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FEYNMAN DIAGRAMS FOR SPIN 1/2 WAVES IN FERROMAGNETS

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Abstract: A modified Feynman diagram technique for the system of interacting spins 1/2 is developed and applied to the problem of spin waves in ferromagnets. The effective interaction between spins is calculated. The spectral function for this interaction reveals at low temperatures the presence of the elementary excitations, which can be identified with the usual spin wawes. Further, the damping and the interactions between spin waves are studied.

1. Introduction

The solution of the many-body problems in solid state physics has been aided considerably by the application of the field-theoretic techniques developed by Feynman, Schwinger, Dyson, *et al.* for quantum electrodynamics. Unfortunately, a straightforward application of these techniques is possible only for the systems of fermions and bosons. In case of interacting spins one runs into the difficulties because of two reasons. First, the spin operators do not obey the fermion or boson commutation relations. Second, the structure of the spin Hamiltonian is not always appropriate for the perturbation expansions. In order to avoid these difficulties different boson and coupled fermion representations for spin operators were used[1–6]. In the present paper a modified Feynman diagram technique is developed and applied to the problem of spin 1/2 waves in ferromagnets. The scope of the paper is the exposition of the very interesting features and the feasibility of the new approach to the spin wave problem.

In case of spin 1/2 the problem of commutation relations actually does not exist. Pauli spin operators

$$\sigma^+ = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}, \qquad \sigma^- = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}, \qquad \sigma_z = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \tag{1}$$

satisfy the anticommutation relations $[\sigma^+, \sigma^-]_+ = 1$, $[\sigma_z, \sigma_z]_+ = 2$, and $[\sigma^\pm, \sigma_z]_+ = 0$. The fact that the spin operators for different lattice sites commute does not destroy the fermion character of these operators. The spins are mutually distinguishable and therefore can be considered as to belong to different fermion systems.

A little more difficult problem is the form of the Hamiltonian. The diagrammatic expansion is possible only if the individual terms of the Hamiltonian contain an even number of fermion operators for the particular lattice site. We shall investigate as an example the spin waves in a Heisenberg ferromagnet.

2. *Perturbation expansion*

The Hamiltonian of the system to be considered is

$$H = -\frac{1}{2}\sum_{i,j} J_{ij}\left(\sigma_j^- \sigma_i^+ + \sigma_i^- \sigma_j^+\right) - \frac{1}{4}\sum_{i,j} J_{ij}\, \sigma_{i,z}\, \sigma_{j,z} - (g\ \mu_B/2)\, B \sum_i \sigma_{iz}, \tag{2}$$

where J_{ij} is the exchange integral between sites i and j and $(g\,\mu_B/2)$ is the magnetic moment of the spin. The external field B points in z direction. The spin operators are now labeled with the lattice indices. By the use of identities

$$\sigma^- \sigma_z = \sigma^-, \qquad \sigma_z \sigma^+ = \sigma^+, \qquad \sigma_z = 1 - 2\,\sigma^- \sigma^+, \tag{3}$$

we transform (2) in the form

$$H = \omega \sum_i \sigma_i^- \sigma_i^+ - \sum_{i,j} J_{ij}\left(\sigma_i^- \sigma_{iz}, \sigma_{j,z}\, \sigma_j^+ + \sigma_i^- \sigma_i^+ \sigma_j^- \sigma_j^+\right) \tag{4}$$

with

$$\omega = \sum_j J_{ij} + (g\,/\,\mu_B)\ B\ .$$

Here we dropped out the ground state energy term. This type of the Hamiltonian can be obtained also by the use of the coupled fermion representation of spin operators. See for instance the works of Mattis,[3] Kenan[5, 6] and Yung-Li Wang *et al.*[4]. The Hamiltonian (4) is familiar for the system of interacting fermions. The operators σ^- and σ^+ play the role of the creation and annihilation operators, respectively. The operator σ_z is Hermitian and therefore behaves like the creation or annihilation operator, depending on its position in the Hamiltonian.

We can now proceed to develop a perturbation expansion of the thermodynamic Green's functions[7] of spin operators. The functions to be considered are usually one or two particle propagators in the imaginary time τ domain.

