FIZIKA, 2 (1970) 45-52

EXCITATION OF SURFACE WAVES ON PLASMAS

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Received 17 June 1969

Abstract: The launching efficiency, surface wave amplitude and radiation field of an infinite slot antenna above a dielectric — plasma interface are computed using Cullen's method. The results are akin to those found with classical reactive structures: the radiation field near the guide can be minimized by the choice of slot height, wavelength and plasma frequency; launching efficiency has a very broad maximum with variation of plasma dielectric constant.

1. Introduction

A surface wave launched on a plasma-dielectric interface is always associated with radiation and evanescent fields due to the fact that the surface wave alone cannot match the boundary conditions at the launcher. The problem has two practical aspects:

a) the radiation field is adverse to surface wave plasma diagnostics, and

b) the radiation field itself can be used as a diagnostic tool.

Experimental evidence of radiation fields in standing surface wave patterns is presented elsewhere[1]. In this paper we derive launching efficiency and radiation patterns for an infinite slot antenna above a planar plasma-dielectric interface.

2. Formulation

The aim of a theory of surface wave excitation is to derive the total field in plasma and dielectric and to relate surface wave and radiation amplitudes to the voltage of the driving antenna. The method adopted here was first developed by Cullen[2] and applied to surface waves on corrugated and dielectric coated metallic reactive surfaces. The crucial steps of the procedure are as follows: the slot voltage is represented by a delta function of surface magnetic current, which is Fourier analysed allowing the synthesis of the total field from inhomogeneous waves forced by the individual Fourier components of magnetic current density. Two asymptotic forms of the total field are then sought: one, far from the launcher but near to the guide contains the surface wave and the radiation field at the interface, the other gives the radiation pattern and total radiated power.

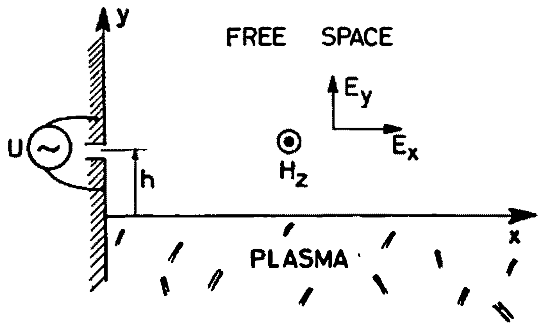

Fig. 1 — The geometry of guide and launcher.

Referring to Fig. 1 for coordinate axes, we have for the H_z component of the total field obtained by the procedure outlined above[2)]

$$H_z = -\frac{j\omega\varepsilon_0 V}{2\pi} \int_{-\infty}^{\infty} \left(e^{uh} + \frac{u+g}{u-g} e^{-uh} \right) \frac{e^{-uy}}{u} e^{-j\beta x} \, d\beta \tag{1}$$

for $y > h$
and

$$H_z = -\frac{j\omega\varepsilon_0 V}{2\pi} \int_{-\infty}^{\infty} \left(e^{uy} + \frac{u+g}{u-g} e^{-uy} \right) \frac{e^{-uh}}{u} e^{-j\beta x} \, d\beta \tag{2}$$

for $0 \leqslant y < h$,
with

$$u = \sqrt{\beta^2 - k^2} \qquad \text{and} \qquad g = -\frac{1}{\varepsilon_p} \sqrt{\beta^2 - k^2} \,.$$

In the above formulae V is the slot voltage, β the phase constant of a component inhomogeneous wave, ε_0 the free space permittivity, ε_p the relative dielectric constant of the plasma, $k^2 = \omega^2 \varepsilon_0 \mu_0$ and $k^2_p = k^2 \varepsilon_p$. An essentially similar expression for H_z holds for the plasma region, $y < 0$, but will not be required here as the plasma has to be opaque ($\varepsilon_p < -1$) to support a surface wave. The other two field components, E_y and E_x of the wave under consideration are obtainable from Maxwell's equations. The requisite boundary conditions at the interface and $y = h$ are already incorporated in (1) and (2).

