FIZIKA, 3 (1971) 175—187

ANALYSIS OF INSTRUMENTAL PROFILE OF AXICON — SCANNED FABRY-PEROT SPECTROMETER*

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Received 5 January 1971; revised manuscript received 28 April 1971

Abstract: Analytical relations among finesses of an Axicon — scanned Fabry — Perot spectrometer are derived. The composition of instrumental profile is discussed and the criterion for an approximation of the step function by a Gaussian one is introduced. In this way numerical computation for each particular case can be avoided and for deconvolution of scanned profile one may use the standard procedure for Voigt profile. An analytical expression for the broadening of the instrumental profile due to the axicon is derived. Experimentally it is shown that the axicon behaves as a linear scanning element.

1. Introduction

The axicon-scanned Fabry-Perot (A-F-P) spectrometer has been first described by Katzenstein[1]. In such a system the axicon is interposed between the imaging lens following the etalon and its focal plane. The axicon has the property[2] of imaging a point source in a line image. The various elements of such line image are formed by rays with different inclinations in respect to the axis, and hence belonging to different spectral intervals of the etalon. The axicon thus maps the resolution intervals from concentric rings of constant area in the focal plane of the imaging lens into circular areas located on the optical axis both in front of and behind the focal plane. By displacing the axicon, various rings in the focal plane will be shrunk to a focal point and can then pass through a circular hole to a photomultiplier. Usually a narrow passband dielectric interference filter is needed as pre-monochromator to isolate the portion of the spectrum which is of the order of the free spectral range. Such a system has all the advantages of an F-P etalon in terms of high luminosity and resolving power[3], as well as the high sensitivity and time resolution of a photoelectric detector.

* This work was partially supported by the Federal Fund for Scientific Work.

The profile scanned by any spectrometer is in principle a convolution of a source function and an instrumental function. Thus if one knows the instrumental parameters it is possible to determine analytically a response of a spectrometer to a line profile. Such an analytical description for the response of a F-P photoelectric spectrometer to a spectral line is given by Hernandez[4], but the response of A-F-P spectrometer to a source line has not yet been analysed. The purpose of this paper is to show the influence of the axicon scanning element on the shape and the width of an instrumental function, and, in some cases, the possibility of deconvolution of the scanned profile obtained by A-F-P spectrometer. We shall consider certain number of characteristics common to all A-F-P spectrometers, in the first place the instrumental line shape function.

2. *Composition of instrumental profile*

The shape of fringes of an ideal F-P etalon can be described by the Airy function[7]. The Airy function itself is a convolution of Dirac comb function with a Lorentzian function. If the halfwidth of an instrumental function is much smaller than the free spectral range the Airy function can be approximated by the Lorentzian function.

In a real F-P etalon the plates are slightly spherically curved and surfaces of plates are not ideally flat and parallel. Thus the real F-P etalon may be conceived as composed of large number of elementary etalons having different spacings that change the free spectral range and therefore cause an increase of the instrumental line halfwidth. The instrumental halfwidth h is related to the free spectral range and reflectance R[8] as

$$h = \frac{\lambda^2}{2\pi t}\frac{1-R}{\sqrt{R}}. \tag{1}$$

By changing t for constant λ and R one changes the halfwidth h. Any imperfection of flatness of the F-P etalon plates, lack of parallelism of the plates or of rays in the incident beam and the finite size of the scanning aperture, contribute to a broadening of the instrumental line. These defects can be summarized as:

— randomly distributed microscopic flatness imperfections which lead to a Gaussian function, and

— spherical curvature of the plates, adjustment errors of the plates and the finite size of the scanning aperture which may be expressed by step functions[7].

Therefore the instrumental function of a real F-P interferometer is not the well known Airy function, but a convolution of several functions introduced by imperfections of F-P plates and the scanning aperture. The instrumental profile is obtained in principle by a convolution of three different functions: Airy function (approximated by the Lorentzian function), Gaussian function and the step function.