In the diagrams the solid directed lines appear which correspond to the contractions between σ^+ and σ^- operators and the undirected dashed lines which describe the contractions between σ_z and σ_z. Further we have two types of the interaction wavy lines. The directed wavy lines correspond to the first and the undirected to the second interaction term in (4). The possible interactions are shown in Fig. 1. Following the Kenan's proposal[6] we call the »directed« interactions also the »spin-flip« interactions and the »undirected« the »longitudinal« interactions.

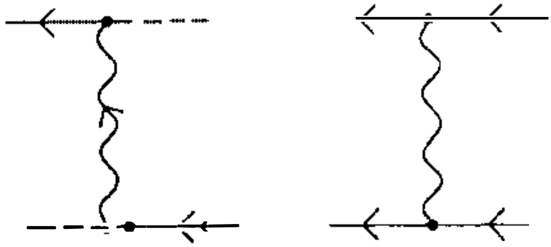

Fig. 1. Allowed spin-spin interactions.

The rules for the calculation of the contribution of the particular diagram are as follows:

1. Each directed particle line gives a factor

$$\frac{1}{z - \omega}\,; \qquad z = (2n + 1)\,\pi i/\beta\,, \qquad \beta = 1/k\,T\,,$$

where z is the frequency associated with the line. This is the Fourier transform of $- \langle T\,\sigma^+(\tau)\,\sigma^-(0)\rangle_0$.

2. Each undirected particle line a factor $2/z$, which is the Fourier transform of $- \langle T\;\sigma_z(\tau)\,\sigma_z(0)\rangle_0$.

3. Each interaction line a factor $- J_{ij}$.

4. At all vertices the sum of the frequencies has to be conserved. The frequency ascribed to the dashed line depends on the choice of the fictitious direction of this line. This choice is completely arbitrary and does not affect the result.

5. To obtain the contribution of the diagram the product of the factors 1–3 is multiplied by $(-1)^{n+f+m}\,\beta^{-n}$ and the summations over the free frequencies are performed. Here n and f are the number of the free frequencies and the fermion loops in the diagram, respectively. The number m is determined as follows. First we consider the diagrams which are build up from the fermion

loops. For each of the loops we choose a direction of the encircling in such a way that it coincides with the direction of one of the directed fermion lines. If we go now around the loop in this direction and we meet another line which points in the opposite direction this contributes the unity to m. The number m is equal to the number of times this happens in all loops. If there are two or more directed lines with the wrong direction which are not separated by the dashed lines their wrong direction is counted only once. Now, if the diagram has also the incoming and outgoing fermion lines, we determine the sign of the diagram by choosing the direction of motion along the chain of the fermion lines in the same way as for the loops.

The essential features of the diagram technique presented here were given by the author[8] in 1965 in connection with the problem of the impurity conduction in semiconductors at high frequencies. This is a problem of the two level system occupied by one electron. Later Yung-Li Wang *et al.*[9, 4] developed a perturbation expansion for spin $^1/_2$ systems, which is a little more complicated than the expansion presented here because of the appearance of the »lock« diagrams. Kenan[6] has given recently a perturbation expansion for the spin $^1/_2$ anti-ferromagnetism, which is based on the coupled fermion representation of spin operators. This method is essentially identical with ours, although the rules for the diagram calculations are to some extent different.

3. *Spin waves*

We now proceed to derive the elementary excitations of the ferromagnet. We shall exploit the formal similarity of the problem to the case of interacting fermions. As an analogy to the dielectric constant problem of the fermion system the question of the effective »spin-flip« interaction between spins at i and j lattice sites, respectively, arises. In addition to the direct interaction $-J_{ij}$ we have also the possibility of the indirect interaction through the intermediate sites l. The low order diagrams for the effective interaction $-J_{ij}$ are shown in Fig. 2. The contribution of the second order diagram is for instance

