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ON THE THREE-NUCLEON GROUND STATE WAVE FUNCTION

II. The Properties of the Radial Part of the Wave Function. The inclusion of the »soft« core. The Electric Charge Form Factor. The μ-meson Capture

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Abstract: Determination of the radial part of the total three-body nuclear wave function which depends on the symmetric scalar argument is discussed. The complete »soft« core is included into the known Irving-Gunn radial function. With this function electric charge form factors for the ^{3}H and ^{3}He nuclei are calculated using different percentage of the S, S′ and D components, as well as dependence of the single nucleon charge form-factor on the momentum transfer. Comparison with the experimental results is discussed. With the same function the μ-meson capture by the ^{3}He nucleus is calculated using the non-relativistic limit of V-A theory. The usually accepted percentages of S, S′ and D-states give very good agreement with the experiment.

1. Introduction

The previous analysis of the total three-body ground state wave function[1]* was based on the group theoretical considerations. There it was noticed that its radial part, which depends on the symmetric scalar argument, cannot be obtained using a pure group theoretical method. In order to determine the plausible form of the radial dependence one starts from a simple form which fits a particular experiment. The included parameters are connected with some, arbitrarily chosen, static physical properties. A functional form by which an experimental result is successfully fitted, usually fails to explain

* We shall refer to this paper as I. The formulae cited from it are always preceeded by I.

other experiments, or the same one but in some other momentum transfer region. Many simple radial dependences were already explored. The Gaussian function with the parameter adjusted to fit the Coulomb energy was used in the form factor calculation[2,3] and good agreement was obtained only for small values of the momentum transfer ($q^2 < 4\,\mathrm{fm}^{-2}$). However, with the same function it was not possible to explain the low energy photodesintegration data[4,5]. The superposition of the two Gaussian functions[6] gives good fit for the photodesintegration processes, but it fails to explain the radiative capture data[7]. The Irving function, with the parameter adjusted to give the correct Coulomb energy value[2,3], also reproduces the electromagnetic form factor values, but for the photodesintegration cross-section one obtains a rather large value for the differential cross-section maximum[8,9]. With the Irving-Gunn function one finds acceptable explanation for the photodesintegration cross-section[5,10], but the form factor values for the momentum-transfer region higher than 5 fm^{-2} are badly reproduced[3]. Gibson[11] has tried to include a part of the soft core into the Irving-Gunn radial form and has succeeded in explaining many data concerning the electromagnetic properties of the three-nucleon bound state system. However, the physical meaning of the Gibson's function is not very clear. On the other hand, it is not difficult to include the complete soft-core part into the Irving-Gunn radial dependences. This function was already used in explaining the thermal neutron radiative capture on deuteron[12] and in the electromagnetic form factor calculations for ^{4}He nucleus [13]. The successes of these calculations are not due to the same physical reasons. In Ref.[12] the function was used to avoid additional parameters which come into play with the unclear nature of the exchange magnetic moment operator, while good agreement in Ref.[13] comes from the fact that the Irving-Gunn radial function with the complete soft core has simultaneously correct behaviour at large and small distances, and in the intermediate region represents a sort of Jastrow's[14] interparticle correlation. In Chapter 2 we shall briefly explain how the complete soft-core function included into the Irving-Gunn radial dependence might be obtained from the general asymptotic properties of the three-nucleon Schrödinger equation with the two particle potential. In Chapter 3 the electric charge form factor calculations for ^{3}He and ^{3}H nuclei are presented. The points of view used in the calculations are explained and prediction for the possible place of the diffraction minimum is given. This prediction comes automatically as a consequence of the very good agreement in the experimentally explored momentum transfer region. In Chapter 4 the μ-meson capture by ^{3}He nuclei is treated. It is shown that the complete soft-core included in the Irving-Gunn radial dependence gives correct value for the capture rate within the experimental errors. Again, comparison with the previously published calculations is given. In Chapter 5 some conclusions, and in the Appendix some useful mathematical formulae are given.

2. *Properties of the Radial Part of the Wave Function. The inclusion of the »soft« core*

We shall now briefly sketch the arguments by which one is led to conclude that the Irving-Gunn functional dependence of the symmetric scalar argument comes from the nucleon-nucleon interaction considerations, and that the inclusion of the core is only the introduction of the known properties of this interaction.

Let us consider two-nucleon interaction,

$$V_{ij} = U(\vec{r}_i - \vec{r}_j)\, O(i, j) \quad , \tag{1}$$

where $O(i, j)$ contains the spin-isospin part, whose matrix element can be written as

$$< s, t \,|\, O(i, j) \,|\, s, t > \; = \lambda_{2s+1,\, 2t+1}\, U_0 \quad , \tag{2}$$

where s and t are the spin and isospin values respectively, λ is the coefficient depending on the total spin and isospin of the state, and U_0 is the scalar coefficient. From the invariance properties of (1) in the spin and isospin space follows the general form of the $O(i, j)$, which is the known superposition of the Wigner, Bartlett, Heisenberg and Majorana forces. The full interaction in the three-particle system is then:

$$\begin{aligned} V &\equiv V_{12} + V_{23} + V_{31} = U_s O_s + U_1 O_1 + U_2 O_2 \;, \\ U_s &= U(\vec{r}_1 - \vec{r}_2) + U(\vec{r}_2 - \vec{r}_3) + U(\vec{r}_3 - \vec{r}_1) \;, \end{aligned} \tag{3}$$