The halfwidth of two convoluted Lorentzian profiles is related to halfwidths of two functions as

$$a = a_1 + a_2, \tag{2}$$

while for the Gaussian functions this can be expressed by

$$g^2 = g_1^2 + g_2^2, \tag{3}$$

where g_1 and g_2 are the halfwidths of equivalent Gaussian functions. According to Chabbal[9] the total halfwidth of two step functions is the halfwidth of the broader function, since

$$f = f_1 \odot f_2, \tag{4}$$

and in the case of $f_1 > f_2$ the resulting halfwidth is $f = f_1$. (The sign $\odot$ means *»addition«* of halfwidths through the convolution of equivalent functions). In general, by convolution of three types of functions, the largest broadening is caused by the Lorentzian profile, a smaller one by the Gaussian, while the smallest broadening is due to the step function.

In the case when after the convolution of a step function with a Gaussian one their ratio is

$$\frac{g}{f \odot g} \leqslant 0.5, \tag{5}$$

then it follows that

$$\frac{f}{f \odot g} \approx 1, \text{ or } f \approx f \odot g. \tag{6}$$

The relation (6) is valid for $f > 2\,g$. This means that in this case the contribution of a Gaussian function to the total halfwidth is negligible. The convolution of $A * F$ ($A =$ Airy function and $F =$ step function) is equivalent with $G * G$, (the sign $*$ means convolution).

In a real F-P interferometer the step functions due to imperfections can be larger than the Gaussian function. If the step function behaves as the Gaussian one in convolution with Airy function an analysis of such profile will be relatively simple. Any step function can be approximated by a Gaussian function only under the conditions

$$\frac{a}{a \odot f} \geqslant 0.6 \quad \text{and} \quad \frac{a}{a \odot g} \geqslant 0.6. \tag{7}$$

The conditions (7) are satisfied for an optimal luminosity[7] when R is not to very high, and for a properly aligned system. The flatness of the F-P plates has also to be high and the scanning aperture must correctly be determined. If the relation (7) is satisfied one can simplify the analysis of instrumental and source profiles. In this case there is a convolution of the Airy function with the Gaussian one. Since the Airy function (for not too low R) can be approximated by the Lorentzian function, one should expect the well known Voigt profile. Deconvolution of such a profile can be done using the convenient tables and graphs for Voigt profiles published elsewhere[10, 11].

From Hernandez's[4] calculations it is evident that instrumental function of a complete Fabry-Perot spectrometer has Voigt profile. Here is assumed that the response of an A-F-P spectrometer to a line profile was similar to any other F-P photoelectric spectrometer. Thus the overall finesse of the A-F-P spectrometer is determined by combination of reflectivity finesse N_R, the defects finesse N_D, and the aperture finesse N_F. A simple relation among these finesses is not known. The reflecting finesse is expressed by

$$N_R = \frac{\pi \sqrt{R}}{1-R} = \frac{\Delta \delta_0}{a}, \tag{8}$$

where $\Delta \delta_0 = (2\,t)^{-1}$ is the free spectral range of the etalon, and a is the halfwidth of the Airy function. The »limiting finesse« is

$$N_D = \frac{m}{2} = \frac{\Delta \delta_0}{d}, \tag{9}$$

where m is given as the flatness of the plates by λ/m, and d is the equivalent halfwidth. If the largest step function is F, then it follows

$$N_F = \frac{\Delta \delta_0}{f}. \tag{10}$$

The total fraction of Gaussian type of functions can be expressed by the finesse N_G as

$$N_G = \frac{\Delta \delta_0}{g}. \tag{11}$$

The relation among finesses of the same type of functions can easily be found. When the conditions (7) are satisfied the resulting halfwidth g is given by

$$g^2 = d^2 + f^2. \tag{12}$$

By substituting d, f and g from equations (9), (10) and (11) into (12) one obtains

$$\frac{1}{N^2_G} = \frac{1}{N^2_D} + \frac{1}{N^2_F}\,. \tag{13}$$

It is shown through the procedure of derivation that the relation (13) can be valid only for Gaussian profiles and approximately for the combination of step and Gaussion functions. The form of Equ. (13) is often wrongly used for the overall finesse of the F-P spectrometer.