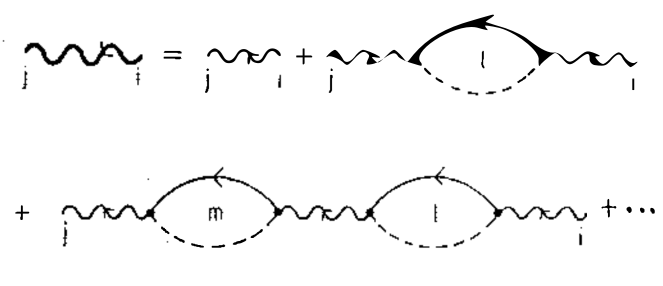

Fig. 2. Low order diagrams for the effective interaction $-J_{ij}$

$$J_{il}\, J_{lj}\, \beta^{-1} \sum_{x} \frac{1}{z + x - \omega} \frac{2}{x} = J_{il}\, J_{lj} \operatorname{tgh}(\beta\omega/2) \frac{1}{z - \omega}, \tag{5}$$

where $z = 2\, n\, \pi\, i/\beta$. The total contribution of the second order diagrams is obtained if we add up the terms (5) corresponding to different l. It is convenient to introduce the Fourier transform of J_{ij}

$$J(\vec{k}) = \sum J_{ij} \exp\left[-i\, \vec{k} \cdot (\vec{r_j} - \vec{r_i})\right], \tag{6}$$

where $\vec{r_i}$ is the lattice vector of the site i and $J_{ii} = 0$. The summation over l is now easy and we get

$$\operatorname{tgh}(\beta\omega/2) \frac{1}{z - \omega} \frac{1}{N} \sum_{k} J^2(\vec{k}) \exp\left[i\, \vec{k} \cdot (\vec{r_j} - \vec{r_i})\right]. \tag{7}$$

In a similar way a contribution of the diagrams of higher orders can be calculated. Therefore

$$\tilde{J}_{ij} = \frac{1}{N} \sum_{k} J(\vec{k}) \exp\left[i\, \vec{k} \cdot (\vec{r_j} - \vec{r_i})\right] \left\{ \sum_{n=0} \left[\frac{J(\vec{k}) \operatorname{tgh}(\beta\omega/2)}{z - \omega} \right]^n \right\} =$$

$$= \frac{1}{N} \sum_{\vec{k}} \frac{J(\vec{k})\,(z - \omega)}{z - \omega(\vec{k})} \exp\left[i\, \vec{k} \cdot (\vec{r_j} - \vec{r_i})\right], \tag{8}$$

where

$$\omega(\vec{k}) = \omega - J(\vec{k}) \operatorname{tgh}(\beta\omega/2). \tag{9}$$

The equivalent to the dielectric constant $\varepsilon(z, \vec{k})$ in case of spins is

$$\frac{1}{\varepsilon(z, \vec{k})} \rightarrow \frac{z - \omega}{z - \omega(\vec{k})}. \tag{10}$$

The pole $z = \omega(\vec{k})$ determines the frequency of the spin wave $\vec{k}$ excitation of the ferromagnet. At low temperature $\beta\,\omega >> 1$ the well known expression for the spin wave frequency $\omega(\vec{k}) = \omega - J(\vec{k})$ is obtained. The Fourier transform of the effective $\tilde{J}_{ij}$ is

$$\tilde{J}(z, k) = J(\vec{k}) \cdot \frac{z - \omega}{z - \omega(\vec{k})} . \tag{11}$$

It is clear now that we can associate the wave vector $\vec{k}$ with the effective interaction lines and consider these lines as the magnon lines. The factor $1/[z - \omega(\vec{k})]$ in (11) is nothing else as the zero approximation for the given spin wave propagator.

One might look also for the similar modification of the »longitudinal« interaction. The polarization buble in this case is

$$\beta^{-1} \sum_{x} \frac{1}{z + x - \omega} \cdot \frac{1}{x - \omega} = \begin{cases} 0, \text{ if } z \neq 0 \\ - \beta \exp(\beta\omega) / [\exp(\beta\omega) + 1]^2, \text{ if } z = 0. \end{cases} \tag{12}$$

The correction appears only at the zero frequency transfer and is small at low temperatures. In the following we shall neglect such correction terms.

4. *Magnon – magnon interactions*

In this section we shall discuss the interactions between magnons and the higher order approximations for the polarization operator of the »spin-flip« interaction. The main interest is in the damping of spin waves.