where the index S corresponds to the symmetric, and 1 and 2 to the mixed representation of the S_3 group. The corresponding function is the superposition of the S and S′ components, since the D-components are being generated from the tensor part of the nucleon-nucleon interaction. Consequently, in this analysis we shall take only S and S′ state and write:

$$\Psi = \Psi_s \Phi_a + \Psi_1 \Phi_2 - \Psi_2 \Phi_1 \;. \tag{4}$$

The function (4) is antisymmetric. The functions Ψ_s, Ψ_1 and Ψ_2 depend on the radial variables, and the functions Φ_a, Φ_1 and Φ_2 on the spin-isospin variables. The indices have evident meaning. A system of coupled equations for the Ψ_s, Ψ_1 and Ψ_2 functions was given by Verde[15]. It is sufficient to consider only the symmetric function for which we have

$$(T - E)\,\Psi_s = -\frac{1}{2}\,(\lambda_{3,1} + \lambda_{1,3})\;U_s\Psi_s - \frac{1}{2}\,(\lambda_{3,1} + \lambda_{1,3})\;(U_1\Psi_1 + U_2\Psi_2)\;. \tag{5}$$

The operator T is the sum of the kinetic energies of the three particles. Using the transformation (I. 4) for the operator T one finds

$$T = -\frac{\hbar^2}{2M}\,(\vec{\nabla}_1^{\,2} + \vec{\nabla}_2^{\,2} + \vec{\nabla}_3^{\,2}) = -\frac{\hbar^2}{6M}\,\vec{\nabla}_{R_0}^{\,2} - \frac{\hbar^2}{2M}\,(\vec{\nabla}_{R_1}^{\,2} + \vec{\nabla}_{R_2}^{\,2})\;\cdot \tag{6}$$

In a dynamical analysis, when only the qualitative behaviour is considered, one can neglect the small S′ components. Once Ψ_s dependence is known it is straightforward to form the Ψ_s radial dependence. When necessary using only the group theoretical arguments. By the requirement of the translational invariance $\vec{R}_0$ is eliminated and Eq. (5) reads:

$$\left[-\frac{\hbar^2}{M}\,(\vec{\nabla}_{R_1}^{\,2} + \vec{\nabla}_{R_2}^{\,2}) + \lambda\,U_s\right]\Psi_s = E\,\Psi_s\;, \tag{7}$$

with

$$\lambda = \frac{1}{2}\,(\lambda_{3,1} + \lambda_{1,3})\;\cdot$$

Let us consider now only the asymptotic properties of the solution of this equation. From the properties of the nucleon-nucleon potential, $U_s \to 0$, $r_{ij} \to \infty$, this corresponds to the free particle wave equation

$$-\frac{\hbar^2}{M}\,(\vec{\nabla}_{R_1}^{\,2} + \vec{\nabla}_{R_2}^{\,2})\;\Psi_f\,(\vec{R}_1, \vec{R}_2) = E\,\Psi_f\,(\vec{R}_1, \vec{R}_2)\;\cdot \tag{8}$$

The function $\Psi_f\,(\vec{R}_1, \vec{R}_2)$ can be thought as a function in a six-dimensional space in wich we can write the solution of the Eq. (8) in the form[15]:

$$\Psi_f\,(R_1, R_2) = \sum_K f_K\,(R)\;\varphi_K\left(\frac{\vec{R}_1}{R}, \frac{\vec{R}_2}{R}\right)\,, \tag{9}$$

where R is the intensity of the vector $\tilde{R}_6 = (\vec{R}_1, \vec{R}_2)$ in the six-dimensional space:

$$R = (R_1^2 + R_2^2)^{1/2} = \left[\frac{2}{3}\sum_{i<j} r_{ij}^2\right]^{1/2}. \tag{10}$$

The function $\varphi_K\left(\frac{\vec{R}_1}{R}\;\frac{\vec{R}_2}{R}\right)$ is a part of the harmonic polynominal

$$P_K\,(\vec{R}_1, \vec{R}_2) = R^K\,\varphi_K\,(\vec{R}_1, \vec{R}_2),$$

which satisfies the equation:

$$(\vec{\nabla}_{R_1}^2 + \vec{\nabla}_{R_2}^2)\, P_K (\vec{R}_1, \vec{R}_2) = 0 \ . \tag{11}$$

From the Eq. (8) follows the equation for the function $f_K(R)$:

$$-\frac{\hbar^2}{M}\frac{1}{R^5}\frac{\partial}{\partial R}\left(R^5 \frac{\partial}{\partial R}\right) f_K (R) - \left[E - \frac{K(K+4)}{R^2}\right] f_K (R) = 0. \tag{12}$$

For $R \to \infty$, and for any K we have:

$$\left(\frac{\partial^2}{\partial R^2} + \varkappa^2\right) f_K (R) = 0 \ , \tag{13}$$

and $f_K(R)$ for the bound state reads:

$$f(R) \underset{R \to \infty}{\sim} \exp\left\{-\sqrt{\frac{|E|M}{\hbar^2}} \cdot R\right\} . \tag{13'}$$