A relation between finesses of different types of functions can in principle be found only through deconvolution of these functions. Therefore the relationship between N_G and N_R can be derived using the standard procedure for deconvolution of the Voigt profile. The halfwidth h of the Voigt profile determines the overall finesse N of the spectrometer

$$N = \frac{\Delta\,\delta_0}{h}\,, \tag{14}$$

and therefore

$$h = a \odot g, \tag{15}$$

where $g = 1.665\ \beta_2$ and β_2 is the constant introduced by Hulst et al.[10]. If one uses the same notations, then

$$\frac{\beta_1}{\beta_2} = \frac{a}{g}\sqrt{\ln 2}\,. \tag{16}$$

where $a = 2\,\beta_1$. From equations (8) and (14) one obtains

$$N = 2\,\frac{\beta_1}{h}\,N_R, \tag{17}$$

and from equations (11) and (14)

$$N = \frac{g}{h}\,N_G. \tag{18}$$

Finally, from relations (17) and (18) it follows

$$\frac{N_R}{N_G} = \frac{\beta_2}{\beta_1}\sqrt{\ln 2}\,. \tag{19}$$

Using the scanned profile of the instrumental function which has the Voigt shape, under the conditions given by (7) one can find the reflectance finesse or reflectivity R of the F-P plates and the total Gaussian finesse. Any source function which has either Gaussian or Lorentzian profile folded by instrumental function can readily be analysed by simple deconvolution of such a scanned profile.

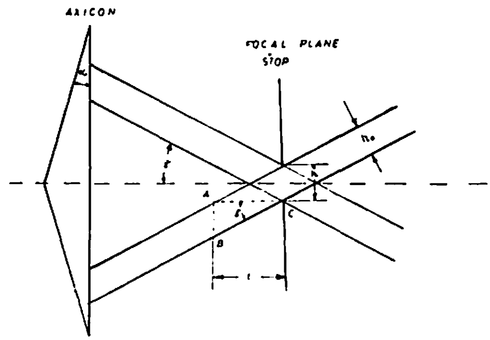

Fig. 1. Ray diagram for the fringe broadening due to an axicon.

Line broadening due to axicon. The axicon as a scanning element can cause further broadening of instrumental profile. According to Fig. 1. the scanned halfwidth of the instrumental function is given by

$$h = \frac{h_0}{\cos\gamma} = h_0 + \Delta h, \tag{20}$$

where h_0 is the halfwidth of the instrumental function (proportional to the width of the fringe), $\cos\gamma$ is the factor of geometrical broadening ot the instrumental function since[6]

$$\cos\gamma = \cos(\mu - 1)\,\alpha, \tag{21}$$

where μ is the index of refraction of the material of the axicon, and α is the vertex angle. Using relations (20) and (21) one obtains

$$\Delta h = h\,[1 - \cos(\mu - 1)\,\alpha]. \tag{22}$$

Since the broadening is linear for any part of instrumental profile one may write

$$\frac{\Delta\beta_1}{\Delta h} = \frac{\beta_1}{h}, \tag{23}$$

or $\Delta a/2\Delta h = \beta_1/h$ and finally

$$\Delta a = 2\beta_1 [1 - \cos(\mu - 1)\alpha]. \tag{24}$$

The relation (24) gives the broadening of the Airy component of the instrumental profile due to the geometry of the axicon.

3. *Axicon as a scanning element*

Diameter of a fringe of the F-P etalon in the focal plane of the imaging lens is given[5] by

$$D_p = 2f\sqrt{\frac{\lambda}{t}(p - 1 + \varepsilon_D)}, \tag{25}$$

where p is the order of the fringe starting from the center ($\varphi = 0$)[6] and $\varepsilon_D = D_1^2/(D_2^2 - D_1^2)$, t is the separation between the reflecting surfaces of the plates, f is the focal length of the imaging lens. The parameter ε_D depends on the spacing of the F-P plates, therefore one can easily adjust the spacing to obtain $\varepsilon_D = 1$. For such set-up the ratio of subsequent fringes is

$$\frac{D_{p_2}}{D_{p_1}} = \sqrt{\frac{p_2}{p_1}}. \tag{26}$$

The axicon[2], supposing it is thin, has the property of changing the inclination of rays with respect to the optical axis without affecting their convergence. Therefore one may assume a linear relation between the displacement L of the axicon, from the focal plane stop, and the diameter of the fringe D,

$$L_p = K \cdot D_p, \tag{27}$$

where K is a constant of the optical system depending on the axicon itself[6]. From the relations (26) and (27) one obtains:

$$\frac{L_{p_2}}{L_{p_1}} = \frac{D_{p_2}}{D_{p_1}} = \sqrt{\frac{p_2}{p_1}}. \tag{28}$$

The response of an A-F-P spectrometer should be similar to the response of the F-P interferometer.

