GREEN'S FUNCTION METHOD IN THE THEORY OF ELECTRON – PHONON INTERACTION

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Abstract: The effect of electron-phonon interaction on electron motion has been calculated with the Green's function method. It is assumed that the whole system is composed of electrons with screened Coulomb interaction and of longitudinal phonons with an energy $\hbar Sq$, where S is the velocity of sound. This effect has been calculated to the second powers in the interaction of the self energy part of electrons. A method is developed which enables one to consider the convenient application of Green's functions to the problem. The calculation of Fourier transform of the Green's function $G(k,\tau)$ has been carried out by using the ordinary perturbation method where annihilation operator is expanded in power series of time. Agreement is shown with a theory that uses Wick's theorem. Such an application of the perturbation method enables us to avoid the adiabatic hypothesis.

A. PERTURBATION CALCULATION OF GREEN'S FUNCTIONS

1. Introduction

The problem of electron-phonon interaction has been investigated by many authors, especially in connection with the problem of superconductivity. Bardeen[1] extended the self consistent field method to take into account the motion of the ions and thus to determine the matrix element for electron-lattice interactions. The electron-electron interactions were treated in the Hartree approximation. Fröhlich[7] used perturbation theory to investigate interaction between electrons and lattice vibrations in a normal metal. He considered an isotropic model of a metal described by the Hamiltonian in which annihilation and creation operators of electrons and phonons appeared. Nakajima[2] presented a simple field-theoretical treatment of the problem using

also the Hartree approximation which gives results essentially equivalent to Bardeen's. A simplified basic Hamiltonian to describe the electron-phonon interaction in metals has been derived by Bardeen and Pines[3]. The energies of the elementary excitations and effective electron interaction for the system were determined in the random-phase approximation. The influence of electron interactions to the longitudinal phonon frequency was investigated by Staver and Bohm[4]. Taking the interaction between ions to be pure Coulomb, the authors obtained simple dispersion relation which was in good agreement with experimental result. Migdal[5] has solved the problem of an electron interacting with a phonon field to order m/M in the approximation where electron-electron interaction was neglected. Migdal solved the self energy equation which corresponded to the sum of all electron-phonon self-energy diagrams. Finally, Pines[6] showed that Migdal's results may be transposed to the case in which eletron-electron interaction is included.

There is nothing new we want to introduce to the problem of electron-phonon interaction, but the method developed in the present article we find of interest in itself. In other words, it seems to us to be interesting to present the problem in somewhat different form. Our approach to the problem of electron-phonon interaction enables one to consider the influence of the electron-phonon interaction to the electron interaction with methods of quantum field theory by introducing the Green's function of electron. We work out the perturbation theory for Fourier transform of electron Green's function $G(k, \tau)$ by expanding annihilation operator in power series of time. This perturbation expansion does not rest on Wick's theorem and on the adiabatic hypothesis. The calculation is carried out to the second order powers of interaction. In result we get the self-energy in Feynman diagrams which are entirely equivalent to those of the general theory in which Wick's theorem is applied.

In the part B. of the present article the method formulated in the part A. is applied. We start with simplified basic Hamiltonian to describe the electron-phonon interaction in metals and a number of approximations are made in order to simplify the calculations. We derive for a monatomic crystal the Hamiltonian expressed in the usual second quantized notation and we use it in subsequent discussion. The result of the part B. shows that the self energy of electrons to the second power in the interaction consists of a contribution of screened Coulomb interaction and of the electron-phonon interaction. An analysis of the expression we get shows that the contribution to the electron-phonon interaction comes from absorption of phonons by electrons. On the basis of these calculations an estimate of the interaction can be done.

