

## Z DEPENDENCE OF LINEAR POLARIZATION IN ELASTIC SCATTERING

B. MOLAK, K. ILAKOVAC and A. LJUBIČIĆ

*Institute »Ruđer Bošković«, Zagreb*

Received 16 March 1971; revised manuscript received 5 July 1971

**Abstract:** A new method for measuring linear polarization was applied to study the Z dependence of the degree of linear polarization in the elastic scattering of 662-keV gamma-rays at 90°. The measurements were performed with five elements: Sn, Sm, Hf, Pb and U. The degree of polarization decreases with increasing Z. The calibration of the efficiency of the polarization analyzer was made by measuring the asymmetry ratio of Compton scattered gamma-rays in graphite.

### *1. Introduction*

Brown et al.<sup>1)</sup> developed the relativistic theory of Rayleigh scattering using second order perturbation theory, and treating exactly the electron intermediate states. The calculations with K-shell electrons turned out to be very elaborate due to the large number of intermediate states which had to be taken into account. They made extensive numerical calculations of the scattering amplitudes for the Rayleigh scattering by K-shell electrons in mercury, at energies of 0.32, 0.64, 1.28 and 2.56 mc<sup>2</sup>. In heavy elements and for gamma-rays of energy higher than about 2 mc<sup>2</sup> other coherent processes are of increasing importance: the nuclear Thomson scattering and at still higher energies the Delbrück scattering. From scattering amplitudes of these processes linear polarization of scattered photons can be calculated<sup>2)</sup>, and also the polarization-transformation matrix.<sup>3)</sup>

At energies of photons above about 0.3 MeV these results differ considerably from the earlier theoretical results<sup>4)</sup> for Rayleigh scattering based on the form factor approximation. In this approximation linear polarization is independent of energy and atomic number, while the calculations of Brown et al.<sup>1)</sup> show a pronounced energy dependence.

Extensive measurements have been performed to study the energy and the  $Z$  dependence of the differential cross sections<sup>5, 6, 7</sup>. The agreement with the theory of Brown *et al.* is very good for scattering in lead at energies up to about 1 MeV. Extrapolations of the theoretical results for the differential cross sections were proposed, based on experimental results obtained with other elements.<sup>5</sup> Recently a detailed measurement with 1.33 MeV  $^{60}\text{Co}$  gamma-rays scattered in lead and uranium<sup>7</sup> showed a discrepancy at large scattering angles, although the Rayleigh, nuclear Thomson and Delbrück scattering amplitudes were taken into account. To explain the discrepancy it was suggested that phase relations of the elastic scattering processes should be modified.

Polarization measurements offer more stringent tests of scattering amplitudes. Linear polarization measurements of the elastic scattering of gamma-rays were previously performed<sup>8, 9, 10</sup> with lead scatterers only, using the method of Metzger and Deutsch<sup>11</sup>. A good agreement was obtained with the results of Brown *et al.* (at 1.33 MeV nuclear Thomson scattering amplitudes must be included). There is no reliable method of extrapolation of the results of Brown *et al.* to other elements, while the calculations are extremely difficult. Therefore, it seemed of interest to acquire a knowledge of the  $Z$  dependence of linear polarization by experimental method. With the development of the new type of polarimeter using a planar Ge(Li) detector<sup>12, 13</sup>, measurement of linear polarization of elastically scattered gamma-rays in medium-heavy elements have become possible. In this paper we report measurements of linear polarization in the elastic scattering of 662-keV gamma-rays at  $90^\circ$  by tin, samarium, hafnium, lead and uranium.

## 2. Measurements and results

The most efficient and very reliable method for determining the degree of linear polarization of photons above 200 keV is the measurement of the asymmetry in Compton scattering. Metzger and Deutsch introduced this method using two scintillation counters. One counter detected the recoil electron and served as the Compton scatterer, while the other detected the secondary Compton photon. Recently a new method<sup>12, 13</sup> was introduced, in which a single planar Ge(Li) detector was used. In the Compton scattering of a linearly polarized beam of photons, secondary photons (and electrons) are preferably scattered in the plane perpendicular to the plane of polarization. Electrons normally stop in the detector volume if such an event occurred in the detector. Secondary photons, however, will be absorbed with a smaller probability if they are scattered perpendicularly to the plane of the detector. A smaller counting rate in the full-energy peak is therefore obtained when the vector of linear polarization is in the plane of the detector and a larger counting rate when the vector of linear polarization is perpendicular to the detector plane.

