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Measurement and Modelling of the Thermal Response of the Resistance Temperature Sensors

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Abstract: Knowledge of the dynamic properties of thermometers is of particular importance for successful automation of process management in real time. The dynamic behaviour, i.e. the thermal response of the thermometers, is primarily determined by the thermal time constants of the thermo sensor and the protective housing in which it is located (thermo immersion probe, thermowell). In this paper, we presented a simple, but for a large number of applications acceptably accurate method of determining the thermal time constants of the temperature sensor. The procedure for determining the thermal time constants of a thermometer is shown on the example of a thermometer with a resistance temperature sensor. The paper presents the thermal model of the resistance temperature sensor and the protective housing of the sensor. By suddenly immersing the immersion probe in a container with heated water, the temperature response was recorded. Based on the presented thermal model, the thermal time constants of the temperature sensor and the protective case were determined.

Keywords: measurement; modelling; resistance temperature sensor; thermal response

1 INTRODUCTION

In all applications of measuring instruments it is necessary to know well the static parameters of instruments. For example: measuring range, accuracy, reading accuracy, offset, hysteresis, resolution, etc. However, in certain applications it is equally important to know the static and dynamic parameters of the instruments [1, 2]. One of such applications of measuring instruments is real-time thermal measurements of certain processes in industry. In this paper, the importance of knowing the dynamic parameters of thermometers based on platinum resistance temperature detector (RTDs) is described in detail [3]. The physical principle on which platinum resistance thermometers are based is described in the paper. Also, the typical structure of platinum resistance thermometers is described. In some cases, the corresponding dynamic parameters of the thermometers with RTDs can be found from the instrument manufacturer's datasheets. When this is not the case, it is necessary to carry out measurements in order to determine the dynamic parameters of platinum resistance thermometers. Therefore, in this paper a simple, but for many applications satisfactorily accurate procedure for determining the dynamic parameters of platinum resistance thermometers is described. The paper also presents a theory and thermal model by which the phenomenon of thermal inertia can be explained. At the same time, with the presented theory and thermal model, the influence of certain physical quantities on the dynamic response of platinum resistance thermometers can be explained. Although this paper is exclusively focused on platinum resistance thermometers, the presented methodology for determining dynamic thermal parameters is also applicable to other types of thermometers.

2 RESISTANCE TEMPERATURE SENSORS AND THERMOMETERS

RTDs are types of sensors with a very simple structure whose resistance changes as its temperature changes [3]. The change in sensor resistance can be described by a general expression [3]:

$$R(T) = R_0 \cdot (1 + \alpha_1 T + \alpha_2 T^2 + ... + \alpha_n T^n), \tag{1}$$

where R_0 is the resistance at the reference temperature (typically at 0 °C), α_1, α_2, …, α_n are temperature coefficients of resistance. Metal type and sensor resistance at 0 °C are marked as follows: PT1000 means a platinum sensor that has a resistance of 1000 Ohms at 0 °C. By injecting a known current through the sensor, the voltage drop on the sensor is proportional to the resistance of the sensor. This voltage is amplified using an amplifier, and this amplified voltage corresponds to the temperature of the sensor in a certain scale. An example of the structure of a thermometer with a resistance temperature sensor is shown in the block diagram in Fig. 1 [4].

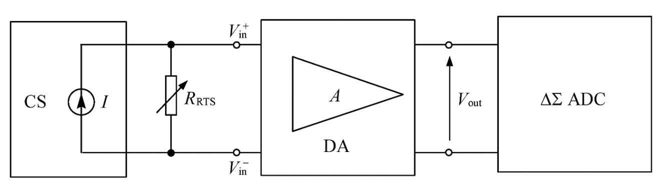

Figure 1 Block diagram of the structure of a thermometer with a resistance temperature sensor

A precise, temperature-stabilized current source (CS) injects a known current (I) into the resistance temperature sensor represented by a temperature-variable resistance (R_{RTS}). The voltage drop at the resistance temperature sensor is taken to the input of a differential amplifier (DA) that has a differential gain of A. At the output of the differential amplifier (DA) there is an amplified voltage $V_{out} = A \cdot (V_{in}^{+} - V_{in}^{-})$, which is taken to the input of the delta-sigma analog-to-digital converter (ΔΣ ADC). From the output of the delta-sigma analog-to-digital converter, the signal can be sent to the display or/and for further processing by a personal computer (PC).