2. The calculation of Green's function to the second order in the interaction h

We introduce the electron Green's function[8]

$$G(k, \tau) = -i < 0 \,|\, T\,[\, a_k(\tau)\, a_k^+(0)\,]\,|\, 0 > \qquad (2.1)$$

$(k \equiv$ wave vector)

and Fourier transform:

$$G(k, k_0) = \int_{-\infty}^{\infty} e^{\,i\,(kx - k_0\tau)}\, G(x, \tau)\, d^3x\, d\tau$$

$$G(x, \tau) = \int_{-\infty}^{\infty} e^{\,-i\,(kx - k_0\tau)}\, G(k, k_0)\, \frac{d^3k\, dk_0}{(2\pi)^4} \qquad (2.2)$$

The use of perturbation method for Green's function is based on two assumptions. If Hamiltonian has a form $H = H_0 + h$ one takes that the ground state of the Hamiltonian H_0 describing bare particles (Bloch electrons or bare phonons) approaches adiabatically to the ground state of a system with interaction, where the interaction is introduced in time adiabatically. The second assumption is that resulting time dependence of the ground state and dependence on τ of the function G, because of the presence of the interaction, can be expanded in series with respect to the interaction. H_0 is simple enough to obtain corresponding Green's function explicitly.

The adiabatic hypothesis enables us to calculate the Green's function of the Hamiltonian H in terms of the Green's function of the Hamiltonian H_0[9].

We will take a different approach. We shall proceed with the calculation of Fourier transform of the Green's function, performing perturbation calculation for the wave function in order to express it in terms of free particle wave functions. Therefore, we shall use Rayleigh-Schrödinger perturbation calculation.

Expanding $a_k(\tau)$ at zero, we get

$$a_k(\tau) = a_k(0) + \dot{a}_k(0)\,\tau + \ddot{a}_k(0)\,\frac{\tau^2}{2!} + \ldots, \qquad (2.3)$$

where

$$\dot{a}_k = -i\,\varepsilon_k\,a_k + i\,[h, a_k]\ ,$$

$$\ddot{a}_k = (-i\,\varepsilon_k)^2\,a_k + (-i\varepsilon_k)\,i\,(h, a_k) + i^2\,[H_0, [h, a_k]] + i^2\,[h, [h, a_k]]\ \text{etc.}$$

$$(2.4)$$

$H = H_0 + h$; the free particle Hamiltonian has the form $H_0 = \sum_k \varepsilon_k a_k^+ a_k$, h is the interaction Hamiltonian. We shall work with a two particle inter-action

$$h = \sum_{k_1 k_2 k_3 k_4} < k_1\,k_2\,|\,h\,(12)|\,k_3\,k_4 > a_{k1}^+\,a_{k2}^+ a_{k1}\,a_{k2}\ , \tag{2.5}$$

so that

$$[h, a_k] = \sum_{k_1 k_2 k_3 k_4} \{< k_1\,k\,|\,h\,(12)\,|\,k_2\,k_3 > - < k\,k_1\,|\,h\,(12)\,|\,k_2\,k_3 >\}\ \cdot \tag{2.6}$$

$$\cdot\ a_{k1}^+\,a_{k_3}\,a_{k_2}\ \cdot$$

If we introduce the notation:

$$< k_1\,k\,|\,h\,(12)\,|\,k_2\,k_3 > - < k\,k_1\,|\,h\,(12)\,|\,k_2\,k_3 > \equiv (1k\,2\,3)\ , \tag{2.7}$$

(and analogous to it), (2.6) will be

$$[h, a_k] = \sum_{k_1 k_2 k_3} (1k\,2\,3)\,a_{k1}^+\,a_{k_3}\,a_{k_2}\ \cdot \tag{2.8}$$

Analogous we have:

$$[H_0, [h, a_k]] = \sum \left\{\varepsilon_{k_1} - \varepsilon_{k_2} - \varepsilon_{k_3}\right\} (1k\,2\,3)\,a_{k1}^+\,a_{k_3}\,a_{k_2}\ ,$$

$$[H_0, [H_0, [h, a_k]]] = \sum_{k_1 k_2 k_3} \left\{\varepsilon_{k_1} - \varepsilon_{k_2} - \varepsilon_{k_3}\right\}^2 (1k\,2\,3)\,\dot{a}_{k1}^+\,a_{k_3}\,a_{k_2},\ \text{etc.} \tag{2.9}$$

