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A NEW SU(3) CLASSIFICATION OF VECTOR MESONS

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Abstract: The Gell-Mann-Okubo mass formula for vectons is examined. It is shown that if we include the B-mesons in the list of vectons, the G-O mass formula is exactly satisfied. This leads us to the existence of two octets of vecton and suggests that the ω — φ mixing can be abandoned. Further consequences of this assumption are considered.

1. Introduction

The Gell-Mann-Okubo[1, 2] formula works very well with baryons and the baryon resonances. Using the fact that hypercharge Y anti-commutes with the charge operator while the mass operator for bosons commutes, we can write the G-O mass relation for bosons as $M = M_0 [A + C \{ (T + 1) — Y^2/4\}]$, where T is the isotopic spin and M_0, A and C are constants. This formula can be used to obtain

$$\frac{M_{T=1} — M_{T=0}}{M_{T=1} — M_{T=1/2}} = \frac{4}{3}, \tag{1}$$

where $M_{T=1}$, $M_{T=0}$ and $M_{T=1/2}$ are the masses of the mesons triplet, singlet, and doublet. If the experimental masses of vector mesons are used, one obtains $x = 2.0$ and 1.14 for the combinations of ρ K* φ and ρ K* ω of the known vector mesons. At the suggestion of Feynman, (mass)^2 of the scalar mesons were used in the G-O formula and the agreement between the theory and experiment improved. In the case of vector mesons we obtain 2.22 and 1.07 when the square of the masses of the combinations ρ K* φ and ρ K* ω is used. To overcome this discrepancy, Sakurai[3] suggested that ω and φ are a mixture of a pure singlet $|1\rangle$ and a pure octet $|8\rangle$ states and given by

$$
\begin{aligned}
|\omega\rangle &= +|1\rangle \cos\theta \pm |8\rangle \sin\theta, \\
|\varphi\rangle &= |1\rangle \cos\theta + |8\rangle \cos\theta.
\end{aligned} \qquad (2)
$$

It is easy to find an angle θ such that the mass of $|8\rangle$ satisfies (1).

In order to make this theory plausible, many theoretical[5] and experimental attempts have been made to justify it. In the next Chapter we discuss these in detail and show that the present experimental results can be explained without any mixing.

2. *Experimental results*

The ω and φ particles are strongly interacting and their dominant decay modes are via strong interactions. Any theoretical calculation for the strong interaction can be challenged for its validity. The ambiguity decreases considerably if we consider the ratio of two decay modes where all the particles in the final and initial states are the same except φ and ω, which are symmetrical. Let us consider such a ratio r where r is

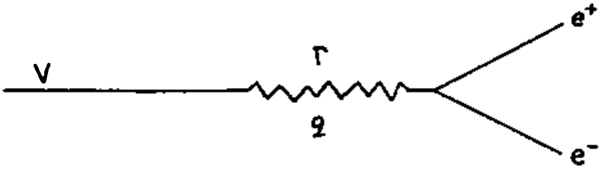
$$
r = \frac{\Gamma(B \to \varphi + \pi)}{\Gamma(B \to \omega + \pi)}. \qquad (3)
$$

If we assume that B and π are related to the representation of SU(3) having different dimensions, then only the $|8\rangle$ content of φ and ω will contribute to the matrix element of these decays. If we denote by K_φ and K_ω the momenta of φ and ω in the rest system of B and by l the orbital angular momentum of the $\pi + \varphi$ and $\pi + \omega$ system, then it follows from (2) that

$$
r = \cot^2\theta \left(\frac{K_\varphi}{K_\omega} \right)^{(2l+1)}, \qquad (4)
$$

where in order to satisfy the mass formula $\cot^2\theta = 1.4$.

Let us analyze this relation in the light of mixing. If the mixing angle changes to 45° the value of $\cot^2\theta = 1$ and the value of r will change by about 30%, which is not bad because the experimental results have an uncertainty of 50%. The value that we obtained by a mixing of 45° is equivalent to the fact that both ω and φ have the same coupling to B and π and belong to the same representation of SU(3), because the singlet does not contribute anyway. If we do not impose the extra condition of equality of the two coupling constants we have one more parameter available to make the prediction agree with the experimental result. Therefore we consider the least ambiguous calculations, the decay of these mesons into two electrons. The Feynman diagram for this process is



and the matrix element is $M = (4)^{3/2} f \alpha \varphi_{\beta} \bar{u} \gamma_{\beta} u$. α is the fine structure constant, φ is the wave function of the meson. The factor f is the only unknown and if we assume it to be a constant, then t, the ratio of the decays of ω and φ mesons is equal to m_{ω}/m_{φ} where m_{ω} and m_{φ} are the masses of ω and φ mesons and $t \simeq 0.76$, which agrees well with the experimental value of 0.69 ± 0.14. In the mixing model, $t = \cot^2 \theta$ (phase space available) ≈ 0.48[7]. It may be remarked that the calculations for such decays are very reliable. Both the calculated and measured values have more uncertainties in all other cases and one can either believe or disbelieve mixing. But the decay into two electrons seems to be the least ambiguous and needs no mixing[8].