3 PROBLEM DESCRIPTION

The phenomenon of thermal inertia of the immersion probe and PT1000 sensor and the influence of these thermal inertias on the dynamic temperature response of the thermometer as a whole system can be explained by a simple thermal model based on Kirchhoff's thermal circuit (Fig. 2 [3]). The immersion probe consists of a cylindrical pipe (∅3 mm) in which a PT1000 sensor is placed at the end of the pipe (Fig. 2a [3]). Around the other end of the cylindrical tube is an ergonomically designed plastic handle with flexible conductors that are connected to a measuring device.

Due to the thermal inertia of the immersion probe and PT1000 sensor, the temperature of the sensor cannot be changed instantly. The sensor temperature changes as a thermal transient.

When the ambient temperature changes, which is described at the model level by a voltage source, the thermal transient of the temperature change of the metal cylindrical tube first begins. After a certain time (deadtime, time delay), the thermal transient of the temperature change of the sensor begins. The entire transient of the temperature change of the sensor can be described as a second-order transient. This will cause an additional error in the reading of the instrument. That is, along with the static error, there will also be a dynamic error (Fig. 3 [3, 5, 6]).

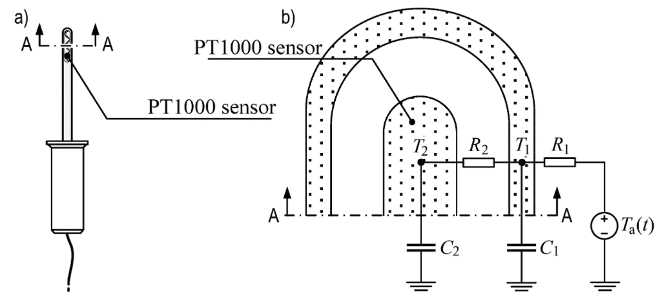

Figure 2 a) Simplified representation of the structure of the thermo probe and PT1000 sensor, b) enlarged detail of the top of the probe and the corresponding Kirchhoff thermal circuit

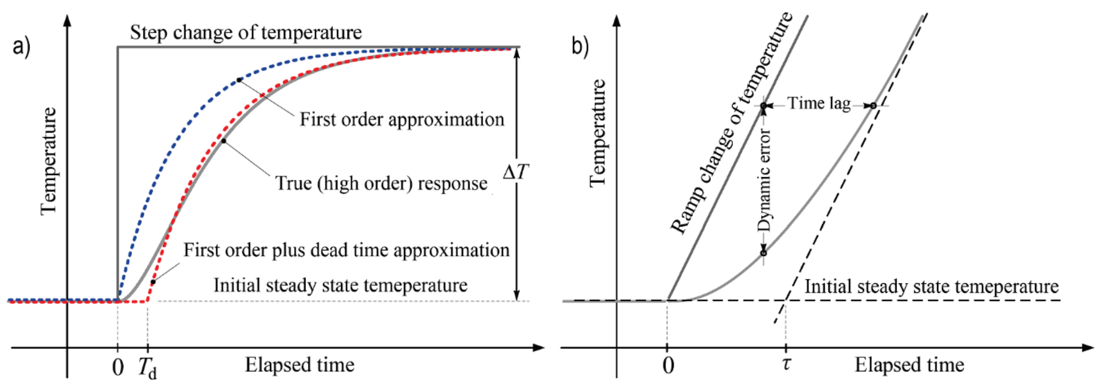

Figure 3 Temperature response (instrument reading) of PT1000 thermometer: a) to a sudden change (step change) in the measured temperature and two common approximations of the response, and b) to a linear change (ramp change) in the measured temperature, definition of dynamic error and time lag.

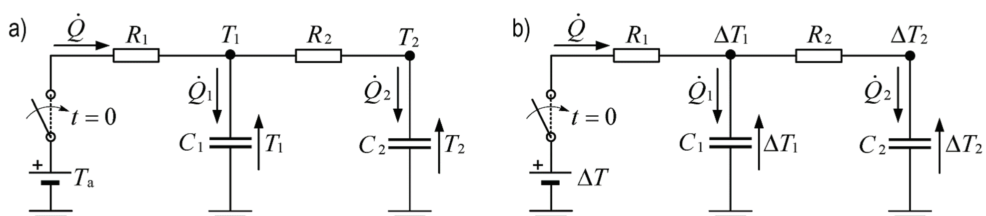

Figure 4 Equivalent Kirchhoff thermal circuits of the probe and the sensor

4 THERMAL MODEL

For further modelling and consideration, the Kirchhoff thermal circuit can be separated from the geometry as shown in Fig. 4.

