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COUPLINGS AND DECAYS OF VECTOR MESONS*

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Abstract: Superconvergent dispersion sum rules for $\rho\rho \to \rho\rho$ and $\rho A_1 \to \rho A_1$ forward scatterings are formulated and their dynamical content within certain saturatin hypotheses is studied in terms of relevant coupling constants. The approximate solution obtained for the coupling constants in the ρA_1 case implies the Schnitzer-Weinberg parameter δ close to —1, in agreement with predictions following from the Veneziano representation and supporting the general idea of universality of the ρ-meson charge coupling. A rough solution of the $\rho\rho$ case sum rules is followed by a mass relation experimentally verified within 10%.

1. Introduction

Vector mesons ρ, ω, φ, B, A_1,... play an important role in the structure of hadronic matter. Vector meson dominance models are well known in particle physics, while in nuclear physics they generate the repulsive component of the force (hard core) which prevents the nucleon from collapsing under the influence of the attractive force generated by pseudoscalar meson exchanges. However, due to the fast decay of vector mesons via strong interactions, there is no direct experimental information about their scattering processes.

This article is based on a general method of studying vector meson parameters (masses, couplings, decay widths,...) by means of superconvergent dispersion sum rules. Following the procedure which has widely been successfully applied to the processes involving lower spin particles[1], we are able to formulate dynamical relations between masses and couplings of vector mesons using general requirements of unitarity and Lorentz co-

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variance, supplemented by the usual postulats about analyticity, asymptotics and intermediate mass spectra. Here we shall limit our considerations to the experimentally unrealizable elastic forward scattering processes $\rho A_1 \to \rho A_1$ and $\rho\rho \to \rho\rho$. Approximate solutions for the coupling constants, accompanied by a mass sum rule, are obtained within certain saturation hypotheses. The Schnitzer-Weinberg parameter[2] δ (anomalous magnetic moment of A_1) is found to be close to —1, in agreement with other theoretical predictions and consistent with the idea of universality of the ρ-meson charge coupling.

2. *Kinematics and definitions*

The general Lorentz invariant form for the elastic scattering amplitude

of vector mesons $V_\alpha(p) + V_\beta(q) \to V_\gamma(p') + V_\delta(q')$ is

$$T_{\gamma\delta\,;\,\alpha\beta}(s,t,u) = \overset{*}{e}_\lambda(p')\,\overset{*}{e}_\sigma(q') \left\{ \sum M^i_{\gamma\delta\,;\,\alpha\beta}(s,t,u) K_i^{\lambda\sigma\,;\,\mu\nu} \right\} e_\mu(p)\, e_\nu(q). \quad (1)$$

Here p, q and p', q' represent the fourmomenta of the incoming and outgoin particles, respectively. The corresponding polarisation fourvectors are denoted by $e(p)$, $e(q)$ and $e^*(p')$, $e^*(q')$. α and γ are the initial and final isospin indices of the ρ meson, while β and δ mean the same for the other meson in question. s, t and u are Mandelstam's scalar variables.

Unessential complications due to the spin are solved by defining the set of kinematical covariants[3] $K_i^{\lambda\sigma\,;\,\mu\nu}(p', q'; p, q)$, $(i = 1, 2, \ldots 25)$, which generate 25 invariant amplitudes $M^i(s,t,u)$. These amplitudes are free of kinematical singularities, contain all of dynamics of systems, and by assumption obey the Mandalstam representation.

The problem of isospin is solved by the expansion

$$M^i_{\gamma\delta\,:\,\alpha\beta} = A^i(\delta_{\alpha\beta}\,\delta_{\gamma\delta} + \delta_{\alpha\delta}\,\delta_{\beta\gamma}) + B^i(\delta_{\alpha\beta}\,\delta_{\gamma\delta} - \delta_{\alpha\delta}\,\delta_{\beta\gamma}) + C^i\,\delta_{\alpha\gamma}\,\delta_{\beta\delta} \quad (2)$$

The amplitudes $A^{(0)}$, $A^{(1)}$ and $A^{(2)}$, corresponding to the total isospin 0, 1, 2, respectively, are linear combinations of A, B and C. These amplitudes do not possess the simple $s \longleftrightarrow u$ crossing symmetry ($p \longleftrightarrow -p'$, $\alpha \longleftrightarrow \gamma$ or $q \longleftrightarrow q'$, $\beta \longleftrightarrow \delta$) as is the case with the amplitudes A, B and C.

3. *Asymptotics and superconvergent relations*

The rigorous upper bounds on the scattering amplitude as the energy tends to infinity are too high to be of practical use. It is more restrictive to argue along the line of the Pomeranchuk theorem [$\sigma_T(\infty) = \text{const.}$] and a reasonable assumption that the total amplitude at high energies does not grow faster than its imaginary part. The diffraction character of elastic scatterings at high energies is a good support to that assumption. It can be shown in this way[3] that the total amplitude grows linearly with the meson lab. energy, which implies the validity of the following superconvergent relations

$$\int_{-\infty}^{\infty} C^i(\nu, t=0)\, d\nu = 0, \quad (i = 4, 5, 13). \tag{3}$$

Here ν denotes the laboratory energy of the incoming meson. The above relation is nontrivial only for the imaginary part of amplitudes because it is trivially satisfied for their real part due to the crossing symmetry. Dynamical relations between masses and coupling constants follow from relation (3) only if a certain hypothesis about its saturation by one-particle intermediate states is adopted.