3. *The distant field near the guide surface*

The amplitude of the surface wave and the radiation field near the interface are derived from (2) by contour integration in the complex β plane. The integrand in (2) has two poles at $u = g$ and four branch points at $\beta = \pm k$ and $\beta = \pm k_p$. We choose the contour as shown in Fig. 2. The condition $u = g$ is equivalent to

$$\beta^2 = k^2 \frac{\omega^2 - \omega^2_p}{2\,\omega^2 - \omega^2_p}\,, \tag{3}$$

which is the dispersion relation for the surface wave; the residue at $u = g$ ($\beta = \pm \beta_s$) is proportional to the surface wave amplitude. The integration along the branch cut starting at $\beta = k$ ($C_2 + C_4$ in Fig. 2) gives radiation

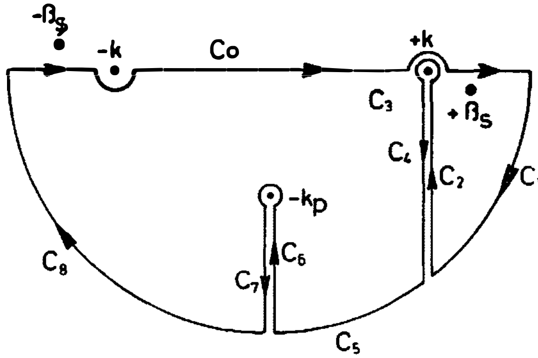

Fig. 2 — Integration contour in complex β plane.

and evanescent fields. This integral contains the large parameter x exponentially and can be treated by the Laplace method to give the radiation field alone. The branch point at $\beta = - k_p$ is peculiar to plasmas as reactive structures; it gives rise to an evanescent field. This line of computation gives for the magnetic field

$$H_z = - \frac{j\,\omega\,\varepsilon_0\,V}{2\,\pi}\, I\,, \tag{4}$$

$$I = \int_{-\infty}^{\infty} f(\beta)\, d\beta\,, \tag{5}$$

$$f(\beta) = \left(e^{uy} + \frac{u+g}{u-g} e^{-uy} \right) \frac{e^{-uh}}{u} e^{-j\beta x}, \tag{6}$$

$$I = -2\pi j \operatorname*{Res}_{\beta \to \beta_s} f(\beta) - \int_{C_2 + C_4} f(\beta)\, d\beta - \int_{C_6 + C_7} f(\beta)\, b\beta \; \cdot \tag{7}$$

For the surface wave amplitude (residue at $\beta = \beta_s$) we get

$$\operatorname*{Res}_{\beta \to \beta_s} f(\beta) = 2 \frac{(-\varepsilon_p)^{3/2}}{\varepsilon_p^2 - 1} e^{-u_s(y+h)} e^{-j\beta_s x},$$

$$u_s = \frac{k}{\sqrt{-(1+\varepsilon_p)}}, \qquad \beta_s = k \sqrt{\frac{\varepsilon_p}{1+\varepsilon_p}} . \tag{8}$$

The radiation field near the guide is determined by

$$\int_{C_2 \cdot C_4} f(\beta)\, d\beta \sim j 2 \sqrt{2\pi}\, e^{-j\left(kx + \frac{\pi}{4}\right)} \cdot \left\{ \frac{F_1(y,h}{(kh)^{3/2}} - -3j \frac{\frac{1}{8} F_1(y,h) + F_2(y,h)}{(kx)^{5/2}} \right\}, \tag{9}$$

$$F_1(y,h) = \frac{k^2}{g^2_o} (1 - g_o y)\,(1 - g_o h)\,, \tag{10}$$

$$g_o = -\frac{k}{\varepsilon_p} \sqrt{1 - \varepsilon_p}\,,$$

$$F_2(y,h) = \frac{k^4 (A^4 - B^4)}{48} - \frac{k^4}{2 g^3_o} \left[\frac{2}{3} - \frac{1}{\varepsilon^2_p} \right] B\,, \tag{11}$$

$$A = y - h \qquad B = y + h - \frac{2}{g_o}\,,$$

in close analogy with the corresponding result in[2]. The radiation field near the guide can be minimized by requiring

$$1 - g_o h = 1 + \frac{k\,h}{\varepsilon_p} \sqrt{1 - \varepsilon_p} = 0 \qquad \text{or} \qquad k\,h \sim \frac{\omega_p}{\omega}.$$

The contour $C_6 + C_7$ contributes for large x

$$\int\limits_{C_6+C_7} f(\beta)\, d\beta \sim - \frac{\sqrt{2\pi}}{(-\varepsilon p)^{3/4}\,(1-\varepsilon_p)^2} \; \frac{e^{-k\sqrt{-\varepsilon_p}\,x}\; e^{-jk\sqrt{1-\varepsilon_p}\,(y+h)}}{(kx)^{3/2}}, \tag{12}$$

which is a field evanescent in the x direction.