The mutual scattering of magnons is produced by the »spin-flip« and »longitudinal« interactions. The lowest order interaction diagram for the »longitudinal interactions is very simple and is shown in Fig. 3. The frequencies z and wave vectors $\vec{k}$ of the incoming and outgoing magnons are labeled from 1 to 4. The interaction takes place at lattice sites i and j. Because of the conservation of the frequencies at the vertices one has $z_1 + z_2 = z_3 + z_4$. The contribution of this diagram without the external magnon lines is

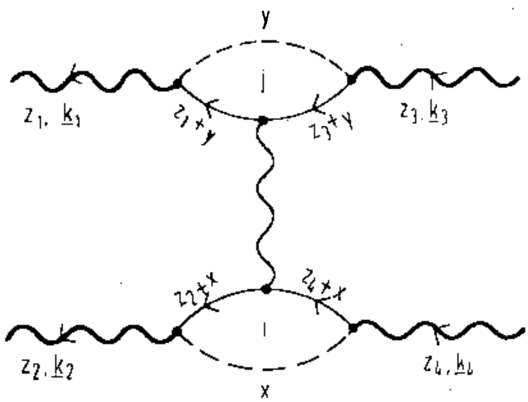

Fig. 3. Diagram for the »longitudinal« magnon-magnon interaction.

$$F(1,2;3,4) = \beta^{-2}\left(\sum_x \frac{1}{z_2+x-\omega}\,\frac{1}{z_4+x-\omega}-\frac{2}{x}\right) J_{ij} \times$$
$$\times\left(\sum_y \frac{1}{z_1+y-\omega}\,\frac{1}{z_3+y-\omega}-\frac{2}{y}\right) = \tag{13}$$
$$= J_{ij}\,\mathrm{tgh}^2(\beta\omega/2)\,\frac{1}{(z_1-\omega)(z_2-\omega)(z_3-\omega)(z_4-\omega)}\,.$$

If we assigne only the factor $-\tilde{J}(z,\vec{k})$ to each magnon line then $F(1,2;3,4)$ should be multiplied by the remaining magnon line phase factors, which correspond to the lattice sites i and j, namely by

$$\exp\left\{i\left[(\vec{k}_3-\vec{k}_1)\cdot\vec{r}_j+(\vec{k}_4-\vec{k}_2)\cdot\vec{r}_i\right]\right\}. \tag{14}$$

We include also a factor $N^{-1/2}$ for each end of the magnon line. The total contribution of the magnon – magnon scattering vertex after the summation over i and j is therefore

$$\Phi_l(1,2;3,4) = \frac{1}{N} J(\vec{k}_1-\vec{k}_3)\,\mathrm{tgh}^2(\beta\omega/2)\,\frac{1}{(z_1-\omega)(z_2-\omega)(z_3-\omega)(z_4-\omega)} \tag{15}$$

together with the crystal momentum conservation requirement $\vec{k}_1+\vec{k}_2-\vec{k}_3-\vec{k}_4=\vec{g}$, where $\vec{g}$ is an arbitrary reciprocal lattice vector.

The possibility of the permutation of the indices 1 and 2 or 3 and 4 requires at the applications the symmetrization of the scattering vertex function (15). The symmetrized form of $\Phi_l(1,2;3,4)$ is

$$\Phi_l(1,2;3,4) = \frac{1}{4N}\Big[J(\vec{k}_1-\vec{k}_3)+J(\vec{k}_1-\vec{k}_4)+J(\vec{k}_2-\vec{k}_3)+$$
$$+J(\vec{k}_2-\vec{k}_4)\Big]\,\mathrm{tgh}^2(\beta\omega/2)\,\frac{1}{(z_1-\omega)(z_2-\omega)(z_3-\omega)(z_4-\omega)}. \tag{16}$$

We proceed to the »spin-flip« type magnon-magnon interactions. Two lowest order diagrams of this kind are shown in Fig. 4. The contributions of the vertices a) and b) are

$$\beta^{-1}\sum_x \frac{1}{x+z_3-\omega}\,\frac{1}{x+z_3-z_2}\,\frac{1}{x+z_1-\omega}\,\frac{2}{x}, \tag{17}$$