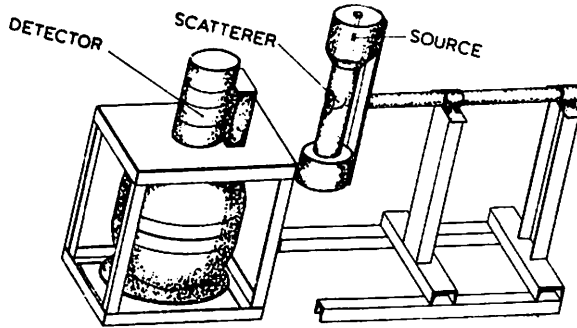


Fig. 1 — Experimental arrangement.

From the counting rates we calculate the asymmetry

$$A = \frac{N_1 - N_2}{N_1 + N_2}. \quad (1)$$

$N_1$  and  $N_2$  are the counting rates when the vector of polarization is perpendicular and parallel to the plane of the detector, respectively. The value of  $A$  depends on the energy of photons for completely linearly polarized photons. It is therefore necessary to calibrate the efficiency of the Ge(Li) planar detector (as also other systems) as a polarization analyzer. This was done in the present measurements by determining the asymmetry  $A_1$  for a beam of photons of the same energy and a known degree of polarization,  $P_1$ . The efficiency of the polarization analyzer,  $\epsilon$ , can be determined from the relation

$$\epsilon = \frac{A_1}{P_1}. \quad (2)$$

The measurements were performed with the arrangement shown in Fig. 1. An Ortec planar Ge(Li) detector, 3 mm thick and 18 mm in diameter, was used as the polarization analyzer. Since it was not possible to rotate the detector, the source and the scatterer, with adequate shields, were mounted on a shaft. Thus the plane of polarization was rotated, ensuring at the same time a very consistent geometry. To reduce systematic errors due to possible small misadjustment (if the centre of the detector's sensitive volume is not on the axis of rotation), counting rates were measured at four positions: two vertical (up and down) and two horizontal (left and right). The counting rates are denoted by  $N_v$ ,  $N_d$ ,  $N_L$  and  $N_R$ , respectively.

$^{60}\text{Co}$  gamma-rays of 1.17 and 1.33 MeV, when Compton scattered at an angle of  $51^\circ$ , have energies of 635 and 680 keV, respectively. The average value of the degree of polarization, calculated from the relation<sup>14)</sup>

$$P_{\text{Compton}} = \frac{-\sin^2 \Theta}{1 + \cos^2 \Theta + (K_0 - K)(1 - \cos \Theta)}, \quad (3)$$

amounts to 0.33. The spectrum of amplitudes from the detector measured by detecting the Compton scattered radiation from a  $^{60}\text{Co}$  source in graphite is shown in Fig. 2. To obtain the number of counts under the peaks, two procedures of subtracting the background were applied. In one procedure a »linear« variation of background was assumed. The average value of the number of counts in twelve channels to the left and twelve channels to the