4. *Apparatus*

The D. C. He-Ne gas laser, as a light source was used giving 0.5 mW at 6328 Å. The Doppler broadened line width of this laser is usually less than 20 kc/s. Such width of the source line can be neglected in comparison with any component of instrumental width. The aperture of the F-P etalon, with glass plates B306 (Hilger and Watts Ltd.), was 20 mm in diameter. The surfaces of the plates are flat matched to $\lambda/50$ and coated with hard, multilayer dielectric coating to give a reflectivity of about 90% at 6328 Å. The free spectral range, with a spacer of 1.08 mm, was 1.85 Å, or 4.63 cm^{-1} at 6328 Å. An imaging lens of 40 cm focal length was placed behind the etalon.

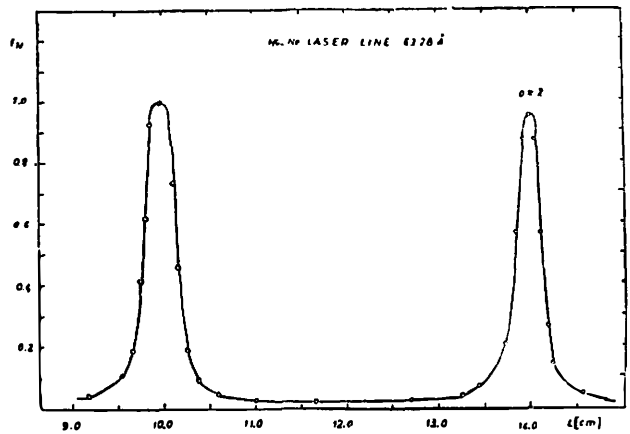

Fig. 2. Scan of two successive orders of the 6328 Å line.

The axicon was made of polished light flint glass, with a vertex angle of 10° and was mounted in a holder on a slide which could be driven along an optical bench. The 0.3 to 0.5 mm stop, placed in the focal plane of the imaging lens was fixed in a holder allowing fine vertical and horizontal displacements. The holder and the stop were connected to a fixed photomultiplier (RCA 7265) housing via light tight bellows.

5. *Results and conclusions*

In order to prove the validity of relation (28) the spacing of the F-P plates was adjusted to give $\varepsilon_D = 1$. To keep the adjusted pattern constant the temperature of the F-P etalon was controlled within $\pm$ 0.1 °C. With such a set up the diameters of fringes in the focal plane were measured without the axicon. First of two successive diameters of fringes were $D_1 = 19.5$ mm and $D_2 = 27.5$ mm. Thus

$$\varepsilon_D = \frac{D_1^2}{D_2^2 - D_1^2} = 1.01. \tag{29}$$

Using the same set-up, but with the axicon inserted between the focal plane stop and the imaging lens, the same two successive fringes were scanned. A set of 8 scanning measurements were performed and the shapes and distances of the fringes have been found to be constant in the limit of experi-