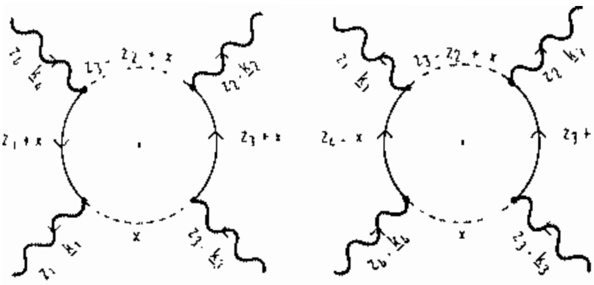

Fig. 4. Diagrams for the »spin-flip« magnon-magnon interactions.

and

$$-\beta^{-1}\sum_{x}\frac{1}{x+z_3-\omega}\,\frac{1}{x+z_3-z_2}\,\frac{1}{z_4-x-\omega}\,\frac{2}{x}, \tag{18}$$

respectively. Note the sign minus in case b) due to the change in the direction of one of the directed fermion lines. The analog to the interaction vertex function Φ_l (1, 2; 3, 4) is here

$$\Phi_a(1,2;3,4)=\frac{2}{N}\,\mathrm{tgh}\,(\beta\omega/2)\,\frac{2\omega-z_1-z_2}{(z_1-\omega)(z_2-\omega)(z_3-\omega)(z_4-\omega)}, \tag{19}$$

and

$$\Phi_b(1,2;3,4)=\frac{2}{N}\,\mathrm{tgh}\,(\beta\omega/2)\,\frac{(z_1-\omega)(z_4-\omega)+(z_2-\omega)(z_3-\omega)}{(z_1-\omega)(z_2-\omega)(z_3-\omega)(z_4-\omega)(z_3+z_4-2\omega)}, \tag{20}$$

respectively for the cases a) and b). The formula (19) is already symmetric with respect to the indices 1 and 2 or 3 and 4, if one takes into account the fact that $z_1+z_2=z_3+z_4$. The symmetrization of (20) gives a simple result

$$\Phi_b(1,2;3,4)=-\frac{1}{2}\,\Phi_a(1,2;3,4). \tag{21}$$

Having derived the vertex functions for the two magnon interactions we can now proceed to the calculation of the magnon damping. Two lowest order diagrams for the polarization operator, which produce the damping, are shown in Fig. 5. Further diagrams are obtained by drawing the corresponding topologically different diagrams. For instance, there are two ways of connecting the vertices i and j with the magnon lines in case a) of Fig. 5. In case b) one can interchange in addition to the magnon lines the longitudinal interaction lines too. We have therefore 2 topologically different diagrams of type a) and 4 of type b). In case a) also the possibility of replacing the vertices of type a) by the vertices of type b) of Fig. 4 exists.

As an illustration we calculate the contribution of the diagram a) without two external lines. We obtain after the summation over all possible positions i and j the following expression for this contribution

$$-\frac{4}{\mathcal{N}^2}\,\mathrm{tgh}^2(\beta\omega/2)\,\beta^{-2}\sum_{z_1,z_2;\,\vec{k}_1,\vec{k}_2} J(\vec{k}_1)\,J(\vec{k}_2)\,J(\vec{k}_3)\times$$

$$\times\frac{(2\omega-z-z_1)^2}{(z-\omega)^2(z_1-\omega)(z_2-\omega)(z_3-\omega)}\times\frac{1}{[z_1-\omega(\vec{k}_1)][z_2-\omega(\vec{k}_2)][z_3-\omega(\vec{k}_3)]}\times$$

$$\times\,\Delta(z+z_1-z_2-z_3)\,\Delta(\vec{k}+\vec{k}_1-\vec{k}_2-\vec{k}_3+\vec{g}),\qquad(22)$$

where Δ is a Kronecker delta symbol. Other diagrams give a similar contribution. The sum $g(z,\vec{k})$ of these contributions is a correction to the lowest order polarization bubble $\mathrm{tgh}\,(\beta\,\omega/2)\,/\,(z-\omega)$. The damping is determined by the imaginary part of $J(\vec{k})\,(z-\omega)\,g(z,\vec{k})$ at pole $z=\omega(\vec{k})+i\,\varepsilon$. At low temperatures only the low frequency magnons are excited and therefore it is enough to consider the singularities produced by the cuts of the diagrams in