4. *Radiation field*

To get launching efficiency the power radiated by the slot antenna has to be evaluated. To this end we introduce $x = r\cos\theta$, $y = r\sin\theta$ in (1) and derive the asymptotic expression for H_z by applying the principle of stationary phase with $r \to \infty$. The result is

$$H_z^{(R)} = - \frac{\omega\,\varepsilon_0}{\sqrt{2\pi}} V e^{-j\left(kr - \frac{\pi}{4}\right)} \cdot \frac{1}{(kr)^{1/2}} \cdot \left[e^{jkh\sin\theta} + \frac{jk\sin\theta + g}{jk\sin\theta - g}\, e^{-jkh\sin\theta} \right],$$

$$g = - \frac{k}{\varepsilon_p} \sqrt{\cos^2\theta - \varepsilon_p} \; . \tag{13}$$

The power radiated from $\theta = 0$ to $\theta = \pi/2$ is

$$P_R = \sqrt{\frac{\mu_0}{\varepsilon_0}} \int_0^{\pi/2} \left| H_z^{(R)} \right|^2 r\, d\theta = \frac{2}{\pi} \sqrt{\frac{\varepsilon_0}{\mu_0}}\; kV^2 \int_0^{\pi/2} F^2(\theta)\, d\theta \,, \tag{14}$$

$$F^2(\theta) = \frac{\sin^2\theta\cos^2(kh\sin\theta) + \frac{1}{\varepsilon_p^2}(\cos^2\theta - \varepsilon_p)\sin^2(kh\sin\theta)}{\sin^2\theta + \frac{1}{\varepsilon_p^2}(\cos^2\theta - \varepsilon_p)} \tag{15}$$

Radiation patterns $F(\theta)$ are presented in Figs. 3 and 4 for $kh = 0$ and $kh = 1$, respectively. The fact that the pattern varies with ε_p suggests that measurements of radiation fields could provide diagnostic data on ω_p. To compute radiated power, the integral in (14) can be evaluated numerically, or by the residue method, which gives:

$$\int_0^{\pi/2} F^2(\theta)\, d\theta = \frac{\pi}{4} + \frac{\pi}{4}\, \frac{\varepsilon_p^2 + 1}{\varepsilon_p^2 - 1} \; .$$

$$\cdot \left\{ J_0(2kh) + \frac{b^2 - a^2}{b\sqrt{b^2-1}} \left[\sum_{l=1}^{\infty} \left(z_2^{2l} - z_1^{2l} \right) J_{2l}(2kh) - \mathrm{ch}\,(2kh\sqrt{b^2-1}) \right] \right\} \qquad (16)$$

with

$$a^2 = \frac{1 + \dfrac{1}{\varepsilon_p}}{1 + \dfrac{1}{\varepsilon_p^2}}, \qquad b^2 = \frac{1}{1 + \dfrac{1}{\varepsilon_p}},$$

$$z_2 = b + \sqrt{b^2 - 1}, \qquad z_1 = b - \sqrt{b^2 - 1}.$$

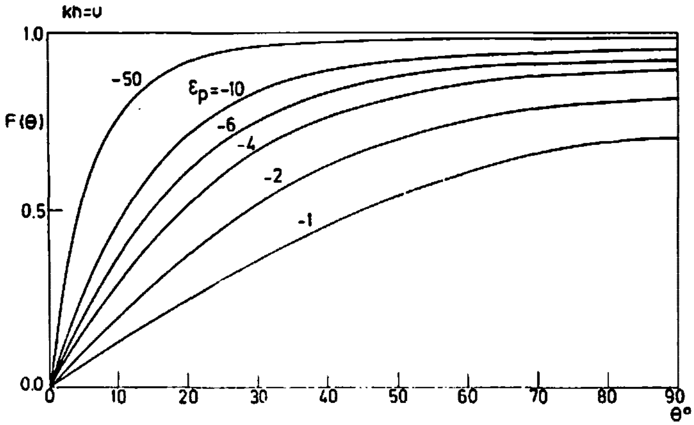

Fig. 3 — Radiation patterns for $kh = 0$. The parameter is the relative plasma dielectric constant.

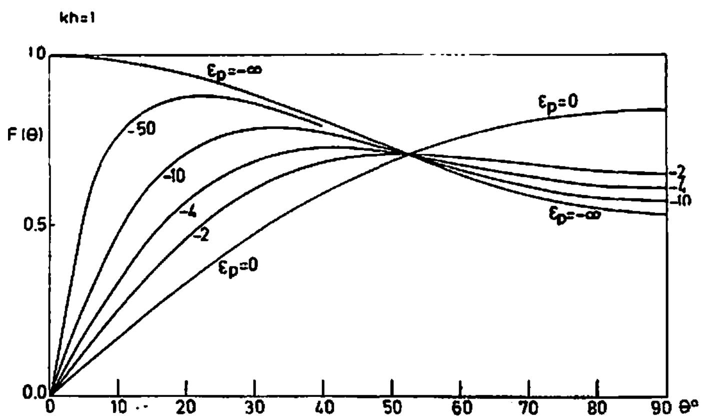

Fig. 4 — Radiation patterns for $kh = 1$.