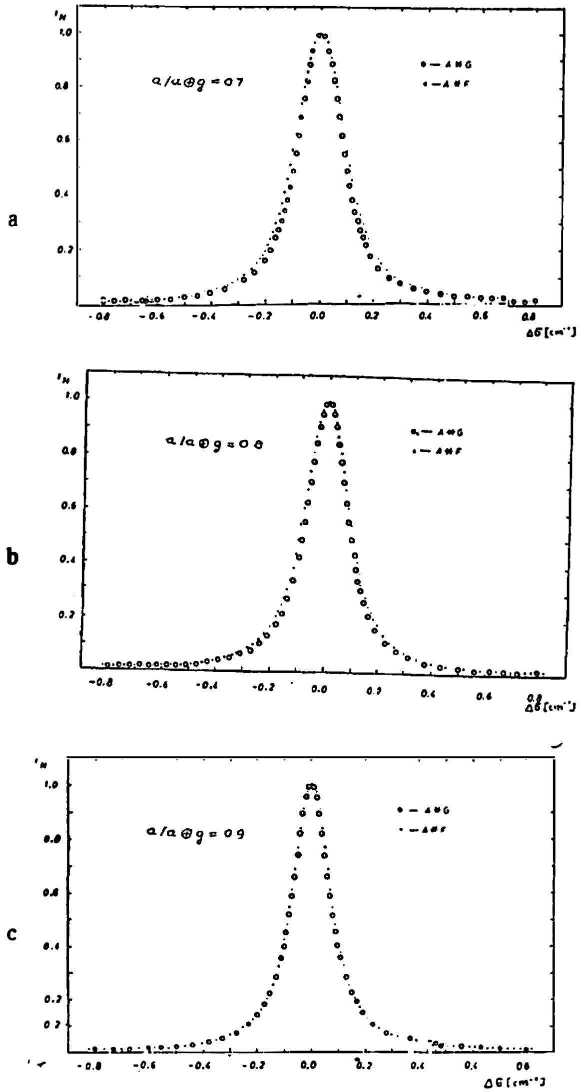

Fig. 3. Theoretical profiles of convoluted Airy-Gaussian and Airy-step functions.

mental error. A set of such profiles is shown in Fig. 2. Equivalent distances of the axicon from the focal plane, for the peaks of the fringes, were $L_1 = 99.5$ mm and $L_2 = 140.0$ mm. This gives

$$\varepsilon_L = \frac{L_1^2}{L_2^2 - L_1^2} = 1.02. \tag{30}$$

The difference of 1% between ε_D and ε_L is within the limits of the experimental error. Thus the relations (27) and (28) are experimentally proved to be valid since $\varepsilon_D = \varepsilon_L$. From these relations one may therefore conclude that the axicon really behaves as a linear scanning element.

It has been shown that the instrumental profile of an A-F-P spectrometer was fully determined by the convolution of Airy, Gaussian and the step function. The step function in such an instrument can be larger than the

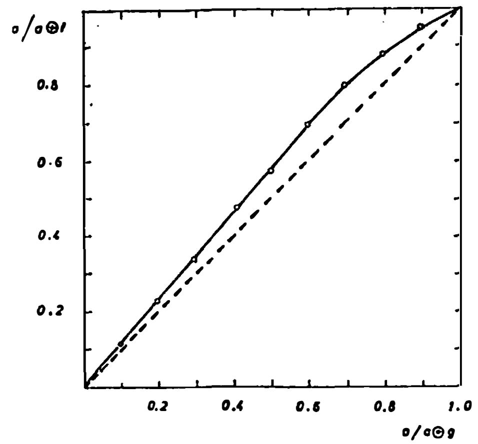

Fig. 4. Deviation of the ratio $a/(a \odot f)$ from the ratio $a/(a \odot g)$ for $g = f$.

Gaussian one. Since $A * F$ is equivalent with $G * G$ and such profile can not be simply analysed through standard deconvolution procedure. There is a domain where F behaves as G in convolution with Airy function and then instrumental function has the Voigt profile. To find this region a programme was prepared for an IBM 360 model 44 computer to calculate the convolution of functions $A * F$ and $A * G$. The halfwidth a of the Airy function was kept constant, and f was set equal to g. A set of profiles was obtained for the region $a/(a \odot g) = 0.10 — 0.90$. When $a/(a \odot g) \geqslant 0.6$ and $a/(a \odot f) \geqslant 0.6$ the profiles $A * G$ can be fitted by the $A * F$ profiles. Very good fit of such functions is shown in Fig. 3. *a, b, c* for $a/(a \odot g) = 0.7, 0.8, 0.9$, respectively. The conditions given by the relation (7) as the criterion for the equivalence of F and G is therefore justified. The calculated functional dependence of $a/(a \odot f)$ is given by the full line in Fig. 4. The dashed line in the same figure represents the ideal or identical behaviour of F as G. Halfwidths of folded functions $A * F$ and $A * G$, for the same a, and $f = g$, are not equal. The halfwidth $a \odot g$ is larger than $a \odot f$. The difference of the widths as the deviation of $a \odot f$ from $a \odot g$, is given in Fig. 5. The maximal error, defined as $\Delta = (a \odot g — a \odot f)/(a \odot g)$, is about 13% and the equation (12) can be used as an approximative formula. From the calculated profiles $A * F$ and