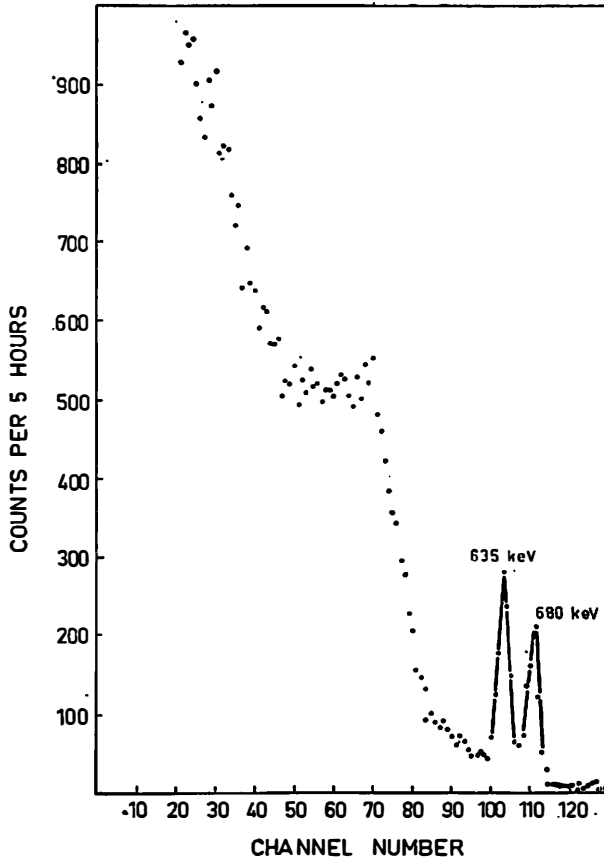


Fig. 2 — Ge(Li) spectrum of radiation from the Compton scattering of  $^{60}\text{Co}$  gamma-rays in graphite at  $51^\circ$ .

right of the two peaks was subtracted from the total number in fifteen channels in the region of the peaks. The other procedure consisted of a least squares fit of an exponential curve to the same pair of twelve channels adjacent to the peaks. Both procedures gave consistent results, but the second procedure seems to be more reliable, and we present the results obtained by this method.

The numbers of counts in both peaks (635 and 680 keV) after subtracting the background were

$$N_1 = N_L + N_R = 50548.36,$$

$$N_2 = N_U + N_D = 48411.42,$$

from which we calculated

$$A_{\text{Compton}} = (2.16 \pm 0.32)\%.$$

The efficiency of the detector for analyzing linear polarization is therefore

$$\varepsilon = \frac{A_{\text{Compton}}}{P_{\text{Compton}}} = \frac{0.0216 \pm 0.0032}{0.33} = (6.54 \pm 0.96)\%.$$

A relatively weak source of  $^{60}\text{Co}$  was available (about 100 mCi), causing lengthy measurements and a somewhat large error.

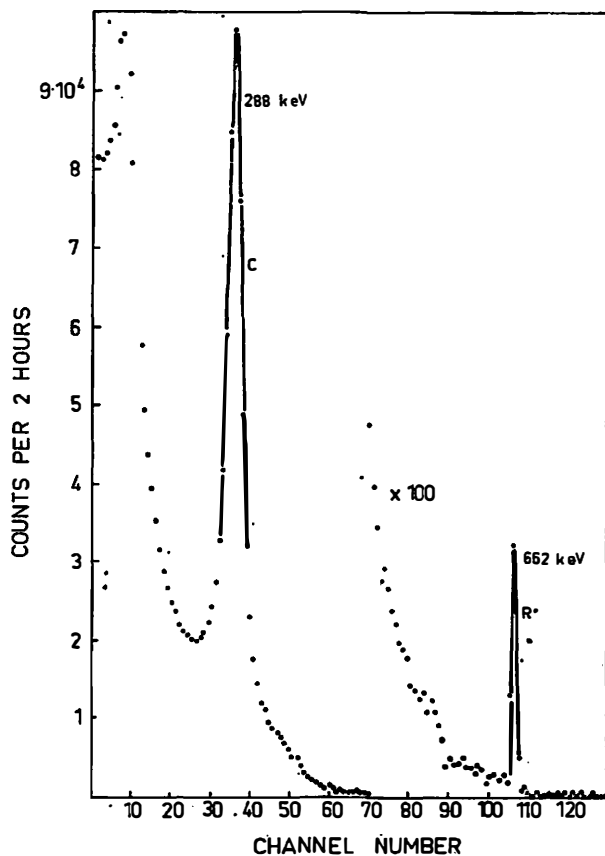


Fig. 3 — Ge(Li) spectrum of radiation from  $90^\circ$  scattering of  $^{137}\text{Cs}$  gamma-rays in uranium.