The circuit elements of the thermal circuit shown in Fig. 4 have the following meanings. R_1 - represents thermal resistance of the probe due to convection of heat from the environment into the probe. R_2 - represents the thermal resistance to heat transfer between the probe and the temperature sensor due to heat convection. C_1 and C_2 represent the thermal capacitance of the thermal mass of the probe and sensor. The event of probe immersion into a liquid substance with a temperature different from the temperature of the probe is modelled with the switch "S". Furthermore, T_a represents the ambient temperature, T_1 represents the temperature of the immersion probe housing, and T_2 represents the temperature of the sensor. Acknowledging the fact that when determining the transfer function, all initial conditions are set to zero, the equivalent thermal scheme that models the temperature transient can be presented as shown in Fig. 4b. The difference compared to the previous scheme (Fig. 4a) is that instead of absolute temperature values, over temperatures are used.

The above will simplify the further mathematical procedure. The resulting expressions will describe the change in overtemperature, and they are related to the absolute temperature by an expression:

$$T_2(t) = T_a + \Delta T_2(t). \tag{2}$$

Based on the scheme shown in Fig. 4, after $t \geq 0$, the temperature transient is described by a system of equations:

$$\Delta T - \dot{Q} \cdot R_1 - \Delta T_1 = 0\,, \tag{3}$$

$$\Delta T_1 - \dot{Q}_2 \cdot R_2 - \Delta T_2 = 0, \tag{4}$$

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2\,. \tag{5}$$

Changes in the internal energy of the heat masses of the probe ($\dot{Q}_1$) and sensor ($\dot{Q}_2$) is determined by the expressions:

$$\dot{Q}_1 = C_1 \cdot \frac{\mathrm{d}(\Delta T_1)}{\mathrm{d}t}, \tag{6}$$

$$\dot{Q}_2 = C_2 \cdot \frac{\mathrm{d}(\Delta T_2)}{\mathrm{d}t}. \tag{7}$$

Since it is objective to assume $\dot{Q}_1 \gg \dot{Q}_2$, then from Eq. (5) follows $\dot{Q} \approx \dot{Q}_1$. Considering the above, the system of Eqs. (3) - (7) which describes the temperature transient is simplified to the form:

$$\Delta T - R_1 C_1 \cdot \frac{\mathrm{d}(\Delta T_1)}{\mathrm{d}t} - \Delta T_1 = 0, \tag{8}$$

$$\Delta T_1 - R_2 C_2 \cdot \frac{\mathrm{d}(\Delta T_2)}{\mathrm{d}t} - \Delta T_2 = 0. \tag{9}$$

By introducing substitutions $\tau_1 = R_1 C_1$ and $\tau_2 = R_2 C_2$ that have the physical meaning of the thermal time constants of the probe and sensor and applying the Laplace transform while setting the initial conditions to zero, the transfer function is obtained:

$$H(s) = \frac{\Delta T_2(s)}{\Delta T(s)} = \frac{1}{(1+\tau_1 \cdot s)\cdot(1+\tau_2 \cdot s)}. \tag{10}$$

Using the obtained transfer function, the temperature response can be determined for any excitation waveform:

$$\Delta T_2(s) = \Delta T(s) \cdot H(s). \tag{11}$$

For a sudden, sharp change in excitation, which corresponds to the momentary immersion of the probe in a liquid substance with an overtemperature ΔT, the excitation is described by a step function:

$$\Delta T(t) = \Delta T \cdot u(t) = \begin{cases} 0, & 0 < t, \\ \Delta T, & t \geq 0. \end{cases} \tag{12}$$

Where $u(t)$ is the Heaviside step (so-called unit) function [7]. Taking the Laplace transform of the previous expression gives:

$$\Delta T(s) = \frac{\Delta T}{s}. \tag{13}$$

Combining the Eqs. (10) - (13) gives an expression for the Laplace transform for the response i.e. sensor overtemperature:

$$\Delta T_2(s) = \Delta T \cdot \frac{1}{s \cdot (1+\tau_1 \cdot s)\cdot(1+\tau_2 \cdot s)}. \tag{14}$$