The angular average of the function $\varphi_K\left(\frac{\vec{R}_1}{R}, \frac{\vec{R}_2}{R}\right)$ gives the constant contribution which can be incorporated in the parameters which are often subject to a trial procedure. Therefore, for the large values of R it follows

$$f(R) = R^{m/2} \exp\{-\beta R\} \ , \tag{14}$$

where β and m are parameters. This is known as the Irving-Gunn function. For small distances it was already suggested[15] that the Eq. (14) has to be modified by the inclusion of the hard core radius

$$f(r_{12}, r_{23}, r_{31}) = \begin{cases} 0 \quad \text{for} \quad r_{ij} < r_c \ , \\ f_0 (r_{12}, r_{23}, r_{31}) \ [(r_{12} - r_c)\ (r_{23} - r_c)\ (r_{31} - r_c)]^{n/2} \ . \end{cases} \tag{15}$$

This is a rather drastic condition which introduces the hard core radius which is not experimentally well established. However, Eqs. (15) can be slightly modified by putting $r_c = 0$ under the condition that the full dependence should be that of the symmetric scalar argument. This is a fraction which we call a complete »soft« core function:

$$\begin{aligned} \Psi_S \text{ (symmetric scalar)} &= f(R)\ [\prod_{i<j} r_{ij}^2]^{n/2} = \\ &= \left\{\frac{1}{16} R_2^2\ [(3R_1^2 + R_2^2)^2 - 12\ (\vec{R}_1 \cdot \vec{R}_2)^2]\right\}^{n/2} R^{m/2} \exp\{-\beta R\} \ . \end{aligned} \tag{16}$$

As it was noticed in the preceeding Chapter, this function has a good behaviour at small and at large distances, and the factor $[\prod_{i<j} r_{ij}^2]^{n/2}$ emphasizes the known particle correlation term for any distances. With this function, for $n = 1$; $m = -2$ fair agreement was already obtained for neutron-deuteron capture cross-section[12], and for the electric-charge form factor of the ^{4}He nucleus[13].

3. *Charge Form Factors for ^{3}H and ^{3}He nuclei*

The expression for the cross-section of the elastic scattering of electrons on nuclei derived from the current-current interaction in the Born approximation (usually called the Rosenbluth formula) apart from the Mott scattering cross-section term, contains the following important dynamical quantities:

1) a combination of the charge form factor of single nucleons,
2) the Fourier transform of the charge distribution in nuclei (to which one refers as to the charge form factor), and
3) the Fourier transform of a magnetic moment distribution (to which one refers as to the magnetic form factor)[18, 17].

Here we shall restrict ourselves to the charge form factor calculations.

Apart from the body form factor calculations, when one supposes that the proton and neutron form factors are 1 and 0, respectively, (what allows one to put the whole dynamics of the electron-nucleus scattering into the quantity which is the Fourier transform of the nuclear wave function), there are two other analyses of the charge distribution in the three-particle nuclei. One of them is reported in the work of Schiff and his collaborators[2, 17, 18] and the other in the work of Srivastava[19]. In the former it is supposed that the neutron charge form factor $F_{ch}^n(q^2)$ is always zero, but the analysis is extended to S, S′ and D components of the nuclear wave function. In the latter, the analysis is done only with the S state function, but the momentum transfer dependence of the neutron charge form-factor was taken into account. Both of them are working hypotheses and neither of them can be disregarded a priori. With function given in (16) we have calculated: α) the body form factor, β) the Schiff like form factor, and γ) the Srivastava-like form factors and its extensions when S′ and D components are included. We shall suppose that the D-state components might be well represented taking only the Ψ_7 function (I. 26).

For this case the expression for the ^{3}He form factor, F_{ch} (^{3}He), using (I. 28) reads:

$$F_{ch}\,(^3\mathrm{He}) = \left(F_{ch}^p + \frac{1}{2}\,F_{ch}^n\right)\left(P_s^2 F_s + P_{s'}^2 F_{s'}\right) - \frac{1}{2}\,P_s P_{s'}\left(F_{ch}^p - F_{ch}^n\right) F_{s-s'} + \cdots$$

$$+ P_D^2 \left[\frac{3}{4} F_{ch}^{p} F_{D_1} + \left(\frac{1}{4} F_{ch}^{p} + \frac{1}{2} F_{ch}^{n} \right) F_{D_2} \right] , \qquad (17)$$

and the corresponding expression for the ^{3}H nucleus is:

$$F_{ch}\,(^3\text{H}) = (F_{ch}^{p} + 2\,F_{ch}^{n})\,(P_s^2 F_s + P_{s'}^2 F_{s'}) + P_s P_{s'}\,(F_{ch}^{p} - F_{ch}^{n})\,F_{s-s'} +$$

$$+ P_D^2 \left[\frac{3}{2} F_{ch}^{n} F_{D_1} + \left(\frac{1}{2} F_{ch}^{n} + F_{ch}^{p} \right) F_{D_2} \right] , \qquad (18)$$

where $F_{ch}^{p,n}$ are the proton and the neutron form factor, respectively, F_s, $F_{s'}$, $F_{s-s'}$ are defined in (I. 31) and (I. 36), F_{D_1} and F_{D_2} are obtained using the definitions (I. 18). In the Appendix we have listed the explicit expressions for these integrals when for the radial function of the symmetric scalar argument the function (16) is used. The calculations are made for (m = 0, —2, —3, —4). While the values obtained for (m = 0, —2, —4) deviate from the experimental points, the values for $m = -3$ are very close to the experimental ones, and oscillate around them depending on the percentage of the S, S′ and D components and on the inclusion or exclusion of the neutron form factor dependence on the momentum transfer.