It is easy now to get first order terms of h in the expansion of $a_k(\tau)$. The Green's function

$$G(k,\tau) = -i < 0 \, | \, T \, [a_k(\tau) \, \overset{+}{a}_k(0)] \, | \, 0 > \tag{2.10}$$

can be therefore obtained to the first order in h, if we express wave function in terms of free system wave functions[9]:

$$|0> = \left(1 + \frac{P_0}{\varepsilon_0 - H_0} \, h + \dots \right) | \Phi_0 > . \tag{2.11}$$

Let us introduce ζ – function

$$\zeta(x) = \int\limits_0^\infty e^{-i \, k \, x} \, dk \; ; \quad \zeta(x) = \lim_{t \to \infty} \frac{1 - e^{-it \, x}}{it} \; ; \tag{2,12}$$

$$\zeta'(x) = \int\limits_0^\infty -ik \, e^{-ikx} \, dk \; .$$

If we now investigate the matrix elements which appear in the Fourier analysis of $G(k,\tau)$ with respect to time, we shall get

$$G(k, k_0) = -i \zeta \left(k_0 - \varepsilon_k - \sum_{k < k_f} \{ (1k \; k1) - (1k \; 1k) \} \right), \quad k < k_F \tag{2.13}$$

$$G(k, k_0) = i \zeta^* \left(k_0 - \varepsilon_k - \sum_{k < k_f} 2 \, (1k \; k1) \right), \; k < k_f$$

$\zeta(o) = \infty$ gives poles:

$$k_0 = \varepsilon_k + \sum_{k < k_f} \{ (1k \; k1) - (1k \; 1k) \}. \tag{2.14}$$

Hence, if we retain only the first power of interaction in the perturbation, we get elementary excitations of infinitely long lifetime with energies

$$k_0 = \varepsilon_k + \sum_k 2 \, \{ < k_1 \, k \, | \, h(12) \, | \, k \, k_1 > - < k \, k_1 \, | \, h(12) \, | \, k \, k_1 > \} \; . \tag{2.15}$$

So we have quasiparticles as stationary motions of system instead of free particles.

In the second order contributions in h in the expression of operator $a_k(\tau)$ the following commutators appear:

$$[\,h,[h,a_k]\,]\,,$$
$$[\,[\,H_0,h\,],[\,h,a_k]\,]\,; \tag{2.16}$$
$$[\,[\,H_0,[\,H_0,h\,]\,],[\,h,a_k]\,]\,,\quad\text{etc.}$$

If we calculate them, we can get the Green's function to the second order in h.

In the expression for the second order of the Fourier transform $G(k,k_0)$, term multiplying $\zeta'(k_0-\varepsilon_k)$ appears. We shall especially concentrate on that term. It has the form:

$$\zeta'(k_0-\varepsilon_k)\left\{\sum_{\substack{k_2,k_3<k_f\\k_1<k_f}}\frac{2(<k_1k\,|\,h\,|\,k_2k_3>\,-\,<kk_1\,|\,h\,|\,k_2k_3>)}{\varepsilon_k+\varepsilon_{k_1}-\varepsilon_{k_2}-\varepsilon_{k_3}}\cdot\right.$$

$$\cdot(<k_2k_3\,|\,h\,|\,k_1k>\,-\,<k_3k_2\,|\,h\,|\,k_1k>)+$$

$$+\sum_{\substack{k_1<k_f\\k_2,k_3>k_f}}\frac{2\,(<k_1k\,|\,h\,|\,k_2k_3>\,-\,<kk_1\,|\,h\,|\,k_2k_3>)}{\varepsilon_k+\varepsilon_{k_1}-\varepsilon_{k_2}-\varepsilon_{k_3}}\cdot \tag{2.17}$$