4. *Saturation hypotheses*

Being the consequence only of analyticity and asymptotics requirements, relation (3), as it stands, contains no dynamical information. Different classes of Hamiltonians can generate the same analyticity and asymptotics of scattering amplitudes. Only the assumption that a certain set of one-particle intermediate states saturates the integral (3) generates a kind of a dynamical equation. Such a saturation hypothesis brings in the information about the intermediate mass spectrum and selection rules.

We shall parallelly consider the elastic forward scatterings $\rho A_1 \to \rho A_1$ and $\rho\rho \to \rho\rho$. The first case has already been treated[4] within the (π, π_N, ω, φ, A_1)-saturation hypothesis. We shall here consider the implications of the simpler hypothesis that only spinless (π and π_N) mesons dominate. In both cases the hypothesis says that the amplitude in the low-energy region is dominated by nearest singularities — lowest mass mesons. Similarly, in the $\rho\rho \to \rho\rho$ process the dominance of ρ and B intermediate mesons is assumed.

In order to calculate one-intermediate-particle contributions to the scattering amplitude, we have to define three-meson couplings. Their most general form is obtained by expanding the meson vertices in terms of isospin and Lorentz covariants, thus defining form factors which become coupling constants when all particles are on their mass shells.

Defining the $\rho\pi A_1$ and $\rho\pi_N A_1$ vertices by

$$\langle A^a(k) \mid \rho^b(p)\,\pi^{c\,'}(q)\rangle = i\varepsilon^{abc}\, m_\rho\, g_{\pi\rho A_1}\, e^*_\nu(k)\, e_\mu(p) \left\{ g^{\nu\mu} + \frac{C}{m^2_\rho} p^\nu k^\mu \right\}, \quad (4)$$

$$\langle A^a(k) \quad \rho^b(p)\,\pi^c_N(q)\rangle = i\varepsilon^{abc}\, \frac{1}{m_\rho}\, g_{\pi_N \rho A_1}\, e^*_\nu(k)\, e_\mu(p)\, \varepsilon^{\mu\nu\alpha\beta}\, p_\alpha\, q_\beta, \quad (5)$$

we obtain their contributions to the invariant amplitudes after lengthy calculations and decomposition according to the kinematical covariants

$$K_i^{\lambda\sigma\,;\,\mu\nu}.$$

For the $\rho\rho\rho$ and $\rho\rho B$ vertices appearing in the $\rho\rho\to\rho\rho$ amplitude, we similarly write

$$\rangle \rho^a(k) \mid \rho^b(p)\,\rho^c(q)\rangle = i\varepsilon^{abc}\, e^*_\alpha(k)\, e_\mu(p)\, e_\nu(q) \left\{ f_1 g^{\nu\alpha} k^\mu + f_2 g^{\mu\alpha} k^\nu + \right.$$
$$\left. + f_3 g^{\mu\nu} p^\alpha + \frac{f_4}{m^2_\rho} k^\mu k^\nu p^\alpha \right\}. \quad (6)$$

$$\langle B^a(k) \mid \rho^b(p)\,\rho^c(q)\rangle = i\varepsilon^{abc}\, e^*_\alpha(k)\, e_\mu(p)\, e_\nu(q) \left\{ d_1\, \varepsilon^{\mu\nu\alpha\beta} p_\beta + d_2\, \varepsilon^{\mu\nu\alpha\beta} q_\beta + \right.$$
$$\left. + \frac{d_3}{m^2_\rho}\, \varepsilon^{\mu\nu\gamma\beta} p_\gamma\, q_\beta\, p^\alpha \right\}. \quad (7)$$

The ρ mass is introduced for dimensionality reasons.

5. *Results and discussions*

Taking into account the transversality property

$$p_\mu\, e^\mu(p) = 0$$

and the formula for the summation over helicities

$$\sum_{\alpha_p} e^{*\,\alpha_p}_\mu(p)\, e^{\alpha_p}_\nu(p) = \frac{p_\mu p_\nu}{m^2} - g_{\mu\nu},$$

our saturation hypothesis for the ρA_1 amplitude leads to equal amplitudes C_4 and C_5 and we obtain only two nontrivial sum rules from relation (3)

$$g^2_{\pi\rho A_1}\, C^2 - g^2_{\pi_N \rho A_1} = 0\,, \tag{8}$$

$$g^2_{\pi\rho A_1}\, C - g^2_{\pi_N \rho A_1}\, \frac{m_\rho^2 + m_{A_1}^2 - m_{\pi_N}^2}{2\, m_\rho^2} = 0\,. \tag{9}$$