In Table 1 we give the results of the calculations for the ^{3}He nucleus, together with the experimental values reported by Colard *et al.*[20]. The experimental values for the proton and neutron form factor are taken from Levinger *et al.*[21]. In column I the values for the body form factor calculations (100 % of the S state) with the mean value of nuclear radius $< r^2 >^{1/2} = 1.8$ fm are given. In column II the Srivastava-like analysis is reproduced again with 100 % of the S state but including the proton and neutron form factor dependence with $< r^2 >^{1/2} = \sqrt{3}$ fm. This value of $< r^2 >^{1/2}$ is the same for both ^{3}He and ^{3}H nuclei as a consequence of the charge symmetry. Columns III, IV, V and VI are calculated for this value of $< r^2 >^{1/2}$. In colums III and IV the values are obtained with 98 % of the S state and 2 % of the S′ state. The former is without neutron form factor values while in the latter this dependence is included. The columns V and VI contain results of the calculation with 92 % of the S state, 2 % of the S′ state and 6 % of the D-state. Again, in first case the neutron form factor is neglected, while in the second it is included. The parameters of the function (16) are taken in such a way as to give the correct value for the nuclear radius which reproduces the Coulomb energy.

In Table 2 corresponding results for the ^{3}H nucleus are reported. Here, in column I the body form factor is calculated for the mean value of nuclear radius $< r^2 >^{1/2} = \sqrt{3}$ fm.

In Table 3 the values for the ratios F_{ch} (^{3}H)/F_{ch} (^{3}He) for all columns are reported.

Table 1

q^2	F_{ch} (^{3}He) exp	F_{ch} (^{3}He) calculated					
		$P_S^2 = 1$		$P_S^2 = 0.98$; $P_{S'}^2 = 0.02$		$P_S^2 = 0.02$; $P_{S'}^2 = 0.02$; $P_D^2 = 0.06$	
		I	II	III	IV	V	VI
1	0.567	0.5869	0.5805	0.5575	0.5642	0.5445	0.5510
2	0.329	0.3477	0.3442	0.3172	0.3269	0.3050	0.3137
3	0.209	0.2064	0.2087	0.1854	0.1949	0.1763	0.2145
4	0.132	0.1216	0.1272	0.1089	0.1181	0.1019	0.1095
5	0.081	0.0701	0.0776	0.0635	0.0707	0.0591	0.0656
6	0.054	0.0387	0.0465	0.0378	0.0415	0.0336	0.0378
8	0.017	0.0079	0.0152	0.0111	0.0131	0.0099	0.0118

Tab. 1. — Momentum transfer dependence of the electric charge form factor of the ^{3}H nucleus. The experimental values are taken from H. Colard *et al.*[20] The values for the single nucleon charge form-factors are due to Levinger and Strivastava[21]. Column I contains the body form-factor (100 % of the S state and $F_{ch}^p = 1$, $F_{ch}^n = 0$), column II the charge form-factor (100 % S state and the corresponding values for F_{ch}^p (q^2) and F_{ch}^n (q^2)). Column III represents F_{ch} (^{3}H) taking S = 98 % and S′ = 2 % and $F_{ch}^p = 1$ and $F_{ch}^n = 0$, while in column IV for the same percentage the F_{ch}^p and F_{ch}^n dependence is taken into account. In column V are the F_{ch} (^{3}H) values obtained with 92 % S, 2 % S′ and 6 % D-state, and for $F_{ch}^p = 1$ and $F_{ch}^n = 0$, while in column VI the same percentage of the S, S′ and D-state are used, but various F_{ch}^p and F_{ch}^n are taken into account.

Table 2

q^2	F_{ch} (^{3}H)* exp.	F_{ch}^{p}**	F_{ch}^{n}**	F_{ch} (^{3}H) calculated					
				$P_S^2 = 1$		$P_S^2 = 0.98$; $P_{S'}^2 = 0.02$		$P_S^2 = 0.92$; $P_{S'}^2 = 0.02$ $P_D^2 = 0.06$	
				I	II	III	IV	V	VI
1	0.622	0.886	0.02	0.6497	0.6000	0.5951	0.6203	0.5810	0.6056
2	0.387	0.794	0.04	0.4228	0.3695	0.3595	0.3916	0.3448	0.3756
3	0.267	0.724	0.06	0.2768	0.2336	0.2183	0.2521	0.2095	0.2385
4	0.175	0.663	0.08	0.1810	0.1490	0.1367	0.1630	0.1280	0.1525
5	0.118	0.610	0.10	0.1175	0.0952	0.0851	0.1052	0.0783	0.0976
6	0.075	0.563	0.11	0.0752	0.0589	0.0522	0.0657	0.0476	0.0608
8	0.029	0.491	0.11	0.0279	0.0198	0.0173	0.0236	0.0169	0.0214

Tab. 2. — Momentum transfer dependence of the electric charge form-factor of the ^{3}He nucleus. The values in column I are obtained using for the nuclear radius $< r^2 >^{1/2} = 1.8$ fm. The quantities in other columns are calculed under the same conditions as in the corresponding columns of Table 1.