3. *Mass relation for vectons*

Thus we abandon the mixing model and examine the mass formula for vector mesons. If (1) is to be satisfied by vectons, M_{T_0} should be equal to 933 MeV. Howewer, if the G-O relation is true for (mass)^2 of the boson the desired value of M_{T_0} is 929 MeV. As today's experiment cannot find the masses of mesons to an accuracy greater than 0.4$^0/_0$, there is no way to tell whether we should adapt the mass or (mass)^2 relation for vectons[9].

Here we look closely at the list of vectons. A controversial vecton is the B-meson of mass about 1220[10]. Its isospin is 1 and G-parity $(G) = +1$. It decays via ω and π strongly followed by the strong decay of ω, and thus the background does not allow the measurement of the angular distribution uniquely. Hence, it is not easy to determine its spin and parity. There have been some evidences[10] that it decays through the S-channel and hence it cannot decay into 2 π's or 2 K's[11, 12]. Now if we go back to the relation (3) and use the expression (4) we get $r = 0.58$ for $J^P = 1^+$ for B-meson and $r = 0.1$ for $J^P = 1^-$. The experimental value for $r = 0.2 \pm 0.1$[10] where the assumption has been made that these decays are allowed by SU(3). If we abandon the mixing, this ratio will depend upon the coupling constants $g_{B\omega\pi}$ and $g_{B\varphi\pi}$ to which we will return later. Thus, we see that the experimental evidence is in favor of $J^P = 1^-$ for B only if we assume that it decays through the S-channel.

Let us examine the value of x. For the combination BK* ω it is exactly 4/3. In order that we can classify all vector mesons in the octet model we need another set of quadruplet vector resonance which we call K*′. It is easy to see that its mass should be 960 MeV if it has nonzero strangeness, and 980 if it is not strange. There are many good candidates for it. Recently[13] evidence has been found for a meson of mass 980 MeV. There has been some doubt regarding the spin in parity of K*′.

4. *Existence of the new meson*

There is another theoretical reason to believe the existence of this new meson. Cabibbo and Chilton[14] have calculated the total cross section for the decays

$$\bar{\nu}_\mu + p \rightarrow \mu^+ + \Lambda,$$

$$\bar{\nu}_\mu + p \rightarrow \mu^+ + \Sigma^-,$$

$$\bar{\nu}_\mu + p \rightarrow \mu^+ + \Sigma^0,$$

based on the SU(3) weak interaction and assuming K* to be the dominant pole. They found a value of $3 \cdot 10^{-40}$ cm^2/nucleon as against the experimental value of $1.98 \cdot 10^{-40}$ cm^2/nucleon. If we assume the existence of another meson K*′ with all the quantum numbers the same as K* of mass 960 MeV, we calculate for the cross-section[15] a value of $2 \cdot 10^{-40}$ cm^2/nucleon in close agreement with the experimental value.

5. *SU(6), quark model and new classification*

Although there is no satisfactory model of SU(6) so far, we want to examine the existence of two octets in its light, because of the success of SU(6) in many predictions[16]. Bosons belong to the *35* and *1* representation[17] of SU(6). If we follow the same process of getting multiplicity through spin, the bosons will belong to the *56* and *1* representation. This classification is not desirable because of the existence of *56*, which contains antibosons different from bosons. Therefore, we follow the suggestion[18] of looking at the problem in terms of their interaction rather than spin. In this model vector mesons have single multiplicity because their interaction with fermions proceeds via p-wave in the momentum space. If we do so, one vecton octet and the presently known nonet of pseudoscalar mesons belong to *35* representations of SU (6). We need another nine pseudoscalar mesons along with the remaining vector octet to form another *35* representation of SU(6)[19]. Obviously this leads to another question. What happens to the quark-model of Gell-Mann? We cannot justify their existence because they cannot give two octets of vector mesons unless there are two sets of them!

6. Conclusion

The unnatural assumption of mixing of ω and φ does not arise if we include the B-meson in the family of vector (not axial vector) mesons. We then have two octets of vector mesons and each satisfies the G-O mass relation. This new classification puts the octets on a fundamental basis and does not need the existence of quarks.

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O JEDNOJ NOVOJ SU(3) KLASIFIKACIJI VEKTORSKIH MEZONA

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Sadržaj

U radu je ispitana Gell-Mann-Okubova masena formula za vektorske mezone. Pokazano je da je ona egzaktno zadovoljena ukoliko se u listu vektorskih mezona uključe i B-mezoni. To dalje vodi na postojanje dva okteta vektorskih mezona pri čemu nije potrebna neprirodna pretpostavka o mešanju ω i φ mezona. Predložena klasifikacija ne zahteva postojanje kvarkova.