The measurements of the asymmetry in elastic scattering were performed in a similar arrangement. The scattering angle was fixed at  $90^\circ$ . A source of 5 Ci of  $^{137}\text{Cs}$  was placed in a lead shield which was mounted on the shaft. Five scatterers were used: 167.6 grams of Sn metal, 14.76 grams of  $\text{Sm}_2\text{O}_3$  powder, 42.4 grams of Hf metal powder, 95.7 grams of Pb metal, and 27.4 grams of  $\text{U}_3\text{O}_8$  powder. The spectrum of radiation scattered from lead is shown in Fig. 3. The figure shows that a well-defined peak due to elastic scattering appears in addition to a somewhat broader peak due to Compton scattering. The results of measurements and the calculated asymmetries are shown in Table 1. Fig. 4 shows the results of asymmetry measurements.

### 3. Discussion

Linear polarization  $P(\Theta)$  in Rayleigh scattering can be expressed by the relation

$$P(\Theta) = \frac{2 \operatorname{Re} [A(\Theta) \cdot B(\Theta)]}{|A(\Theta)|^2 + |B(\Theta)|^2}, \quad (4)$$

where  $A(\Theta)$  and  $B(\Theta)$  are the amplitude with and the amplitude without polarization change, respectively. Calculations of both amplitudes were made in the form factor approximation and in the formalism of Brown *et al.* In the former theory, linear polarization  $P(\Theta)$  reaches its maximum value of 1 for photons elastically scattered at  $90^\circ$  and is independent of the photon energy  $E$  and of the atomic number  $Z$  of the scatterer. This means that the amplitudes  $A(\Theta)$  and  $B(\Theta)$  are equal at  $90^\circ$  and that the  $Z$  and  $E$  dependence is the same for both amplitudes.

Brown *et al.* have shown that the form factor approximation is inadequate for calculating the amplitude  $B(\Theta)$ . This amplitude is considerably smaller

Table 1.  
RESULTS OF POLARIZATION MEASUREMENTS ( $\Theta = 90^\circ$ )

Z	50	62	72	82	92
T (hours)	528	688	280	140	164
$N_1$	6499.1	6102.3	12908.1	16414.5	12232.1
$N_2$	5803.3	5350.2	11463.9	14715.2	11266.1
A (%)	$5.66 \pm 0.90$	$6.57 \pm 0.93$	$5.93 \pm 0.64$	$5.46 \pm 0.57$	$4.11 \pm 0.65$
P (%)	$86.4 \pm 18.8$	$100.4 \pm 20.6$	$90.6 \pm 16.6$	$83.4 \pm 15.0$	$62.8 \pm 13.6$

than predicted by the form factor approximation and this discrepancy increases with increasing momentum transfer. However, the form factor

approximation yields good results for the amplitude  $A(\Theta)$ . Consequently, the peak of the linear polarization curve is shifted towards lower scattering angles. As mentioned earlier, the results of theory of Brown et al. are in very good agreement with the extensive measurements of the differential cross sections, and with the measurements of the degree of linear polarization of gamma-rays elastically scattered in lead. Unfortunately, the calculations of Brown et al. were made only for the scattering from K-shell electrons in mercury at a few specific gamma-ray energies. The  $Z$  dependence of scattering amplitudes cannot be obtained from the expressions given by Brown et al. without extensive computational labour.

Our result for the degree of linear polarization of 662-keV gamma-rays, elastically scattered in lead at  $90^\circ$  of  $(83.4 \pm 15)\%$ , is in good agreement with the result of Sood<sup>2)</sup> and Singh et al.<sup>9)</sup> of  $(83 \pm 13)\%$  and  $(78 \pm 8)\%$ , respectively, and with the theoretical value of  $78.5\%$  derived from the theory of