* Results from Ref.[20] ** Results from Ref.[21]

One can see that best agreement is found in columns IV for both cases. That reflects the importance of the neutron form factor dependence, as well as the ambiguities connected with the D-state function, especially, with the D-state radius:

$$< R_D^2 > \equiv \frac{1}{3} N_D^2 P_D^2 \int \Psi_D r_{23}^2 \Psi_D \, dV_G = P_D^2 \frac{1}{36 \beta^2} (16 + m)(17 + m).$$

The experimental minimum which appears for $q^2 = 6 \text{ fm}^{-2}$ (see Table 3) is probably accidental. It is intriguing that the theoretical minimum appears only in column VI. It might happen that a reasonable change of the D-state radius would shift it to the right position.

Table 3

F_{ch} (^{3}H)/F_{ch} (^{3}He)

q^2	exp.	I	II	III	IV	V	VI
1	1.0970	1.1070	1.0336	1.0674	1.0994	1.0670	1.0991
2	1.1763	1.2160	1.0735	1.0735	1.1979	1.1305	1.1973
3	1.2775	1.3411	1.1181	1.1775	1.2935	1.1883	1.1119
4	1.3258	1.4885	1.1714	1.2553	1.3802	1.2561	1.3927
5	1.4568	1.6762	1.2268	1.3402	1.4880	1.3349	1.4878
6	1.3889	1.9432	1.2667	1.3810	1.5831	1.4167	1.6085
8	1.7059	3.5316	1.3026	1.5586	1.8015	1.7071	1.8136

Tab. 3. — Ratio of the electric charge form-factors F_{ch} (^{3}He)/F_{ch} (^{3}H) for different momentum transfers. The values are obtained as the ratios of the quantities taken from the corresponding columns of Table 1 and 2.

The calculated charge form factors both for ^3He and ^3H nuclei tend to zero for the momentum transfer values of about 10 fm^{-2}. These minima are similar to those already found in the charge form factor of the ^4He nucleus[22]. Unfortunately, measurements for values of the momentum transfer $q^2 > 8$ fm^{-2} do not exist.

4. μ-meson capture by ^3He nuclei

The capture rate, Λ, for the reaction

$$\mu^- + {}^3He \rightarrow {}^3H + \nu \tag{19}$$

was measured by several groups[23]. The result[24] to which one usually refers in the theoretical considerations is: $\Lambda = (1410 \pm 140)\ s^{-1}$. This reaction was considered as one of the possible tests for the induced terms in the weak interaction[25], in which case the theoretical expression for the capture rate contains the relativistic corrections. The determination of the induced terms presumes that the radial part of the nuclear wave function is well known. Here, however, we want to calculate the capture rate with the new function, and since the relativistic corrections are of the same order of magnitude as the experimental errors, it will be sufficient to make the calculations in the nonrelativistic limit.

Within the framework of the V-A theory of the weak interaction, the capture rate for the reaction (19) is[26]

$$\Lambda = \frac{W}{(2\pi)^3}\frac{9}{2}\int d\nu \sum_{I_t = \pm\frac{1}{2}} \sum_{I_t = \pm\frac{1}{2}} \left\{ G_v^2 \left|\int 1\right|^2 + \Gamma^2 \left|\int \vec{\sigma}\right|^2 \right\}, \tag{20}$$

with

$$W = \left(\frac{2\, m'_\mu}{137}\right)^3 \frac{\nu^2}{1 + \frac{\nu}{M_{^3H}}},$$

where

$$G_v = g_v\left(1 + \frac{\nu}{M}\right) + g_s,\ \Gamma^2 = G_A^2 + \frac{1}{3}\left(G_P^2 - 2\, G_A G_P\right),$$

and

$$G_P = \frac{\nu}{2M}\left(g_P - g_A - g_V - g_M + g_T\right),\ G_A = g_A - \left(g_V + g_M\right)\frac{\nu}{2M}$$

are the constants which characterize the weak interaction; m'_μ and M are the reduced μ-meson and proton mass, respectively, ν is the neutrino impulse, and φ_{μ}, the S-state μ-mesic function, is taken to be a constant over nuclear volume. $\tau_l^{(-)}$ is the isospin operator for the particle l with which the elementary proces $\mu^- + p \rightarrow n + \nu$ takes place, and $\vec{\sigma}_l$ is the spin operator acting on this particle. For the wave function Ψ (^{3}H) and Ψ (^{3}He) as in the preceeding chapter we shall suppose that they are composed of the S, S′ and one of the D-components (Ψ_7).

The quantities $\int 1$ and $\int \vec{\sigma}$ are then:

$$\int 1 = \frac{1}{\sqrt{2}} \left[2 P_s^2 F_s + 4\sqrt{2}\, P_s P_{s'} P_{s-s'} + 2 P_{s'}^2 F_{s'} + P_D^2 (3 F_{D_1} - F_{D_2}) \right], \tag{22}$$

$$\int \vec{\sigma} = \frac{1}{\sqrt{2}} \sum_{\rho} (-)^{\frac{1}{2} - I_t + \rho}\, \vec{e}_{-\rho} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ -I_t & I'_t & \rho \end{pmatrix} \sqrt{6}\, \cdot$$
$$\cdot \left[-2 P_s^2 F_s + P_{s'}^2 I_{s'} + P_D^2 I_D \right], \tag{23}$$

where P_s^2, $P_{s'}^2$ and P_D^2 are the percentage of the S, S′ and D components in the total wave function. When Ψ (sym. scalar) is defined by (16) all other quantities are the integrals listed in the Appendix.