$$\left.\cdot(<k_2k_3\,|\,h\,|\,k_1k>\,-\,<k_3k_2\,|\,h\,|\,k_1k>)\right\}\,.$$

3. Self energy

Let us express the function $G(k,\tau)$ in the form of series:

$$G(k,\tau)=G_0(k,\tau)+G_1(k,\tau)+G_2(k,\tau)+\cdots\,,$$

$$G(k,k_0)=\int_{-\infty}^{\infty}e^{ik_0\tau}G(k,\tau)\,d\tau=\int_{-\infty}^{\infty}e^{ik_0\tau}G_0(k,\tau)\,d\tau+\int_{-\infty}^{\infty}e^{ik_0\tau}G_1(k,\tau)\,d\tau+$$

$$\tag{3.1}$$

$$+\int_{-\infty}^{\infty}e^{ik_0\tau}G_2(k,\tau)\,d\tau+\cdots\,;$$

$$G(k,k_0)=G_0(k,k_0)+G_1(k,k_0)+G_2(k,k_0)+\cdots\,.$$

Here G_0, G_1, G_2, etc. are the Green's functions of zeroth, first, second etc. order, respectively, in the interaction h. According to the general theory of Green's functions[8], if there are quasiparticles in the system, their energies and lifetimes are given by real and imaginary part of poles of the Green's function $G(k, k_0)$. Assuming that there are quasiparticles in the neighbourhood of some pole, the Green's function can be written in the form

$$G(k, k_0) = \frac{1}{k_0 - \sigma_0 - \Sigma + i\eta} \,, \qquad \text{or}$$

$$G(k, k_0) = \frac{1}{k_0 - \sigma_0 - (Re\,\Sigma + i\,Im\Sigma) + i\eta} \,. \qquad (3.2)$$

Let us now expand the self energy Σ in series of interaction. We express it as a sum of self energies of the first, second, etc. order in the interaction

$$\Sigma \to \sigma_1 + \sigma \cdots \,.$$
$$(h^1)\,(h^2) \qquad\qquad (3.3)$$

We get

$$\frac{1}{k_0\,\sigma_0 + i\eta + (\sigma_1 + \sigma_2 + \cdots)} \cdot \frac{1}{k_0 \cdot \sigma_0\,i\eta} - \frac{\sigma_1 + \sigma_2 + \cdots}{(k_0 - \sigma_0 + i\eta)^2} +$$

$$+ \frac{(\sigma_1 + \sigma_2 + \cdots)^2}{(k_0 - \sigma_0 + i\eta)^3} - \cdots = G(k; k_0) \,. \qquad (3.4)$$

Finally

$$G(k, k_0) = i\,\zeta\,(k_0 - \sigma_0) + i\xi'\,(k_0 - \sigma_0)\,(\sigma_1 + \sigma_2) + \cdots \,. \qquad (3.5)$$

Now we can point out the expansion (3.5). In the second order terms of the expansion of the Green's function $G(k, k_0)$ next to $\zeta'(k_0 - \sigma_0)$ we get $\sigma_1 + \sigma_2$. Therefore, the expression we have obtained next to $\zeta'(k_0 - \sigma_0)$ in the expansion of Green's function to the second order in the interaction is just the self energy σ_2. Since the expressions (2.13) for $G(k, k_0)$ can be written in the form:

$$G(k, k_0) = i\zeta\,(k_0 - \varepsilon_k)) + i\zeta'\,(k_0 - \varepsilon_k) \sum_{k < k_f} \{(1k\ 1k) - (1k\ k\ 1)\} \,,$$

$$G(k, k_0) = i\zeta*(k_0 - \varepsilon_k) + i\zeta*'(k_0 - \varepsilon_k) \sum_{k > k_f} \{(1k\ k1) - (1k\ 1k)\}\ ,$$

(2.13*)

we get the self energy of the first order in the interaction.

