gives the following equation:

$$\begin{aligned}&L_{-5}\mathrm{e}^{-5\mathrm{i}\varphi} + L_{-4}\mathrm{e}^{-4\mathrm{i}\varphi} + L_{-3}\mathrm{e}^{-3\mathrm{i}\varphi} + L_{-2}\mathrm{e}^{-2\mathrm{i}\varphi} + L_{-1}\mathrm{e}^{-\mathrm{i}\varphi}\\&+L_0 + L_1\mathrm{e}^{\mathrm{i}\varphi} + L_2\mathrm{e}^{2\mathrm{i}\varphi} + L_3\mathrm{e}^{3\mathrm{i}\varphi} + L_4\mathrm{e}^{4\mathrm{i}\varphi} + L_5\mathrm{e}^{5\mathrm{i}\varphi} = 0\end{aligned} \qquad (9)$$

where $L_{-5} = l_{-4}c_{-4} + l_{-3}\overline{c}_4$; $L_{-4} = l_1c_{-4} + l_{-4}c_{-3} + l_{-3}\overline{c}_3 + l_{-1}\overline{c}_4$;

$L_{-3} = l_3c_{-4} + l_1c_{-3} + l_{-4}c_{-2} + l_{-3}\overline{c}_2 + l_{-1}\overline{c}_3 + l_4\overline{c}_4$;

$L_{-2} = l_3c_{-3} + l_1c_{-2} + l_{-4}c_{-1} + l_{-3}\overline{c}_1 + l_{-1}\overline{c}_2 + l_4\overline{c}_3$;

$L_{-1} = l_{-2} + l_3c_{-2} + l_1c_{-1} + l_{-4}c_0 + l_{-3}\overline{c}_0 + l_{-1}\overline{c}_1 + l_4\overline{c}_2$;

$L_0 = l_0 + l_3c_{-1} + l_1c_0 + l_{-4}c_1 + l_{-3}\overline{c}_{-1} + l_{-1}\overline{c}_0 + l_4\overline{c}_1$;

$L_1 = l_2 + l_3c_0 + l_1c_1 + l_{-4}c_2 + l_{-3}\overline{c}_{-2} + l_{-1}\overline{c}_{-1} + l_4\overline{c}_0$;

$L_2 = l_3c_1 + l_1c_2 + l_{-4}c_3 + l_{-3}\overline{c}_{-3} + l_{-1}\overline{c}_{-2} + l_4\overline{c}_{-1}$;

$L_3 = l_3c_2 + l_1c_3 + l_{-4}c_4 + l_{-3}\overline{c}_{-4} + l_{-1}\overline{c}_{-3} + l_4\overline{c}_{-2}$;

$L_4 = l_3c_3 + l_1c_4 + l_{-1}\overline{c}_{-4} + l_4\overline{c}_{-3}$; $L_5 = l_3c_4 + l_4\overline{c}_{-4}$;.

Analyzing Equation (9) , it can be found that the expression of L_{-3}, L_{-2}, L_{-1}, L_0, L_1, L_2, L_3 in equation no longer changes when and taking higher harmonic numbers, according to the properties of complex exponential functions, it can be obtained that if Equation (9) is equal to 0, then L_i (i = –3,–2,..., 2,3) should all be 0. Also, in order to eliminate the periodicity of the complex exponential and trigonometric functions, it is defined that $cos\alpha_4 = (1 - t^2)/(1 + t^2)$, $\cot\alpha_3 = q$, $\mathrm{e}^{\mathrm{i}\psi_0} = u$, $\mathrm{e}^{\mathrm{i}\psi_0} = v$, $t = \tan(\alpha_4/2)$, $\sin\alpha_4 = 2t/(1 + t^2)$, $\cot\alpha_1 = r$, which leads to the following equation:

$$\begin{aligned}&2l_0uv - 2l_0t^2uv - t^2u^2v^2c_{-1} - 2qtu^2vc_0 + u^2c_1\\&-t^2\overline{c}_{-1} - 2qtv\overline{c}_0 + v^2\overline{c}_1 = 0\end{aligned} \qquad (10\text{-}1)$$

$$\begin{aligned}&-2rtuv^2 - u^2v^2c_0 - 2qtu^2vc_1 + t^2u^2c_2 - \overline{c}_{-2}\\&-2qtv\overline{c}_{-1} + t^2v^2\overline{c_0} = 0\end{aligned} \qquad (10\text{-}2)$$