It is interesting to note that while the quantity (22) is directly related to the form factor integrals, the quantity (23) can be expressed through them only for the S state component.

Table 4

	exp.	$P_s^2 = 1$	$P_s^2 = 0.98$ $P_{s'}^2 = 0.02$	$P_s^2 = 0.92$ $P_{s'}^2 = 0.02$ $P_D^2 = 0.06$
Λ (s^{-1})	1410 ± 140	1555	1515	1401

Tab. 4. — The calculated capture rate, Λ, for the reaction $\mu^- + {}^3\text{He} \rightarrow {}^3\text{H} + \nu$. The experimental value is taken from Falomkin *et al.*[24] The constants which characterize the weak interaction process are taken as: $\left(g\frac{\beta}{A} / g\frac{\beta}{V}\right) = -1.18$; $g_P = 7 g_A$; $g\frac{\mu}{V} = 0.972\, g\frac{\beta}{V}$; $g\frac{\mu}{A} = 0.999\, g\frac{\beta}{A}$; $g_M = 3.7\, g\frac{\beta}{V}$; $g_S = g_T = 0$; $g\frac{\beta}{V} = 1.1473 \cdot 10^{-11}$ MeV^{-2}.

We have calculated the capture rate supposing first 100 % of the S state in the final and initial nuclei and then including 2 % of the S′ state and 6 % of the D-state. Results are given in Table 4. The obtained value given in the last column is in very good agreement with the experimental results.

5. *Conclusion*

The agreement with the experiments, obtained by including a »soft« core into the known radial dependences for the three nucleon bound state wave function, strongly suggests that the »core« part is important both for the small and high momentum transfer regions. Moreover, it is a unique quantity, which in a stronger form reproduces and predicts the diffraction minima that appear an electron scattering on very light nuclei.* The determination of the percentages of different components is a delicate question. It seems definitely that without introducing new parameters, S′-state is about 2 %. The analysis of the D-state implies a new parameter, its radius, which interplays with the value of its percentage, or more precisely, from the very complicate nature of the D-state it appears that the whole D-state function is a parameter.

Appendix

For the reduction of the integrals F_S, $F_{S'}$ and $F_{S-S'}$ to the form (I. 32) the relation:

$$\int d\Omega_1 \, d\Omega_2 \, (\vec{R}_1 \vec{R}_2)^{2K} \exp\{i \vec{q} \vec{R}_1\} = \frac{(4\pi)^2}{2K+1} (R_1 R_2)^{2K} j_0 (q R_1) \tag{A.1}$$

is used. For the reduction of the integrals F_{D_1} and F_{D_2} to the same form, we have used the following relations:

$$\int D_1 D_1^* (\vec{R}_1 \vec{R}_2)^2 \exp\{\vec{q} \vec{R}_1\} \, d\Omega_1 \, d\Omega_2 = \frac{4 (4\pi)^2}{9} R_1^2 R_2^2 \cdot$$

$$\cdot \left(R_1^4 + R_2^4 - \frac{4}{5} R_1^2 R_2^2 \right) j_0 (q R_1)$$

$$\int D_2 D_2^* (\vec{R}_1 \vec{R}_2) \exp\{\vec{q} \vec{R}_1\} \, d\Omega_1 \, d\Omega_2 = \frac{8 (4\pi)^2}{5} (R_1 R_2)^4 j_0 (q R_1) \cdot \tag{A.2}$$

The integrals in (17) are:

$$F_{S'} = (1+z)^{\frac{17}{2}+m} \left[\frac{18}{5} F \left(-\frac{m+5}{2}; -\frac{m+4}{2}; 4; -z \right) - \right.$$

* *Note added in proof*. The experiment of electron scattering on ^{3}He nucleus for higher momentum transfer, up to $q^2 = 20 \text{ fm}^{-2}$, is performed by J. S. McCarthy *et al.* Stanford and published in HELP — 635, July 1970. The diffraction minimum appears at $q^2 = 11.6 \text{ fm}^{-2}$.

$$- 4(1+z)\, F\left(-\frac{m+3}{2}; -\frac{m+2}{2}; 5; -z\right) +$$

$$+ \frac{7}{5}(1+z)^2\, F\left(-\frac{m+1}{2}; -\frac{m}{2}; 6; -z\right), \qquad \text{A.3)}$$

$$F_{S-S'} = -2\sqrt{\frac{2}{5}}\sqrt{\frac{(12+m)(13+m)}{(14+m)(15+m)}}\,(1+z)^{\frac{21}{2}+m}$$

$$\cdot F\left[9F\left(-\frac{m+7}{2}; -\frac{m+6}{2}; 4; -z\right) - \frac{85}{4}(1+z)\,\cdot\right.$$

$$\cdot F\left(-\frac{m+5}{2}; -\frac{m+4}{2}; 5; -z\right) + \frac{35}{2}(1+z)^2\,\cdot$$

$$\cdot F\left(-\frac{m+3}{2}; -\frac{m+2}{2}; 6; -z\right) - \frac{21}{4}(1+z)^3\,\cdot$$

$$\left.\cdot F\left(-\frac{m+1}{2}; -\frac{m}{2}; 7; -z\right)\right]. \qquad \text{A.4)}$$

$$F_{S'} = (1+z)^{\frac{25}{2}+m}\left[9F\left(-\frac{m+9}{2}; -\frac{m+8}{2}; 4; -z\right) - 25(1+z)\,\cdot\right.$$