Applying the inverse Laplace transform [7] to the previous expression gives:

$$\Delta T_2(t) = \Delta T \cdot \left[1 + \left(\frac{\tau_1}{\tau_2 - \tau_1} \cdot \mathrm{e}^{-\frac{t}{\tau_1}} - \frac{\tau_2}{\tau_2 - \tau_1} \cdot \mathrm{e}^{-\frac{t}{\tau_2}}\right)\right]. \tag{15}$$

Substituting Eq. (15) into Eq. (2) gives:

$$T_2(t) = T_\mathrm{a} + \Delta T \cdot \left[1 + \left(\frac{\tau_1}{\tau_2 - \tau_1} \cdot \mathrm{e}^{-\frac{t}{\tau_1}} - \frac{\tau_2}{\tau_2 - \tau_1} \cdot \mathrm{e}^{-\frac{t}{\tau_2}}\right)\right]. \tag{16}$$

Eq. (16) is an exact expression that describes the transient phenomenon of temperature change of the temperature sensor and enclosures around it. However, this expression is not practical for determining temperature time constants. At the same time, the influence of the time constants on the waveform of the temperature change is not intuitive. For this reason, it is more convenient to find a simpler and satisfactorily accurate function that describes the change in the sensor's temperature. Two approximations are typically used for this purpose. Both approximations refer to the system that is overdumped, i.e. without oscillations, such as the system analysed in this article.

If the description of the system is not critical, then a very rough approximation can be applied. That is, a second-order system can be approximated by a first-order system (FO) [2]:

$$T_2(t) \approx T_\mathrm{a} + \Delta T \cdot \left(1 - \mathrm{e}^{-\frac{t}{\tau'}}\right), \tag{17}$$

where τ' is equivalent thermal time constant.

If greater accuracy is required, the approximation of the second-order transfer function with the first-order transfer function plus dead time (FOPDT) can be applied. According to the approximation proposed by Sigurd Skogestad [8], a second order system that is overdumped, i.e. without oscillation, can be approximated as a first order system plus dead time (time delay) [8]:

$$\frac{1}{(1+\tau_1 \cdot s)\cdot(1+\tau_2 \cdot s)} \approx \frac{\mathrm{e}^{-t_\mathrm{d} \cdot s}}{s \cdot \tau + 1}, \tag{18}$$

Where are the time constant τ and dead time (time delay) t_d specified by expressions [8]:

$$\tau = \tau_2 + 0.5 \cdot \tau_1, \tag{19}$$

$$t_d = 0.5 \cdot \tau_1. \tag{20}$$

This method is applicable as long as the time constants differ by a factor greater than 1.5 [8]. Inserting Eq. (18) into the Eq. (11) gives:

$$\Delta T_2(s) = \Delta T \cdot \frac{1}{s \cdot (1+\tau_1 \cdot s) \cdot (1+\tau_2 \cdot s)} \approx \Delta T \cdot \frac{1}{s} \cdot \frac{\mathrm{e}^{-t_d \cdot s}}{s \cdot \tau + 1}. \tag{21}$$

Applying the inverse Laplace transform [7] to the previous expression gives an expression for the overtemperature in the time domain:

$$\Delta T_2(t) = \Delta T \cdot \left(1 - \mathrm{e}^{-\frac{(t-t_d)}{\tau}}\right) \cdot u(t-t_d). \tag{22}$$

Where $u(t-t_d)$ is the delayed Heaviside step function [7]. Combining Eq. (2) with Eq. (22) gives:

$$T_2(t) = T_a + \Delta T \cdot \left(1 - \mathrm{e}^{-\frac{(t-t_d)}{\tau}}\right) \cdot u(t-t_d). \tag{23}$$