$$\begin{aligned}&-2rtu - u^2v^2c_{-2} - 2qtu^2vc_{-1} + t^2u^2c_0 - \overline{c}_0\\&-2qtv\overline{c}_1 + t^2v^2\overline{c}_2 = 0\end{aligned} \qquad (10\text{-}3)$$

$$\begin{aligned}&-u^2v^2c_2 - 2qtu^2vc_3 + t^2u^2c_4 - \overline{c}_{-4} - 2qtv\overline{c}_{-3}\\&+t^2v^2\overline{c}_{-2} = 0\end{aligned} \qquad (10\text{-}4)$$

$$\begin{aligned}&-u^2v^2c_1 - 2qtu^2vc_2 + t^2u^2c_3 - \overline{c}_{-3} - 2qtv\overline{c}_{-2}\\&+t^2v^2\overline{c}_{-1} = 0\end{aligned} \qquad (10\text{-}5)$$

$$-u^2v^2c_{-3} - 2qtu^2vc_{-2} + t^2u^2c_{-1} - \overline{c}_1 - 2qtv\overline{c}_2 + t^2v^2\overline{c}_3 = 0 \quad (10\text{-}6)$$

$$-u^2v^2c_{-4} - 2qtu^2vc_{-3} + t^2u^2c_{-2} - \overline{c}_2 - 2qtv\overline{c}_3 + t^2v^2\overline{c}_4 = 0 \quad (10\text{-}7)$$

Equations (10-4) to (10-7) are containing only the unknown quantity u, v, t, q. The system of equations is built by associating the equations, and the value of u, v, t, q can be obtained by solving with the Mathematica program, and the value of φ_0, Ψ_0, α_3, α_4 can be obtained by solving with the following equations:

$$\alpha_3 = \operatorname{arccot} q, \alpha_4 = \arctan t,$$

$$\varphi_0 = -\mathrm{i}\ln v, \psi_0 = -\mathrm{i}\ln u$$

Based on the known u, v, t, q the associative Equations (10-1) and (10-2) form a system of equations, and the solutions of l_0 and r can be obtained by solving the system of equations:

$$r = (-u^2v^2c_0 - 2qtu^2vc_1 + t^2u^2c_2 - \overline{c}_{-2} - 2qtv\overline{c}_{-1} + t^2v^2\overline{c}_0) / 2tu$$

$$l_0 = (u^2v^2c_{-1} + 2qtu^2vc_0 - t^2u^2c_1 + \overline{c}_{-1} + 2qtv\overline{c}_0 - t^2v^2\overline{c}_1) / [2(1+t^2)uv]$$

α_1 and α_2 can be calculated according to the following equation:

$$\alpha_1 = \operatorname{arccot} r$$

$$\alpha_2 = \arccos[(\cos\alpha_4 \cot\alpha_1 \cot\alpha_3 - l_0)/(\csc\alpha_1 \csc\alpha_3)]$$

The output displacement S_1 of the mechanism is related to the angles θ_2 and θ_3 as follows:

$$S_4 \sin\theta_3 \sin\alpha_3 + a_1\cos\theta_2 + a_2 + a_3\cos\theta_3 + a_4(\cos(\varphi+\varphi_0)\cos\theta_3 - \sin(\varphi+\varphi_0)\sin\theta_3\cos\alpha_3) + S_1\sin\alpha_1\sin\theta_2 = 0 \quad (11)$$

$$\sin\theta_2 = [\sin\alpha_3\sin(\varphi+\varphi_0)\cos(\psi+\psi_0) + \sin\alpha_4 \cos\alpha_3\sin(\psi+\psi_0) + \cos\alpha_4\sin\alpha_3\cos(\varphi+\varphi_0) \sin(\psi+\psi_0)]/\sin\alpha_2 = 0 \quad (12)$$

$$\sin\theta_3 = [\sin\alpha_1\sin(\psi+\psi_0)\cos(\varphi+\varphi_0) + \sin\alpha_4 \cos\alpha_1\sin(\varphi+\varphi_0) + \cos\alpha_4\sin\alpha_1\cos(\psi+\psi_0) \sin(\varphi+\varphi_0)]/\sin\alpha_2 = 0 \quad (13)$$

