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USE OF THE SAHA EQUATION FOR THE SIMULTANEOUS DETERMINATION OF TEMPERATURE AND ELECTRON CONCENTRATION*

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Abstract: The Saha equation was applied for the simultaneous determination of temperature and electron concentration using a suitable representation of the relative line intensities of spectral lines emitted by ionized and neutral atoms. An illustration of the method is shown.

The Saha equation describes the thermal equilibrium between neutral and ionized atoms for any particular element in a mixture of elements, with temperature and electron concentration as variables. Where no thermal equilibrium exists, i. e. temperature of gas atoms, temperature of free electrons and temperature corresponding to the level populations of atoms are not equal, the ratio of neutral and ionized atoms defines the ionization temperature T_i which corresponds to the given electron concentration N_e. The ionization temperature differs from other temperatures until the local thermal equilibrium is approached[1].

The Saha equation reads:

$$\frac{N^+ u^0}{N^0 u^+} N_e = 10^{15.68} T_i^{3/2} 10^{-5040 \chi / T_i} \tag{1a}$$

or

$$Y(T_i, \chi) = \log \frac{N^+ u^0}{N^0 u^+} = 15.68 - \log N_e + 1.5 \log T_i - \frac{5040}{T_i} \chi = $$
$$= Y_0 (N_e, T_i) - a(T_i) \cdot \chi, \tag{1b}$$

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where χ is the ionization energy in eV, N^+ and N^0 are the concentrations of ionized and neutral atoms, and u^+ and u^0 their partition functions.

For evaluation of T_i and N_e one has to establish the dependence between Y and χ using different elements and knowing the percentage of their ionizations. This method was applied to the data given in Ref.[2]. In this reference, the ionization percentage was measured by means of the relative line intensities of spectral lines I^+ and I^0, emitted by ionized and neutral atoms of the same species:

$$\frac{N^+ u^0}{N^0 u^+} = \frac{(\lambda^3 I)^+ (gf)^0}{(\lambda^3 I)^0 (gf)^+} 10^{-5040 (E^0 - E^+)/T}, \tag{2}$$

where g is the statistical weight of the lower level of transition, f the oscillator strength, λ the wavelength, and E (eV) the excitation energy of the upper level of transition.

Combining Eqs. (1) and (2) with $T = T_i$ we obtain

$$\Theta (N_e, T_i) = \log \frac{(\lambda^3 I)^+ (gf)^0}{(\lambda^3 I)^0 (gf)^+} = 15.68 - \log N_e + 1.5 \log T_i - \\ - \frac{5040}{T_i} (\chi - E^0 + E^+) = \Theta_0 (N_e, T_i) - a (T_i) \Phi. \tag{3}$$

The method of least squares helps in finding the best fit of the straight line, Eq. (3), and the points observed (Fig. 1). For this, we have to determine the following sum, for various temperatures:

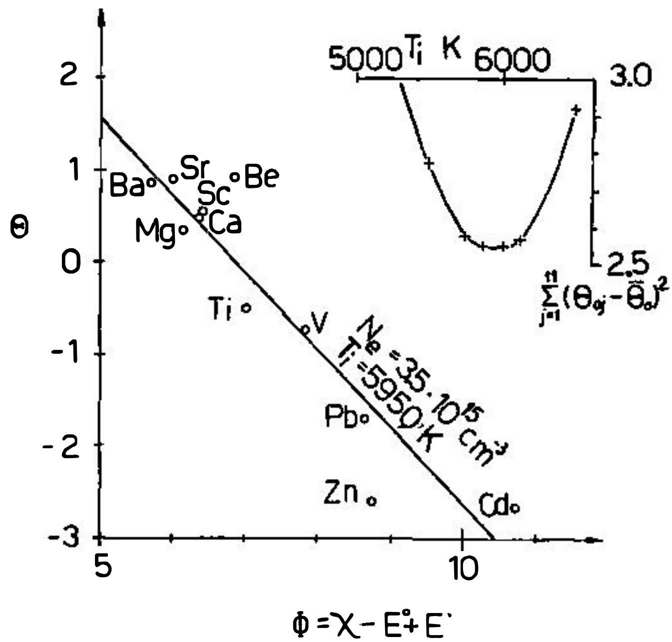
$$\sum_{j=1}^{n} (\Theta_{0j} - \overline{\Theta}_0)^2 \ldots, \tag{4}$$

T being the parameter.

Θ_{0j} is the intercept for the straight line which passes through a particular point in Fig. 1 (j-th element) and it was evaluated using Eq. (3) and taking all the required data (λ, I, gf, χ, E) from Table 4 in Ref. [2]

$\overline{\Theta}_0$ is the mean intercept equal to $\sum_{j=1}^{n} \Theta_{0j}/n$. The sum (4) was then plotted (insert in Fig. 1) as a function of temperature, choising that temperature and that electron concentration which corresponded to the minimum of the sum of squares.

The following results were obtained: $T_i = 5950$ K, $N_e = 3.5 \times 10^{15}$ cm^{-3}. In Ref.[2] the electron concentration equal to 2.4×10^{14} cm^{-3} was determined by presuming a temperature of 5100 K, which means that the potentialities of



the Saha equation were not fully utilized. (The procedure applied in[2] was reproduced in[3]; the representation of physical quantities was slightly different:

$$\log \frac{N^+ u^0}{N^0 u^+}$$

versus χ).

The deviations of data points from the mean straight line which warn against the use of a small number of spectral-line pairs, may have been caused by several factors, e, g. innacurate gf-values and departures from thermal equilibrium. The population of neutral and ionized atoms in Eq. (2) is described by a single temperature only in case of thermal equilibrium. In a real source, the temperatures describing the neutral atom population and the population of ions could be different, making the equation (2) invalid; when using Eq. (2), the error introduced by a mean temperature would be smaller for a smaller difference of $|E^+ - E^0|$. In inspecting originial data in Ref.[2], a large difference $|E^+ - E^0|$ was found for Be(20 000 cm^{-1}) and Cd(12 000 cm^{-1}), i. e. for elements which show marked deviations from the straight line in Fig. 1, but no general conclusion can be drawn from this fact. The dependence of the measured T_i, N_e on the difference in excitation energies, could be used for checking local thermal equilibrium.

References

1) W. Lochte-Holtgreven, ed., Plasma Diagnostics, North Holland Publ. Co., Amsterdam 1968, p. 140;
2) C. H. Corliss and W. R. Bozman, NBS Monograph **53** (1962);
3) P. W. J. M. Boumans, Theory of Spectrochemical Excitation, Hilger and Watts, London 1966, **172** (Fig. 7.4).

SIMULTANO ODREĐIVANJE TEMPERATURE I KONCENTRACIJE ELEKTRONA UPOTREBOM JEDNADŽBE SAHE

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Sadržaj

Jednadžba Sahe primijenjena je za određivanje temperature i koncentracije elektrona korišćenjem spektralnih linija ioniziranih i neutralnih atoma različitih elemenata. Pokazano je da se iz odsječka na ordinati i nagiba pravca Θ (Φ), jedn. (3), istovremeno određuje i temperatura i koncentracija elektrona. Eksperimentalni pravac izveden je iz mjerenih vrijednosti metodom najmanjih kvadrata.