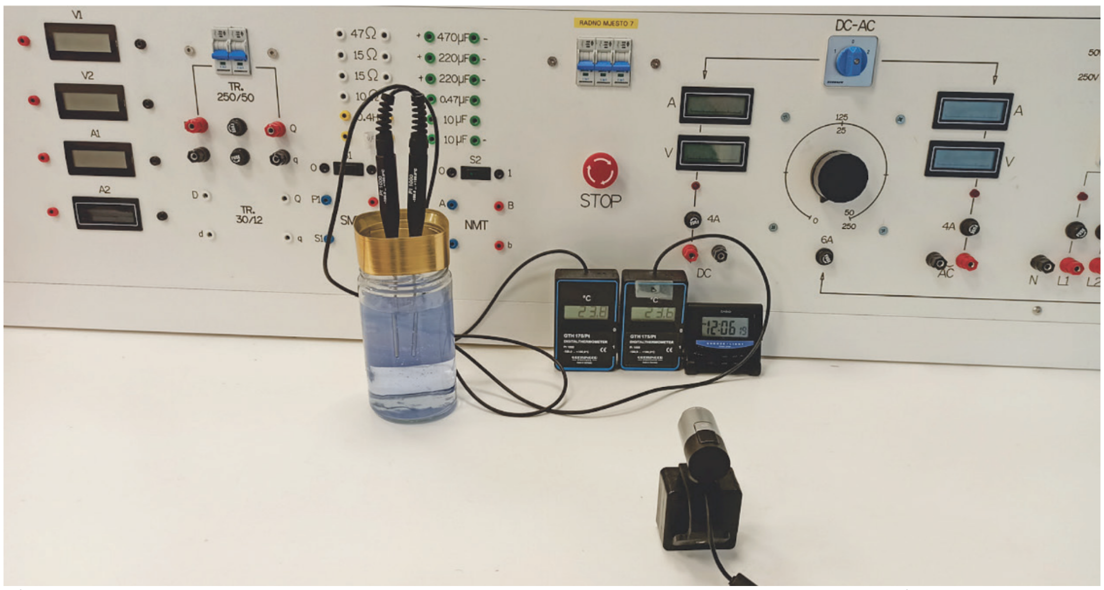

Figure 5 Photo of the measuring setup. Thermometer with resistance temperature sensor, container with heated water and camera for recording transient phenomena.

5 MEASUREMENT RESULTS AND DISCUSSION

The measurement of the temperature response was carried out for a sudden change (step change) in temperature. This was achieved by immersion probes being suddenly immersed in pre-heated water. In order for the recorded graphs to be smooth, two thermometers were used simultaneously (Fig. 5 [9]). The measurements were carried out using GTH 175 PT1000 thermometers with RTDs [10]. The measured values were recorded with a web camera and transferred from the video to an excel file for further processing. Model-based parameters were determined using the Mathcad 14 program [11]. For this purpose, the method of least squares was applied [7]. The obtained parameter values correspond to those that minimize the error between the model function and the measured values (Tab. 1). Temperature responses of the thermometer for different overtemperatures are shown in Fig. 6 [9].

According to the data in Tab. 1, certain trends are noticeable. By increasing the overtemperature ΔT, the time delay decreases t_d, also the time constant τ_1, which is related to it by the expression, $\tau_1 = 2t_d$ decreases, the time constant τ decreases, and the thermal time constant τ_2 remains constant. The obtained results are consistent with the theoretical prediction.

Table 1 Table title aligned centre

ΔT (K)	4.15	8.65	11.10	15.25	20.38
t_d (s)	0.526	0.431	0.332	0.306	0.228
τ (s)	1.848	1.763	1.675	1.599	1.532
$\tau_1 = 2t_d$	1.052	0.862	0.665	0.612	0.457
$\tau_2 = \tau - \tau_1/2$	1.322	1.332	1.342	1.293	1.304

*The average value of the thermal time constant τ_2 is $\overline{\tau}_2 = 1.3186\ \mathrm{s} \approx 1.3\ \mathrm{s}$

According to the theory of heat transfer [12], an increase in overtemperature causes an increase in the heat convection coefficient. By increasing the convection coefficient, the thermal resistance caused by heat convection R_1 is decreasing [12]. Given that C_1 has a constant value for metals (immersion probe), and taking into account that the thermal time constant is determined by the product of the thermal resistance and the thermal capacity C_1, i.e. $\tau_1 = R_1C_1$, the time constant τ_1 decreases as the resistance R_1 decreases, the thermal time constant τ_2 remains constant because it is determined by the product $\tau_2 = R_2C_2$ in which both values are constant. Namely, the thermal resistance R_2 due to heat conduction is constant, as well as the thermal capacity of the sensor C_2.