$$\cos\theta_2 = [-\cos\alpha_3\cos\alpha_4\sin\alpha_1 - \cos\alpha_1\cos\alpha_4 \cos(\psi+\psi_0)\cos(\varphi+\varphi_0)\sin\alpha_3 - \cos\alpha_1\cos\alpha_3 \cos(\psi+\psi_0)\sin\alpha_4 + \cos(\varphi+\varphi_0)\sin\alpha_1\sin\alpha_3 \sin\alpha_4 + \cos\alpha_1\sin\alpha_3\sin(\psi+\psi_0)\sin(\varphi+\varphi_0)]/\sin\alpha_2 \quad (14)$$

$$\cos\theta_3 = [-\cos\alpha_1\cos\alpha_4\sin\alpha_3 - \cos\alpha_3\cos\alpha_4 \cos(\psi+\psi_0)\cos(\varphi+\varphi_0)\sin\alpha_1 - \cos\alpha_3\cos\alpha_1 \cos(\psi+\psi_0)\sin\alpha_4 + \cos(\psi+\psi_0)\sin\alpha_3\sin\alpha_1 \sin\alpha_4 + \cos\alpha_3\sin\alpha_1\sin(\psi+\psi_0)\sin(\varphi+\varphi_0)]/\sin\alpha_2 \quad (15)$$

According to Equation (3), the n^{th} harmonic term is taken to express S_1 as a Fourier series expansion form:

$$S_1 = c_0' + c_1'\mathrm{e}^{\mathrm{i}\varphi} + c_{-1}'\mathrm{e}^{-\mathrm{i}\varphi} + c_2'\mathrm{e}^{2\mathrm{i}\varphi} + c_{-2}'\mathrm{e}^{-2\mathrm{i}\varphi} + \cdots + c_n'\mathrm{e}^{n\mathrm{i}\varphi} + c_{-n}'\mathrm{e}^{-n\mathrm{i}\varphi} \quad (16)$$

Bringing Equations (7), (8), (12) to (16) into (11) and using Euler's formula to expand $\sin(\varphi+\varphi_0)$, $\cos(\varphi+\varphi_0)$, $\sin(\psi+\psi_0)$, $\cos(\psi+\psi_0)$ therein, the following equation can be obtained after combining and simplifying them:

$$T_0 + T_1\mathrm{e}^{\mathrm{i}\varphi} + T_{-1}\mathrm{e}^{-\mathrm{i}\varphi} + T_2\mathrm{e}^{2\mathrm{i}\varphi} + T_{-2}\mathrm{e}^{-2\mathrm{i}\varphi} + T_3\mathrm{e}^{3\mathrm{i}\varphi} + T_{-3}\mathrm{e}^{-3\mathrm{i}\varphi} + \cdots + T_{2n+1}\mathrm{e}^{(2n+1)\mathrm{i}\varphi} + T_{-(2n+1)}\mathrm{e}^{-(2n+1)\mathrm{i}\varphi} = 0 \quad (17)$$

By the nature of the complex exponential, T_n should all be 0. Since the lower harmonics are able to fit the original function better, the associations T_{-2} to T_2 form a system of equations which is a quintic system of equations. Solving the system of equations yields the values of the remaining design parameters α_1, α_2, α_3, α_4 and S_4.

EXAMPLE OF FUNCTION SYNTHESIS

By this method the mechanism is synthesized so that the mechanism input and output angle outputs are in order to satisfy the functional relationship in Figure 2.

Sampling the objective function with $2\pi/64$ as the period, 64 points of the objective function can be obtained, and the Fourier series of the output angle as well as the Fourier series of the output displacement can be obtained by calculation, as shown in Table1.

The value of the obtained Fourier series is brought into the functional synthesis equation of the spatial