This system uniquely determines the constant C which gives the S to D wave ratio of the $A_1 \to \rho\,\pi$ decay

$$C = \frac{2\, m_\rho^2}{m_\rho^2 + m_{A_1}^2 - m_{\pi_N}^2} \approx 1.74. \tag{10}$$

According to our definition (4), the ratio of S to D wave coupling is

$$\frac{g_S}{g_D} = \frac{m_\rho\, g_{\pi\rho A_1}}{m_\rho\, g_{\pi\rho A_1}\, C/m_\rho^2} = \frac{1}{2}(m_\rho^2 + m_{A_1}^2 - m_{\pi_N}^2) \approx 0.55\, m_\rho^2 \tag{11}$$

This result is in good agreement with the results obtained by applying the Veneziano representation to the processes $\pi^-\pi^+ \to \pi^- A_1^+$ and $\pi A_1 \to \pi\,\omega$

$$\frac{g_S}{g_D} = \frac{1}{2}(3\, m_\rho^2 - m_{A_1}^2 - 3\, m_\pi^2) \approx 0.5\, m_\rho^2\,, \qquad \text{(Ref. 5)}\,, \tag{12}$$

$$= \frac{1}{2}(m_{A_1}^2 - m_\rho^2 + m_\pi^2) \approx 0.5\, m_\rho^2\,, \qquad \text{(Ref. 6)}\,. \tag{13}$$

The above solution for C fixes the Schnitzer-Weinberg parameter[2] δ to the values around —1,

$$C = \frac{2\,\delta}{2+\delta} \approx 1.74 \to \delta \approx -0.93.$$

This is attractive from the theoretical point of view for it is consistent with the idea of universality[7] of the ρ-meson charge coupling. The experimental total $A_1 \to \rho\,\pi$ decay width is compatible with $\delta \approx -1$, while the angular distribution measurements still do not allow a clear cut conclusion[4] about the ratio of the S to D wave fraction of the decay.

In the case of $\rho\rho$ scattering we obtain the following set of equations

$$(f_1 + f_2) f_4 + (d_1 - d_2) d_3 = 0, \tag{14}$$

$$L^2(f) - f_4^2 - N(d) = 0, \tag{15}$$

$$f_1(f_2 + f_3) + f_3 f_4 - \frac{1}{2} f_3 L(f) + d_1 d_2 - d_2 d_3 \left(\frac{m_B^2}{m_\rho^2} - 2\right) +$$
$$+ \left(\frac{m_B^2}{2m_\rho^2} - 1\right) N(d) = 0, \tag{16}$$

where we introduced the shorthands

$$L(f) = f_1 + f_2 + \frac{1}{2} f_4,$$
$$N(d) = - d_3^2 + \frac{m_\rho^2}{m_B^2} \left\{ d_1 - d_2 - d_3 \left(\frac{m_B^2}{2m_\rho^2} - 1\right) \right\}^2.$$

To solve approximately the system (14) — (16), we first neglect the higher angular momentum couplings[3], which is equivalent to the linearity and smoothness assumptions of vertex functions in Ref. [2]. Further, assuming the universality of the ρ coupling, we obtain the following approximate solution

$$f_1 = f_2 = f_3, \quad f_4 = 0; \quad d_1 = -d_2, \quad d_3 = 0. \tag{17}$$

This solution reduces one of the equations to a mass sum rule

$$m_B^2 - 3m_\rho^2 = 0, \tag{18}$$

which is experimentally satisfied within 10%. Thus far this has been the only test for the validity of the solution (17).

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References

1) F. J. Gilman and H. Harari, Phys. Rev. **165** (1968) 1803;
2) H. Schnitzer and S. Weinberg, Phys. Rev. **164** (1967) 1828;
3) N. Zovko and I. Andrić, Z. Phys. **226** (1969) 34;
4) N. Zovko, Nuclear Physics **B18** (1970) 215;
5) Fayyazuddin and Riazuddin, Phys. Letters **28B** (1969) 561;
6) G. P. Srivastawa, Northeastern University Preprint (1969);
7) J. J. Sakurai, Annals of Phys. **11** (1960) 1.

VEZANJA I RASPADI VEKTORSKIH MEZONA

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Sadržaj

Na temelju općih zahtjeva unitarnosti i relativističke invarijantnosti te dodatnih uobičajenih pretpostavki o analitičnosti, asimptotskom ponašanju i spektru međustanja formulirane su superkonvergentne disperzione sumacione relacije i primijenjene na elastična raspršenja $\rho\rho \to \rho\rho$ i $\rho A_1 \to \rho A_1$ u smjeru prema naprijed. Nađena su približna rješenja za relevantne konstante vezanja i jedna masena relacija.

U okviru određene saturacione hipoteze izračunat je parametar koji opisuje omjer S i D valne frakcije raspada $A_1 \to \rho\pi$ kao i njegovu totalnu širinu.