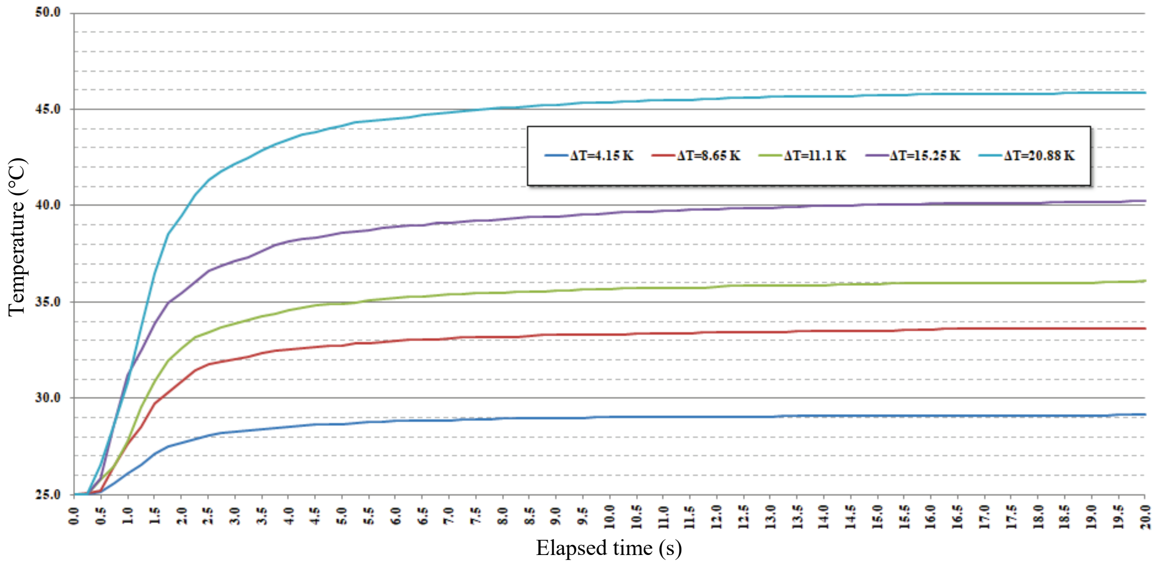

Figure 6 Temperature responses (instrument readings) of PT1000 thermometer to a sudden change (step change) in the measured temperature

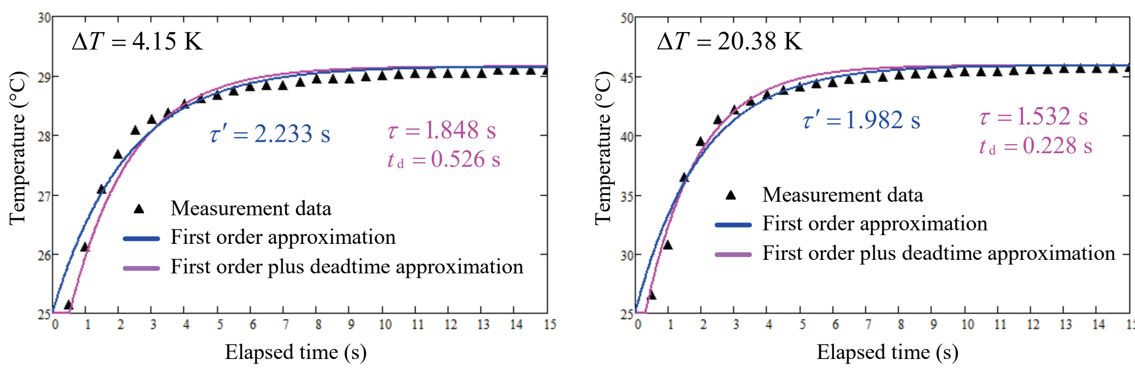

Figure 7 Measured temperature response of PT1000 thermometer to a sudden change in the measured temperature and response approximations

6 RECAPITULATION ANNOTATION

Automating the management of industrial processes in real time requires knowledge of the actual temperatures in real time. For this purpose, it is important to know the dynamic characteristics of thermometers. The paper presents a relatively simple, but for many applications acceptably accurate model by which electric thermometers can be modelled. Also, the paper presents one of the procedures that can be used for determining the parameters of such models. Although the selected example in the paper refers to a laboratory instrument, where the immersion probe has a relatively small diameter, and therefore a small thermal mass, i.e. time constant, the presented procedure is also applicable for industrial applications when thermowells are used. In that case, significantly longer time delays can be expected, i.e. the importance of their consideration is greater. Given that the fluid convection coefficient depends on the type of fluid and the temperature of the fluid, the following recommendations are given. We recommend that, if possible, the procedure for determining the dynamic parameters of the thermometer uses the fluid that is present at the place of installation of the thermometer. We also recommend determining the dynamic parameters in the temperature range expected in the process where the thermometer will be installed.

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