Therefore:

$$\sigma_1 = \sum_k \{< k_1 k\,|\,h\,|\,k_1\,k > + < k\,k_1\,|\,h\,|\,k_1\,k >\},$$

$$\sigma_2 = \sum_{\substack{k_2, k_3 < k_f \\ k_1 > k_f}} \frac{3\,(k_1 k\ k_2\ k_3)}{\varepsilon_k + \varepsilon_{k_1} - \varepsilon_{k_2} - \varepsilon_{k_3}} + \sum_{\substack{k_1 < k_f \\ k_2, k_3 > k_f}} \frac{2\,(k_1\ k\ k_2\ k_3)\,(k_2\ k_3\ k_1\ k)}{\varepsilon_k + \varepsilon_{k_1} - \varepsilon_{k_2} - \varepsilon_{k_3}}\ .$$

(3.6)

In Feynman diagrams self energy to the first order in the interaction is represented by diagrams

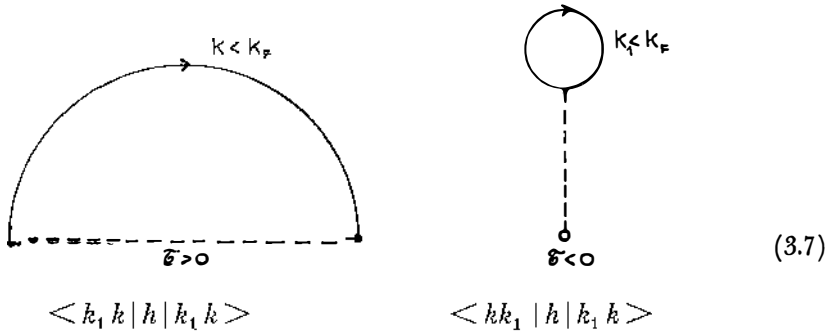$$< k_1\,k\,|\,h\,|\,k_1\,k >$$ $$< k\,k_1\,|\,h\,|\,k_1\,k >$$

Fig. 1.

(3.7)

The lines that correspond to the lower point of the second diagram are k − lines. In the theory based on Wick's theorem they correspond to free propagator with energy ε_k.
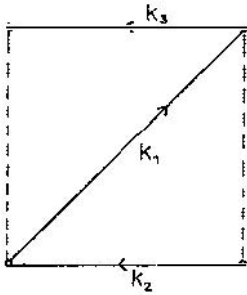
Matrix element of this diagram in the same theory is $< k k_1\,|\,h\,|\,k_1\,k >$, which is the same as in our expression (3.6)

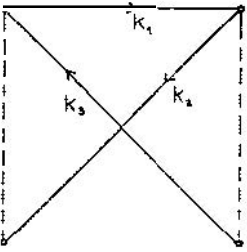The same applies for the second order diagram.
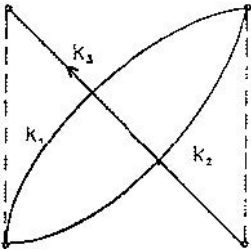
Our σ_2, represented by diagrams, gives:
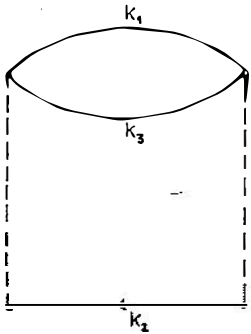
The corresponding matrix elements in (3.6) are:

$$\langle k_1 k \mid h \mid k_3 k_2 \rangle \quad \langle k_2 k_3 \mid h \mid k_1 k \rangle \tag{1}$$

$$\langle k k_1 \mid h \mid k_3 k_2 \rangle \quad \langle k_3 k_2 \mid h \mid k_1 k \rangle \tag{2}$$

$$\langle k k_1 \mid h \mid k_3 k \rangle \quad \langle k_2 k_3 \mid h \mid k_1 k \rangle \tag{3}$$

$$\langle k k_1 \mid h \mid k_3 k_2 \rangle \cdot \langle k_2 k_3 \mid h \mid k_1 k \rangle \tag{4}$$

Fig. 2. (3.8)

The comparison with results of the general theory where Wick's theorem is applied shows that our treatment gives entirely equivalent results, avoiding adiabatic hypothesis.