5. *Launching efficiency*

Launching efficiency is defined by

$$\eta = \frac{P_S}{P_S + P_R}, \tag{17}$$

where P_S is the power carried by the surface wave and P_R the radiated power given by eq. (14) in the previous section. Performing the necessary Poynting vector integration of the surface wave field (8) gives

$$P_S = 2k \sqrt{\frac{\varepsilon_0}{\mu_0}}\, V^2 (-\varepsilon_p)^{3/2}\, e^{-\frac{2kh}{\sqrt{-(1+\varepsilon_p)}}} \tag{18}$$

where V is the r.m.s. voltage at the radiating slot. For any fixed value of kh $P_S(-\varepsilon_p)$ has a maximum which occurs at values of ε_p satisfying the relation:

$$kh = \frac{1}{2}\,\frac{(\varepsilon_p^2 + 3)\sqrt{-\varepsilon_p - 1}}{-\varepsilon_p(-\varepsilon_p + 1)}. \tag{19}$$

A plot of (19) is given in Fig. 5. Launching efficiency versus plasma dielectric constant for $kh = 0$ and $kh = 1$ is shown in Fig. 6. The particular case $kh = 0$ is amenable to the simple expression

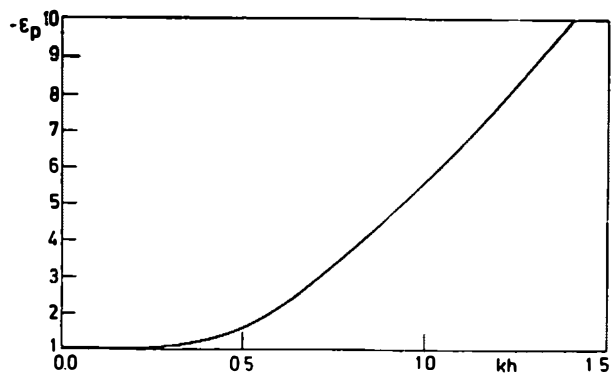

Fig. 5 — For a specified value of slot height surface wave power is largest at the value of ε_p given by (19) and plotted above.

$$\eta_{(kh=0)} = \frac{1}{1 + \frac{1}{2}(\sqrt{-\varepsilon_p} - 1)}. \tag{20}$$

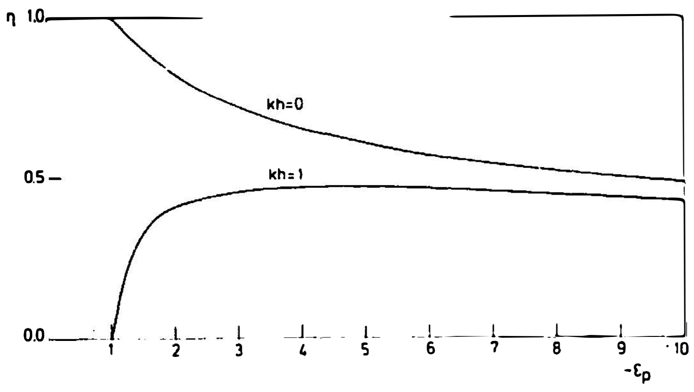

Fig. 6 — Launching efficiency vs. plasma dielectric constant.

It can be seen from Fig. 6 that launching efficiencies of the order of 0.5 are to be expected in a wide range of ε_p.

References

1) B. Aničin and D. Ilić, Third European Conference on Controlled Fusion and Plasma Physics — Symposium on Beam Plasma Interactions;

2) A. L. Cullen, Proc. I. E. E. **101**, part IV, (1954) 225.

EKSITACIJA POVRŠINSKIH TALASA NA PLAZMI

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Sadržaj

Cullen-ovim postupkom određena je efikasnost eksitacije, amplituda površinskog talasa i radijaciono polje za fiktivnu magnetnu struju iznad razdvojne površine plazma — dielektrik. Rezultati su slični onima koji se dobivaju za klasične reaktivne strukture: radijaciono polje uz vodicu može da se svede na minimum izborom visine antenskog procepa (magnetne struje), talasne dužine i plazma učestanosti. Efikasnost eksitacije malo se menja s dielektričnom konstantom plazme.