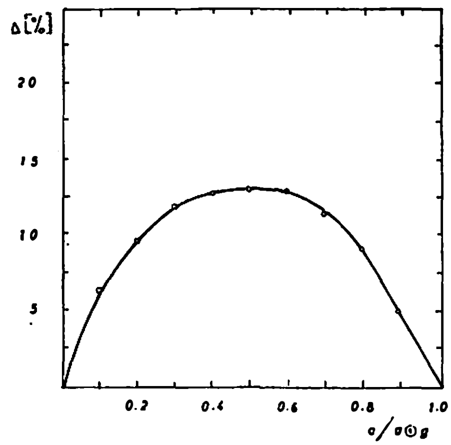

Fig. 5. Errors distribution when step function is approximated by Gaussian function.

$A * G$, when (7) is satisfied, one can evaluate the halfwidth of Lorentzian fraction of the instrumental profile, using the standard procedure for deconvolution of Voigt profiles[10, 11] with the error smaller than 5%. Using the same procedure the evaluated halfwidth f is smaller than g. Therefore the analysis of any dispersive source function can be done in this way with even smaller error.

The relation (13) among finesses of Gaussian type functions is derived in order to show the applicability of such form of equation. Very frequently this form of relation is wrongly used for overall finesse in which N_R finesse is included. Therefore the overall finesse is derived using standard procedure for deconvolution of Voigt profile.

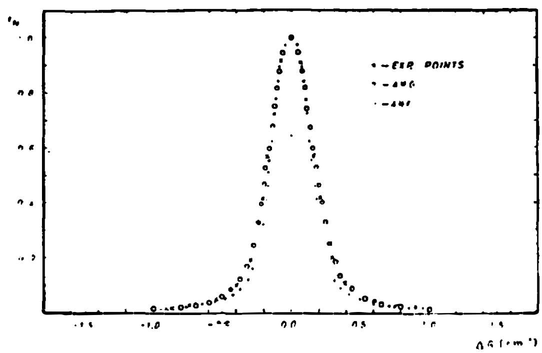

Fig. 6. Instrumental profile of A-F-P spectrometer for $a/(a \odot g) = 0.4$ compared by $A * G$ and $A * F$ profiles.

It has been shown that the axicon causes a broadening of instrumental function:

— due to geometry of a fringe in the focal plane stop as it is given by the derived relation (22), and

— because all the light rays incident to the axicon are not complanar with the plane in which lays optical axis.

In general, the instrumental profile of the A-F-P spectrometer can be approximated by the Airy function only when the flatness and parallelism of the F-P plates have relatively high degree, low reflectivity and when the focal plane stop has the optimal aperture. The reflectivity of the F-P etalon plates, of the used instrument, was relatively high and therefore the halfwidth of the Lorentzian fraction small. By changing the size of the scanning aperture the halfwidth of the step function was varied and the instrumental functions varied from Voigt profiles to pure Gaussian ones. Fig. 6. shows a typical instrumental profile of the A-F-P spectrometer. Experimental data are represented by crosses, calculated values of $A * G$ with circles and $A * F$ with points. There is some scatter of calculated values $A * F$ from the scanned profile since $a/(a \odot f)$ was 0.4 and the criterion (7) is not satisfied.

The A-F-P spectrometer has the same response to a line profile and the same structure of the instrumental function as any other F-P photoelectric spectrometer.

Acknowledgement

We wish to express our gratitude to Dr. J. Labat and to Dr. N. Konjević for their helpful remarks on the manuscript, to Dr. Lj. Ćirković for useful discussion, and to M. Sc. P. Grujić for preparing the computer programme.

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ANALIZA INSTRUMENTALNOG PROFILA AKSIKON FABRY-PEROT SPEKTROMETRA

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S a d r ž a j

Izvedene su analitičke relacije između finesa Aksikon Fabry-Perot spektrometra. Diskutovana je kompozicija instrumentalnog profila i izveden kriterijum za aproksimaciju step funkcije Gauss-ovom funkcijom, čime se izbegava numeričko računanje za svaki poseban slučaj pa se može koristiti standardna procedura za dekonvoluciju Voigt-ovog profila.

Izveden je analitički izraz za širenje instrumentalnog profila aksikonom i eksperimentalno je pokazano da se aksikon ponaša kao linearni elemenat za skaniranje.