B. APPLICATION OF PERTURBATION CALCULATION

4. Hamiltonian of the system

We shall formulate the problem of electron-phonon interaction by using the Green's function method discussed in the previous section. We are interested in the effect of the electron-phonon interaction on electron motion. In other words, we concentrate on the Green's function of electron containing in this expansion the part which corresponds to the electron-phonon interaction, besides the electron-electron-interaction. Therefore we start from the Hamiltonian which has been separated in parts corresponding to: free electrons, interacting electrons with screened Coulomb interaction, free phonons and effective electron-phonon interaction. Therefore, our Hamiltonian has the following form

$$H = H_o^e + H_{ph} + H_{eph} + H_{ee} \tag{4.1}$$

where

$$H_0^e = \sum_k \varepsilon_k \, a_k^+ \, a_k \; ,$$

$$H_{ph} = \sum_q \hbar \, w_q \, b_q^+ b_q \; ,$$

$$H_{ee} = \sum_{k_1 k_2 k_3 k_4} < k_1 \; k_2 \,|\, V^{sc} \,|\, k_3 \; k_4 > a_{k_1} a_{k_2} a_{k_4} a_{k_3} \; ,$$

$$H_{ep} = \sum_{q, \, k} i \, \{ \, D \, (q) \, b_q \, a_{k-q} \, a_k - D^* \, (q) \, b_q \, a_{k-q} \, a_k \, \} \; ; \tag{4.2}$$

$$D \, (q) = \sqrt{\frac{\hbar q}{2 M V S}} \int u^*_{k-q} \; W u_k \, d^3 u \; , \tag{4.3}$$

where

W is the potential energy of electron in lattice, V is the volume of the system

$$u_{k-q} \quad \text{and} \quad u^*_{k-q}$$

are electron wave functions,

$$a_k, \ a_k^+ \ \text{and} \ b_q, \ b_q^+$$

are electron and phonon annihilation and creation operators, respectively. S is the velocity of the longitudinal sound waves which correspond to the wave vector q, since only longitudinal waves contribute in this case.

$$w_q = S_q \tag{4.4}$$

The operator H_{int} has been obtained under the assumption that the ions in the lattice move as a whole, that $D(q)$ depends of q only and that ion oscillations in the lattice are divided into longitudinal and transverse for all q. Therefore the interaction H_{int} takes place with the longitudinal phonons only. The free system is now represented by the Hamiltonian

$$H_0^e + H_{ph} \ . \tag{4.5}$$

It satisfies the following equation

$$\left(H_0^e + H_{ph} \right) \Phi_0 > = \left(\varepsilon_0 + E_0^{ph} \right) \Phi_0 > \ . \tag{4.6}$$

The wave functions of the free system $|\Phi_0>$ are here defined in the Hilbert space which makes the union of spaces of free electron and phonon wave functions. Therefore we write:

$$|\Phi_0> = |\Phi_{oel}>|\varphi_{oph}> \ , \tag{4.7}$$

where the energy ε_0 corresponds to the function $|\varphi_{oel}>$ and E_0^{ph} corresponds to the function $|\psi_0{}^{ph}>$.

Analogous to the method used in the section A., we shall carry out the perturbation method. We have

$$\left(H_0^e + H_{ph} + H_{ee} + H_{eph} \right) 0 > = E^0 0 > \ ,$$

$$0> = \left(1 + \frac{P_0}{\varepsilon_0 + E_0^{ph} - (H_0 + H_{ph})} (H_{ee} + H_{eph} + \cdots) \right) |\Phi_0> \ . \tag{4.8}$$

Here $\varepsilon_0 + E_0^{ph}$ corresponds to the value ε_0 in the previous analysis, and $H_0^e + H_{ph}$ is analogous to H_0 . We had for the perturbation $H_{ee} \equiv h$, now we have additional perturbation $H_{eph} = h_1$.