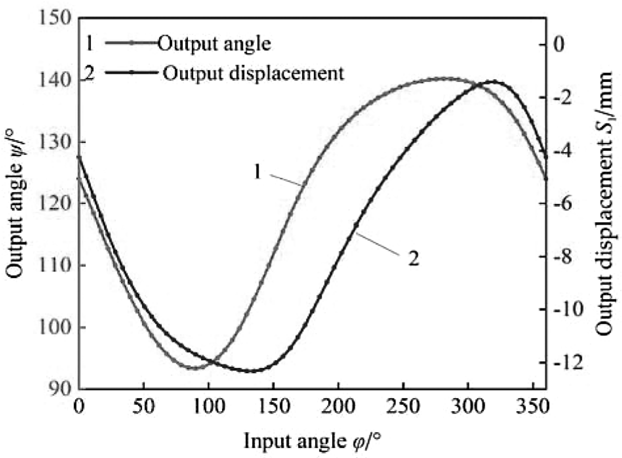

Figure 2 Given output angle and displacement functions

Table 1 **Fourier coefficients of target angle and displacement functions**

Harmonic number	Fourier coefficient c_n	Fourier coefficient c'_n
-4	-0,0024+0,0011i	0,0051-0,0184i
-3	-0,0058+0,0024i	0,0891-0,2231i
-2	-0,0436+0,0074i	0,0500-0,2024i
-1	-0,0930+0,1771i	1,3779-2,3697i
0	-0,4881+0,8228i	-0,7255
1	0,0999-0,1767i	1,3779+2,3690i
2	-0,0350-0,0067i	0,0500+0,2024i
3	0,0096+0,0030i	0,0891+0,2231i
4	0,0999-0,1767i	0,0051+0,0184i

Table 2 **Synthesized design parameters**

Parameters	Value	Parameters	Value	Parameters	Value
α_1	60	φ_0	20	α_3	5,0003
α_2	30	Ψ_0	30	α_4	12,0018
α_3	20	α_1	7,0018	S_4	-0,0008
α_4	65	α_2	10,0005		

RCCC mechanism, and the design parameters of the mechanism can be obtained by solving the system of equations, as shown in Table 2.

The synthesized results obtained are brought into the motion simulation program of the spatial RCCC mechanism to verify the synthesized results. Figure 3 shows the comparison between the synthesized mechanism generated function curves and the given function curves, and Figure 4 shows the error curves of the generated function and the original function of the synthesized mechanism, which show that the synthesized mechanism is able to realize the generation of the given function within the error tolerance, where the angular error and displacement error are denoted by Δa and Δb, respectively.

CONCLUSION

An algebraic solution method for multi-point function synthesis of spatial RCCC mechanism is established, which overcomes the deficiency of the traditional analytical method that cannot accomplish multi-point function synthesis due to the limitation of the number of unknown points. The method is characterized by fast calculation speed, high synthesis precision and easy programming, which improves the efficiency of function synthesis of spatial RCCC mechanism and provides a theoretical basis for the wide application of spatial RCCC mechanism in metallurgical equipment.

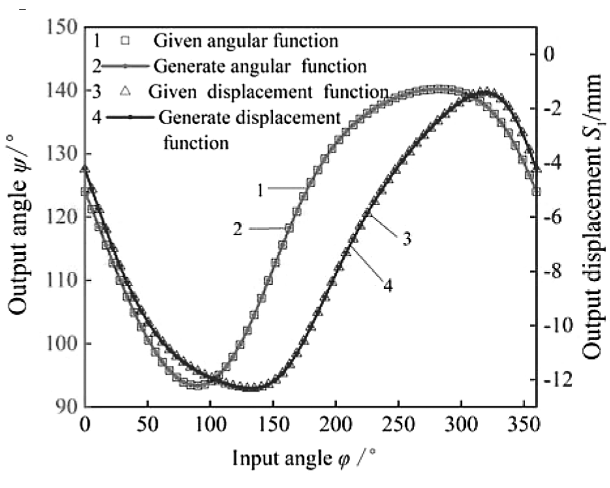

Figure 3 Comprehensive institutional output function curve versus target output function curve

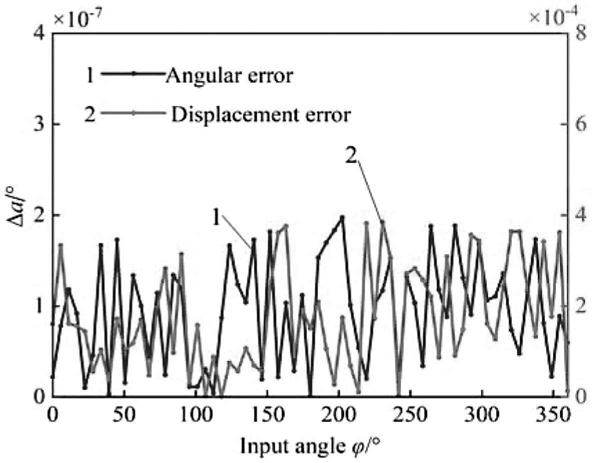

Figure 4 Error curve of synthesized mechanism output function curve versus target output function curve

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