$$\cdot F\left(-\frac{m+7}{2}; -\frac{m+6}{2}; 5; -z\right) + 7\,\frac{22}{5}(1+z)^2\,\cdot$$

$$\cdot F\left(-\frac{m+5}{2}; -\frac{m+4}{2}; 6; -z\right) - \frac{7\cdot 13}{5}(1+z)^3\,\cdot$$

$$\cdot F\left(-\frac{m+3}{2}; -\frac{m+2}{2}; 7; -z\right) + \frac{22}{5}(1+z)^4\,\cdot$$

$$\left.\cdot F\left(-\frac{m+1}{2}; -\frac{m}{2}; 8; -z\right)\right]. \qquad \text{A.5)}$$

$$F_{D_1} = (1+z)^{\frac{25}{2}+m}\left[\frac{360}{65}F\left(-\frac{m+9}{2}; -\frac{m+8}{2}; 4; -z\right) - \frac{714}{65}(1+z)\,\cdot\right.$$

$$\cdot F\left(-\frac{m+7}{2}; -\frac{m+6}{2}; 5; -z\right) + \frac{850}{85}(1+z)^2\,\cdot$$

$$\cdot F\left(-\frac{m+5}{2};-\frac{m+4}{2};6;-z\right)-\frac{1071}{65}(1+z)^3\cdot$$

$$\cdot F\left(-\frac{m+3}{2};-\frac{m+2}{2};7;-z\right)+\frac{198}{65}(1+z)^4\cdot$$

$$\cdot F\left(-\frac{m+1}{2};-\frac{m}{2};8;-z\right)\Bigg] \qquad \text{A.6)}$$

$$F_{D_2}=(1+z)^{\frac{25}{2}+m}\left[\frac{750}{65}(1+z)\,F\left(-\frac{m+7}{2};-\frac{m+6}{2};5;-z\right)-\right.$$

$$-\frac{1477}{65}(1+z)^2 F\left(-\frac{m+5}{2};-\frac{m+4}{2};6;-z\right)+$$

$$+\frac{1078}{65}(1+z)^3 F\left(-\frac{m+3}{2};-\frac{m+2}{2};7;-z\right)-$$

$$\left.-\frac{286}{65}(1+z)^4 F\left(-\frac{m+1}{2};-\frac{m}{2};8;-z\right)\right]. \qquad \text{A.7)}$$

Where $F(l;p;q;x)$ is a Gauss hypergeometric function, and $z = q^2/12\beta^2$. The integrals in (23) are:

$$I_s' = I_{s'1} + I_{s'2}\,,$$

$$I_{s'1}=2\left(1+\frac{\nu^2}{12\beta^2}\right)^{\frac{25}{2}+m}\left[9\,F\left(-\frac{m+9}{2};-\frac{m+8}{2};4;-\frac{\nu^2}{12\beta^2}\right)-\right.$$

$$-\frac{65}{2}\left(1+\frac{\nu^2}{12\beta^2}\right)F\left(-\frac{m+7}{2};-\frac{m+6}{2};5;-\frac{\nu^2}{12\beta^2}\right)+$$

$$+\frac{189}{4}\left(1+\frac{\nu^2}{12\beta^2}\right)^2 F\left(-\frac{m+5}{2};-\frac{m+4}{2};6;-\frac{\nu^2}{12\beta^2}\right)-$$

$$-\frac{63}{2}\left(1+\frac{\nu^2}{12\beta^2}\right)^3 F\left(-\frac{m+3}{2};-\frac{m+2}{2};7;-\frac{\nu^2}{12\beta^2}\right)+$$

$$\left.+\frac{33}{4}\left(1+\frac{\nu^2}{12\beta^2}\right)^4 F\left(-\frac{m+1}{2}\cdot-\frac{m}{2};8;-\frac{\nu^2}{12\beta^2}\right)\right], \qquad \text{A.8)}$$

$$I_{S_2} = \frac{10}{3}\left(1+\frac{\nu^2}{12\,\beta^2}\right)^{\frac{25}{2}+m}\left[\frac{15}{2}\left(1+\frac{\nu^2}{12\,\beta^2}\right)\cdot\right.$$

$$\cdot F\left(-\frac{m+7}{2};-\frac{m+6}{2};5;-\frac{\nu^2}{12\,\beta^2}\right)-\frac{329}{20}\left(1+\frac{\nu^2}{12\,\beta^2}\right)^2\cdot$$

$$\cdot F\left(-\frac{m+5}{2};-\frac{m+4}{2};6;-\frac{\nu^2}{12\,\beta^2}\right)+\frac{133}{10}\left(1+\frac{\nu^2}{12\,\beta^2}\right)^3\cdot$$

$$\cdot F\left(-\frac{m+3}{2};-\frac{m+2}{2};7;-\frac{\nu^2}{12\,\beta^2}\right)-\frac{77}{20}\left(1+\frac{\nu^2}{12\,\beta^2}\right)^4\cdot$$

$$\left.\cdot F\left(-\frac{m+1}{2};-\frac{m}{2};8;-\frac{\nu^2}{12\,\beta^2}\right)\right], \qquad \text{A.9)}$$

$$I_D = I_{D_1} + I_{D_2},$$

$$I_{D_1} = \frac{6}{13}\left(1+\frac{\nu^2}{12\,\beta^2}\right)^{\frac{25}{2}+m}\left[9\,F\left(-\frac{m+9}{2};-\frac{m+8}{2};4;-\frac{\nu^2}{12\,\beta^2}\right)-\right.$$