We must bare in mind that the projection operator P_0 is defined in the space of free electrons and free phonons.

5. *The contributions of the second order terms in the interaction*
$$h + h_1$$

In the expansion of $a_k(\tau)$ in terms of the interaction $h + h_1$, a set of new commutators appear. They can be split in two parts. The first part are just the commutators which have been calculated in section $A.$, and the second part represent commutators with quantities h_1 and H_{ph}. We can carry out the calculation for the terms with these commutators. They represent additional terms to the contribution of h.

Since

$$h_1 = \sum_{q, k} \{A_q \, a_{k-q} \, a_k^+ + B_q \, a_{k-q}^+ \, a_k\} \, , \tag{5.1}$$

where the following abbreviations are used

$$A_q \equiv i \, D(q) \, b_q \text{ and } B_q \equiv i \, D^*(q) \, b_q \, , \tag{5.2}$$

we obtain

$$[h_1, a_k] = \sum_q (A_q \, a_{k-q} + B_q \, a_{k+q}) \, , \tag{5.3}$$

$$[H_0, [h_1, a_k]] = \sum_q (A_q \, \varepsilon_{k-q} \, a_{k-q} + B_q \, \varepsilon_{k+q} \, a_{k+q}) \, .$$

Analogously we can calculate the other commutators.

To write down the Green's function to the first order terms in $h + h_1$, we have to put:

$$|0> = \left\{ 1 + \frac{P_0}{(\varepsilon_0 + E_0 - (H_0 + H_{ph})} (h + h_1 + \cdots \right\} | \Phi_0 > \, , \tag{5.4}$$

$$|\Phi_0 > = | \varphi_{ofreeel} > | \varphi_{ofreeph} > \, .$$

If we use the same procedure for h_1 as in the section $A.$, we can obtain the part of $G(k, k_0)$ that corresponds to perturbation h_1.

The calculation shows that there is no contribution from the interaction h_1 for the first order terms in the expansion of electron Green's function.

We can proceed the method used in the previous section to get the contributions of the second order terms in the interaction $h + h_1$. Therefore the correction of self energy that comes from electron-phonon interaction is given as the factor of $\zeta'(k_0 - \varepsilon_k)$ — function. The calculation gives that it has the form:

$$-\zeta'(k_0 - \varepsilon_k)\left\{ \sum_q < 0 \frac{D(q) D^*(q) b_q b_q^+ a_k^+ a_k}{\varepsilon_{k+q} - \varepsilon_k - w_q} \mid 0 > \cdots \right.$$

(5.5)

$$\cdots < 0 \mid \frac{D(q)^* D(q) b_q b_q^+ a_k^+ a_k}{\varepsilon_{k-q} - \varepsilon_k - w_q} \mid 0 > \Big\} .$$

Since $b_q \mid \Phi_{0ph} > = 0$, the second term in brackets dissapear.
Contribution of the first term is

$$\sum_q \frac{\mid D(q) \mid^2}{\varepsilon_{k+q} - \varepsilon_k - \omega_q} , \quad k > k_f .$$

(5.6)

Substituting the value of $D(q)$ and changing sum for an integral, we get

$$\frac{1}{(2\pi)^3} \frac{1}{2 MS} \overline{W}^2 \int \frac{q}{\varepsilon_{k+q} - \varepsilon_k - w_q} d^3q .$$

(5.7)

Here we take that the integral in the expression for $D(q)$ does not depend on q; $V = 1$. $\overline{W}$ is the value of such integral.
Since

$$\varepsilon_{k+q} = \frac{\hbar^2}{2m} (k+q)^2, \quad \varepsilon_k = \frac{\hbar^2}{2m} k^2, \quad wq = Sq$$

(5.8)

integration over q gives:

$$\int \frac{q \, dq}{\frac{1}{m} k \cdot q + \frac{q^2}{2m} - Sq} .$$

(5.9)