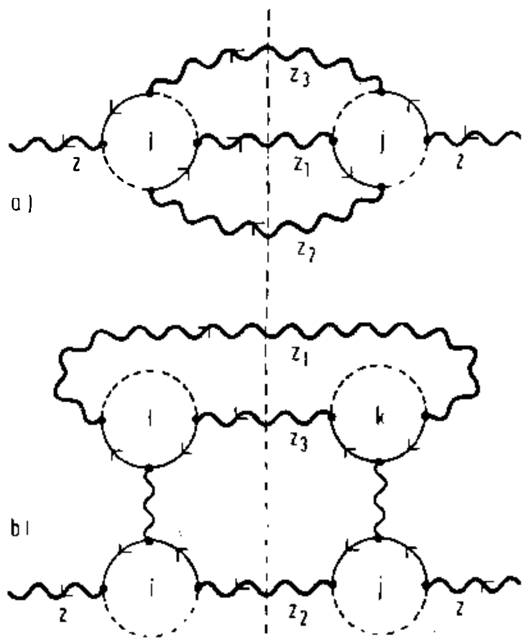

Fig. 5. Lowest order diagrams for the magnon damping.

Fig. 5 along the dotted lines. This means that we are allowed to substitute in the first fraction after the summation sign in (22) for frequencies the values $z_1 = \omega(\vec{k}_1)$, $z_2 = \omega(\vec{k}_2)$, and $z_3 = \omega(\vec{k}_3)$. Therefore the formula (22) is replaced by

$$\frac{4}{N^2}\frac{1}{(z-\omega)^2}\sum_{\vec{k}_1,\vec{k}_2}[J(\vec{k})+J(\vec{k}_1)]^2\frac{n_1(n_2+1)(n_3+1)-n_2n_3(n_1+1)}{z+\omega(\vec{k}_1)-\omega(\vec{k}_2)-\omega(\vec{k}_3)}\times$$
$$\times\Delta(\vec{k}+\vec{k}_1-\vec{k}_2-\vec{k}_3+\vec{g}), \tag{23}$$

where $n_i = \{\exp[\beta\,\omega(\vec{k}_i)]-1\}^{-1}$.

The calculation of the damping constant $\Gamma(\vec{k})$ is now straightforward and we obtain after the collection of the contributions of all possible diagrams the following formula

$$\Gamma(\vec{k}) = \lim_{z\to\omega(\vec{k})+i\varepsilon} J(\vec{k})(z-\omega)\,g(z,\vec{k}) = \frac{\pi}{2N^2}\sum_{\vec{k}_1,\vec{k}_2}[J(\vec{k})+J(\vec{k}_1)+$$
$$+J(\vec{k}_2)+J(\vec{k}_3)-J(\vec{k}-\vec{k}_2)-J(\vec{k}-\vec{k}_3)-J(\vec{k}_1-\vec{k}_2)-J(\vec{k}_1-\vec{k}_3)]^2\times$$
$$\times[n_1(n_2+1)(n_3+1)-n_2n_3(n_1+1)]\,\delta[\omega(\vec{k})+\omega(\vec{k}_1)-\omega(\vec{k}_2)-\omega(\vec{k}_3)]\times$$
$$\times\Delta(\vec{k}+\vec{k}_1-\vec{k}_2-\vec{k}_3+\vec{g}). \tag{24}$$

This result agrees with the calculations[10] based on the Holstein-Primakoff representation of spin operators. In the boson representation the lowest order coupling between magnons is given by

$$H_{int} = \frac{1}{4N}\sum_{\vec{k},\vec{k}_1,\vec{k}_2,\vec{k}_3}[J(\vec{k})+J(\vec{k}_1)+J(\vec{k}_2)+J(\vec{k}_3)-J(\vec{k}-\vec{k}_2)$$
$$-J(\vec{k}-\vec{k}_3)-J(\vec{k}_1-\vec{k}_2)-J(\vec{k}_1-\vec{k}_3)]\,a^+(\vec{k}_3)\,a^+(\vec{k}_2)\,a(\vec{k}_1)\,a(\vec{k})\times$$
$$\times\Delta(\vec{k}+\vec{k}_1-\vec{k}_2-\vec{k}_3+\vec{g}), \tag{25}$$

where $a^+(\vec{k})$ and $a(\vec{k})$ are boson creation and annihilation operators, respectively. The use of (25) in the rate equation for the magnon decay gives for $\Gamma(\vec{k})$ the expression (24).