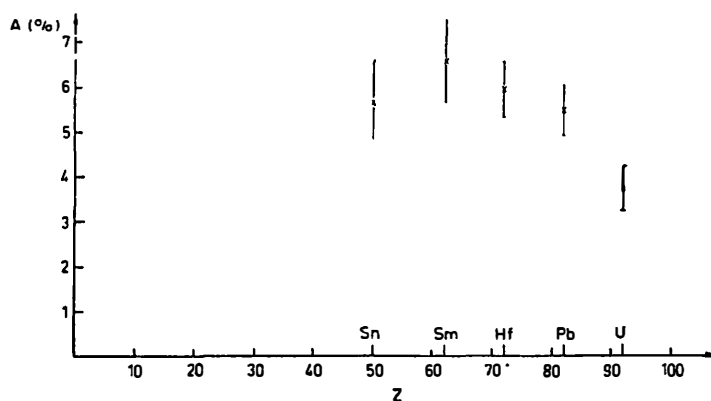


Fig. 4 — Results for the asymmetry in the elastic scattering of 662-keV gamma-rays in Sn, Sm, Hf, Pb and U at  $90^\circ$ .

Brown et al. The errors in our results on the absolute values of the degree of linear polarization are mainly due to the error in the measurement of the efficiency of the polarimeter. This error does not enter when the results on asymmetries for different elements are compared.

Our results, as shown in Fig. 4., indicate that the scattering amplitudes  $A(\Theta)$  and  $B(\Theta)$  are different functions of the atomic number. The degree of linear polarization of 662-keV gamma-rays elastically scattered at  $90^\circ$  seems to be a steadily decreasing function of  $Z$ , apparently starting at low values of  $Z$  from the limiting value of close to  $100\%$  polarization.

## References

- 1) G. E. Brown, R. E. Peierls, F. R. S. and J. B. Woodward, Proc. Roy. Soc. **A227**, (1955) 51;  
S. Brenner, G. E. Brown and J. B. Woodward, Proc. Roy. Soc. **A227**, (1955) 59;  
G. E. Brown and D. F. Mayers, Proc. Roy. Soc. **A234**, (1956) 387;  
G. E. Brown and D. F. Mayers, Proc. Roy. Soc. **A242**, (1957) 89;
- 2) B. S. Sood, Proc. Roy. Soc. **A247**, (1958) 375;
- 3) G. Böbel and G. Passatore, Nuovo Cim. **15**, (1960) 979;
- 4) W. Franz, Z. Physik **95**, (1935) 652; **98**, (1936) 314;
- 5) S. Anand and B. S. Sood, Nucl. Phys. **73**, (1965) 368;
- 6) A. K. Mann, Phys. Rev. **101**, (1956) 4;
- 7) G. Hardie, W. J. Mellow and D. R. Schwandt, Phys. Rev. **C1**, (1970) 714;
- 8) G. Manuzio and S. Vitale, Nuovo Cim. **20**, (1961) 638;
- 9) M. Singh, S. Anand and B. S. Sood, Nuovo Cim. **35**, (1965) 1047;
- 10) R. A. Williams and K. G. McNeill, Can. J. Phys. **43**, (1965) 1078;
- 11) F. R. Metzger and M. Deutsch, Phys. Rev. **78**, (1950) 551;
- 12) J. Honzátko and J. Kajfosz, Československa Akad. Véd, Report UJF 2113-F (1968);  
Check. J. Phys. **B19**, 1281 (1969);
- 13) G. T. Ewan, G. I. Andersson, G. A. Bartholomew and A. E. Litherland, Phys. Let. **29B**, (1969) 352;
- 14) e. g., U. Fano, J. Opt. Soc. Am. **39**, (1949) 859.

OVISNOST LINEARNE POLARIZACIJE  
O REDNOM BROJU U ELASTIČNOM RASPRŠENJU

B. MOLAK, K. ILAKOVAC i A. LJUBIČIĆ

*Institut »Ruđer Bošković«, Zagreb*

S a d r Ź a j

Izvršena su mjerenja Z-ovisnosti stupnja linearne polarizacije u elastičnom raspršenju gama-zraka energije 662 keV na kutu od 90°. Kao polarimetar korišten je planarni Ge(Li) detektor. Mjerenja su izvršena s pet elemenata: Sn, Sm, Hf, Pb i U, i nađeno je da stupanj polarizacije pada s porastom rednog broja. Kalibracija efikasnosti polarimetra načinjena je mjerenjem asimetrijskog odnosa u Comptonovom raspršenju <sup>60</sup>Co gama-zraka u grafitu.