It can be transformed into the form

$$I = 2\pi \int \frac{q \sin \Theta \, d\vartheta q^2 \, d q}{\frac{1}{m} k q \cos \vartheta + \frac{q^2}{2m} - Sq} .$$

(5.10)

By solving, we reduce this to the integral

$$I = 2\pi \int_0^{q_{max}} q^2 \, \frac{m}{k} \, \ln \left| \frac{\frac{q}{2m} - S + \frac{k}{m}}{\frac{q}{2m} - S - \frac{k}{m}} \right| \, dq \, , \tag{5.11}$$

which gives the following result:

$$I = \frac{2m^2}{k} \left\{ \frac{1}{3} \frac{q^2_{max}}{(2m)^3} \, \ln \left| \frac{\frac{q_{max}}{2m} + \frac{k}{m} - S}{\frac{q_{max}}{2m} - \frac{k}{m} - S} \right| + \right.$$

$$+ \left(\frac{k}{m} - S \right) \frac{1}{3} \left[\frac{q^3_{max}}{8m^2} - \left(\frac{k}{m} - S \right) \frac{q}{2m} + \right.$$

$$\left. + \left(\frac{k}{m} - S \right)^2 \ln \left| \frac{q_{max}}{2m} + \frac{k}{m} - S \right| \right] +$$

$$+ \frac{\frac{k}{m} + S}{3} \left[\frac{q^2_{max}}{8m^2} + \left(\frac{k}{m} + S \right) \frac{q_{max}}{2m} + \right.$$

$$\left. + \left(\frac{k}{m} + S \right)^2 \ln \left| \frac{q_{max}}{2m} - \frac{k}{m} - S \right| \right] - \tag{5.12}$$

$$- \frac{1}{3} \left(\frac{k}{m} - S \right)^3 \ln \left(\frac{k}{m} - S \right) - \frac{1}{3} \left(\frac{k}{m} + S \right) \ln \left| - \frac{k}{m} - S \right| \right\} .$$

q_{max} is of the order of $\frac{2\pi}{\lambda}$, $\lambda = 2a$, where a is interatomic distance.

Introducing the result (5.12) into the expression (5.7) we finally get the value for the change of electron energy caused by the interaction with phonons.

6. Conclusion

The self energy of electrons to the second power in the interaction consists of a contribution of screened Coulomb interaction and electron-phonon interaction. An analysis of the expression (5. 6) shows that the contribution to the electron-phonon interaction comes from absorption of phonons by electrons. An estimate of electron phonon interaction shows that it is of the same order of magnitude as the energy of free electrons, as well as an estimate of electron-electron interaction term shows the contribution of the same order of magnitude.

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METODA GREENOVIH FUNKCIJA U TEORIJI ELEKTRON-FONON MEĐUDJELOVANJA

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Sadržaj

Promatrao se je utjecaj elektron-fonon međudjelovanja na kretanje elektrona pomoću metode kvantne teorije polja posredstvom Greenove funkcije elektrona. Za proračun Fourierovog transformata upotrebljen je običan račun smetnje za valnu funkciju osnovnog stanja i razvoj operatora poništavanja po vremenu. Račun koji je proveden do drugog reda po međudjelovanju dao je za vlastitu energiju rezultat koji je potpuno ekvivalentan rezultatu opće teorije sa Wick-ovim teoremom (3.6); vidi sl. 1 i 2.

Metoda razvijena u dijelu *A.* primijenjena je na sistem sastavljen od elektrona sa zasjenjenim Coulombskim međudjelovanjem kao i longitudinalnih fonona sa energijom $\hbar Sq$, gdje je S brzina zvuka. Analiza rezultata (5.6) pokazuje, da doprinos elektron-fonon međudjelovanju dolazi od apsorpcije fonona elektronima. Ocjena elektron-fonon međudjelovanja je utvrdila da je ono istog reda veličine kao i energija slobodnih elektrona. Također i ocjena člana elektron-elektron međudjelovanja pokazuje, da je to doprinos istog reda veličine.