$$-\frac{34\cdot 5!!}{4!}\left(1+\frac{\nu^2}{12\,\beta^2}\right)F\left(-\frac{m+7}{2};-\frac{m+6}{2};5;-\frac{\nu^2}{12\,\beta^2}\right)+$$

$$+\frac{58\cdot 7!!}{2\cdot 5!}\left(1+\frac{\nu^2}{12\,\beta^2}\right)^2F\left(-\frac{m+5}{2};-\frac{m+4}{2};6;-\frac{\nu^2}{12\,\beta^2}\right)-$$

$$-\frac{48\cdot 9!!}{4\cdot 6!}\left(1+\frac{\nu^2}{12\,\beta^2}\right)^3F\left(-\frac{m+3}{2};-\frac{m+2}{2};7;-\frac{\nu^2}{12\,\beta^2}\right)+$$

$$\left.+\frac{16\cdot 11!!}{8\cdot 7!}\left(1+\frac{\nu^2}{12\,\beta^2}\right)^4F\left(-\frac{m+1}{2};-\frac{m}{2};-\frac{\nu^2}{12\,\beta^2}\right)\right], \qquad \text{A.10)}$$

$$I_{D_2} = \frac{6}{13}\left(1+\frac{\nu^2}{12\,\beta^2}\right)^{\frac{25}{2}+m}\left[9\;\frac{5!!}{4!}\left(1+\frac{\nu^2}{12\,\beta^2}\right)\cdot\right.$$

$$\cdot F\left(-\frac{m+7}{2};-\frac{m+6}{2};5;-\frac{\nu^2}{12\,\beta^2}\right)-\frac{633}{25}\;\frac{7!!}{2\cdot 5!}\left(1+\frac{\nu^2}{12\,\beta^2}\right)\cdot$$

$$\cdot F\left(-\frac{m+5}{2};-\frac{m+4}{2};6;-\frac{\nu^2}{12\,\beta^2}\right)+\frac{616}{25}\;\frac{9!!}{4\cdot 6!}\left(1+\frac{\nu^2}{12\,\beta^2}\right)\cdot$$

$$\cdot F\left(-\frac{m+3}{2};-\frac{m+2}{2};7;-\frac{\nu^2}{12\beta^2}\right)-\frac{208}{25}\frac{11!!}{8\cdot 7!}\left(1+\frac{\nu^2}{12\beta^2}\right)\cdot$$

$$\cdot F\left(-\frac{m+1}{2};-\frac{m}{2};8;-\frac{\nu^2}{12\beta^2}\right)\Bigg]. \qquad \text{A.11)}$$

The parameters β and m are related to the root mean square radius and Coulomb energy by the formulae:

$$\langle r^2 \rangle = \frac{1}{6\beta^2}\left[P_S^2(12+m)(13+m)+P_{S'}^2(16+m)(17+m)+\right.$$
$$\left.+P_D^2(16+m)(17+m)\right], \qquad \text{A.12)}$$

$$E_c = e^2\frac{\beta}{\pi}\frac{3\cdot 2^6}{9!!}\left\{\frac{2\cdot 61}{5(11+m)}P_S^2+\frac{6\cdot 29}{11}\cdot\right.$$
$$\cdot\left[\frac{12+m}{10(13+m)(14+m)(15+m)}\right]^{1/2}P_S P_{S'}+\frac{27\cdot 137}{11\cdot 13}\frac{1}{15+m}P_{S'}^2+$$
$$\left.+\frac{64\cdot 53}{11\cdot 13}\frac{1}{15+m}P_D^2\right\}. \qquad \text{A.13)}$$

and for $\begin{matrix} m=-3; \\ n=1 \end{matrix}$ $\langle r^2 \rangle = 3\ \text{fm}^2$; $\beta = \sqrt{\frac{3}{2}}\,0.4945\ \text{fm}^{-1}$; $E_c = 0.6255$ MeV.

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O TALASNOJ FUNKCIJI OSNOVNOG STANJA TROČESTIČNIH NUKLEARNIH SISTEMA

II. Svojstva radijalnog dela talasne funkcije. Uvođenje »soft-core«. Form-faktor naelektrisanja. Zahvat μ-mezona.

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Sadržaj

U radu je diskutovano određivanje radijalnog dela talasne funkcije tronukleonskog sistema. U publikaciji[1] pokazano je da je argument ove funkcije simetrični skalar.

U poglavlju 2 diskutovani su asimptotski uslovi koje funkcija treba da zadovoljava (relacija 13′) dok je u relaciji 16 uveden kompletan »soft-core«.

U poglavlju 3 diskutovan je form factor jezgara ^3H i ^3He dat u (relaciji 17 i 18). Numerički rezultati za različite parametre funkcije (relacija 16) dati su u tablicama 1, 2 i 3 dok su analitički rezultati dati u apendiksu.

U poglavlju 4 diskutovan je zahvat μ-mezona na jezgru ^3He. Analitički izrazi za verovatnoću procesa dati su relacijama 20, 22 i 23, dok su numerički rezultati dati u tablici 4.

Dobro slaganje računatih podataka sa eksperimentalnim opravdava definisanje funkcije relacijom 16, i ističe potrebu razmatranja korelacija kratkog dosega — »soft-core«.