5. Conclusion

It has been shown in previous chapters that there exists a striking formal resemblance between the problems of interacting spins and interacting fermions. This analogy could be pursued further. For instance, the problem which can be solved without difficulty is the Hartree approximation for the spin system. This essentially leads to the Weiss internal field approximation. The Weiss field is produced only by the »longitudinal« interactions. On the contrary the Fock exchange scattering is due to the »spin-flip« interactions. We shall not enter into these problems here.

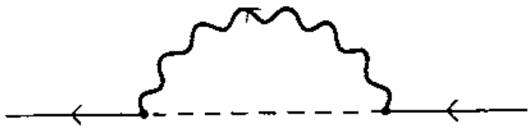

Fig. 6. Diagram for the magnon contribution to the magnetization.

Let us treat a little more in detail only the magnetization at low temperatures. One has to study the thermodynamic average $\langle \sigma^- \sigma^+ \rangle$ which can be obtained from the Green's function

$$g(\tau) = -\langle T \sigma^+ (\tau)\, \sigma^- (0) \rangle \,. \tag{26}$$

The contribution of the magnon excitations to the magnetization is calculated from the second order diagram for $g(\tau)$, as shown in Fig. 6. The value of this diagram is

$$\frac{1}{N} \sum_{\vec{k}} \operatorname{ctgh} [\beta\, \omega (\vec{k})/2]\, J^2 (k) \, \frac{1}{(z-\omega)^2 [z - \omega (\vec{k})]} \,. \tag{27}$$

The zero and the second order diagram lead therefore to the following result

$$g(\tau = -0) = \langle \sigma^- \sigma^+ \rangle = [\exp(\beta\omega) + 1]^{-1} + \frac{1}{N} \sum_{\vec{k}} \operatorname{ctgh} [\beta\omega (\vec{k})/2] \times$$
$$\times \left\{ \{\exp [\beta\omega (\vec{k})] + 1\}^{-1} - [\exp(\beta\omega) + 1]^{-1} \right\} \simeq \frac{1}{N} \sum_{k} \{\exp [\beta\omega (\vec{k})] - 1\}^{-1}. \tag{28}$$

This is a usual spin-wave result.

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FEYNMANOVI DIJAGRAMI ZA SPIN 1/2 VALOVE V FEROMAGNETIKIH

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Vsebina

Posebna metoda modificiranih Feynmanskih diagramov je uporabljena pri študiju spinskih valov v feromagnetikih s spinom 1/2. Komutacijska pravila za spinske operatorje in možnost zapisa Hamiltonove funkcije (2) v bilinearni obliki (4) dopuščajo uporabo diagramske tehnike, ki se le malo razlikuje od tehnike Feynmanskih diagramov za fermionske sisteme. S to metodo je najprej izračunana efektivna interakcija med različnimi spini v kristalu in njena spektralna funkcija. Nekateri diagrami nižjega reda za interakcijo so pokazani na sl. 2. Spektralna funkcija (8) in njena krajevna Fourierova transformacija (11) kažeta na prisotnost elementarnih ekscitacij, ki imajo značaj valov in jih je mogoče identificirati kot spinske valove. Izračunana frekvenca (9) se pri nizkih temperaturah ujema s klasičnimi rezultati.

Opisana metoda da že v najnižjem redu temperaturno odvisnost frekvence. V nadaljevanju so izračunani tudi popravki višjih redov za spektralno funkcijo z upoštevanjem interakcij med spinskimi valovi. Osnovni diagrami za interakcijo med spinskimi valovi so razvidni iz sl. 3 in 4. Upoštevanje takih interakcij omogoča izračunati dušenje spinskih valov. Sl. 5 kaže ustrezne diagrame, formula (24) pa predstavlja izraz za dušenje. V nizko temperaturni aproksimaciji se naši rezultati ujemajo s klasičnimi.

V zaključku je predložena metoda uporabljena za račun temperaturne odvisnosti magnetizacije.

V delu je pokazano, da obstaja izredna formalna analogija med problemi interagirajočih fermijonov na eni strani in interagirajočih spinov na drugi strani. Dalje omogoča ta metoda sistematičen študij lastnosti feromagnetikov s spinom 1/2 tudi pri višjih temperaturah.