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THE DISTORTION OF THE DEUTERON BY THE COULOMB FIELD*

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Abstract: The deformation of the internal deuteron wave function by the Coulomb field of the nucleus has been considered. The relevant characteristic physical quantities have been evaluated. The influence on the angular distribution of the outgoing particles in deuteron stripping reactions has been calculated. The results are given in the form which allows a rough estimation of this effect without performing numerical calculations in each particular case.

1. Introduction

The theoretical treatment of a collision process involving composite particles leads to considerable mathematical difficulties. These are mainly associated with the structure of a composite particle. As long as a composite particle represents a compact, strongly bound system which remains unaffected by the collision process, the ordinary two-body collision theory can be applied with a high degree of accuracy. However, when the internal structure of a composite particle is strongly affected by the collision process, this theoretical treatment allows only an approximate description of the reaction mechanism. In such a case the perturbation theory has been commonly used, and even then with various simplifications in which less important, small effects are discarded. Such an effect is the distortion of the deuteron by the Coulomb field of a charged particle entering into collision with the deuteron (usually the atomic nucleus). The deuteron is particularly liable to this effect since its constituents, the proton and the neutron, are relatively weakly bound in the deuteron ground state[1-3]. The Coulomb force acting only on the proton causes the deformation of the deuteron. Sometimes this effect has been termed the polarization of the deuteron in the Coulomb field. It has been

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regarded as a small effect because the Coulomb force is weak in comparison with the nuclear force, although it can hardly be assumed negligible in the case of heavier nuclei (with higher charge). This effect has most frequently been studied in nuclear stripping reactions[4-7]. There the capture of one deuteron constituent by the nucleus was expected to be affected by the deuteron polarization, and the capture of the neutron was supposed to be preferred to the capture of the proton. That qualitative effect was based on the purely classical picture of a stripping reaction with a few attempts to make it quantitative in the framework of quantum mechanics. The reason for that may be found in the fact that the total cross section of a stripping reaction is theoretically very badly reproduced, so that the analysis is concentrated only on the angular distribution. A refined description of the cross section, which takes into account the polarization of the deuteron, appeared immaterial under those circumstances, especially if the description required elaborate numerical calculations[8].

The theoretical approach in this paper is not concerned with any particular nuclear process. The wave function of the deformed deuteron is evaluated using the semiclassical theory[9]. This theory, which is based on the quantum mechanical time-dependent perturbation theory and the concept of classical trajectory, provides an approximate internal wave function of the deuteron at the instant of nuclear collision. The shape and properties of the wave function thus obtained are discussed and the relevant physical quantities are calculated. Applications to deuteron stripping reactions are included and the influence on the angular distribution evaluated. Two numerical tables are added to enable the reader to promptly evaluate the effect over the periodic system of atomic nuclei entering numerical calculations.

2. *Theory*

We consider the collision of the deuteron and the particle with charge Z. The relative coordinates of the particle Z with respect to the centre of mass of the deuteron are given by the vector $\vec{\rho} = \vec{r}_Z - \frac{1}{2}(\vec{r}_p + \vec{r}_n)$. The deuteron internal variables are represented by the vector $\vec{r} = \vec{r}_p - \vec{r}_n$. The Coulomb interaction between the particle Z and the deuteron can be separated into two parts

$$V_C(\vec{r}_p) = \frac{Z e^2}{r_p} = \frac{Z e^2}{\rho} + Z e^2 \left(\frac{1}{r_p} - \frac{1}{\rho}\right). \tag{1}$$

The first term leads to the Coulomb scattering of the deuteron, or it contributes the Coulomb part to the distortion of the incoming wave in the entrance channel of a nuclear reaction induced by the deuteron[6,7]. This term is not the object of our considerations. We concentrate our attention on the second term

$$V(\vec{r},\vec{\rho}) = Ze^2\left(\frac{1}{\left|\frac{\vec{r}}{2}-\vec{\rho}\right|}-\frac{1}{\rho}\right), \tag{2}$$

which in its $\vec{r}$ dependence represents an additional interaction to the nuclear proton-neutron interaction. The interaction (2) causes the distortion of the deuteron internal wave function describing the polarization of the deuteron in the Coulomb field.

We apply the classical treatment to the motion described by the variables $\vec{\rho}$ and introduce the classical trajectory $\vec{\rho}=\vec{\rho}(t)$[9]. Then the interaction $V\{\vec{r},\vec{\rho}(t)\}$ will play the role of time-dependent perturbation in a quantum mechanical system of two particles, a proton and a neutron. This semiclassical treatment of a quantum mechanical system has already found its application in the Coulomb excitation of the atomic nucleus[9,10], where the hyperbolic trajectory was used. There are two points in which our problem differs from the Coulomb excitation. First, in a collision process in which the breakup of the deuteron occurs, only part of the trajectory is relevant, and secondly, there are no excited bound states of the proton — neutron system.

We apply the time-dependent perturbation theory. The initial proton — neutron state at time $t=-\infty$ ($\rho=\infty$), representing a free deuteron, develops in time under the influence of the time-dependent interaction $V\{\vec{r},\vec{\rho}(t)\}$ until $t=t_0$, when strong interactions take place followed by the disintegration of the deuteron. We are interested in the wave function of the proton-neutron system at time t_0.

The problem is mathematically too complicated to be treated exactly and therefore we simplify it. First of all, we write a multipole expansion of the interaction (2) taking into account that the part of the configuration space defined by $2\rho \geq r$ is dominant.

$$V(\vec{r},\vec{\rho}) = 4\pi\frac{Ze^2}{\rho}\sum_{\lambda=1}^{\infty}\sum_{\mu=-\lambda}^{\lambda}(2\lambda+1)^{-1}\left(\frac{r}{2\rho}\right)^{\lambda}Y_{\lambda}^{\mu*}(\hat{\rho})\,Y_{\lambda}^{\mu}(\hat{r}), \tag{3}$$

and consider only the first multipole (dipole)

$$V_1(\vec{r},\vec{\rho}) = \frac{2\pi}{3}Ze^2\frac{r}{\rho^2}\sum_{\mu=-1}^{1}Y_1^{\mu*}(\hat{\rho})\,Y_1^{\mu}(\hat{r}) = \tag{4}$$

$$= \frac{1}{2} Z e^2 \frac{\vec{r} \cdot \vec{\rho}}{\rho^3} .$$

A complete set of functions of the proton-neutron system consists of a bound s-state wave function $\phi_0(r)$ and scattering states $\phi_{\vec{k}}(\vec{r})$. The dipole interaction allows transitions from the initial s-state ϕ_0 to the p-wave component of the state $\phi_{\vec{k}}$, i. e., in the partial-wave decomposition of the scattering state

$$\phi_{\vec{k}}(\vec{r}) = 4\pi \sum i^l u_l(k, r) Y_l^{m*}(\hat{k}) Y_l^m(\hat{r}) \tag{5}$$

only the function $u_1(k, r)$ has to be specified; it can be chosen arbitrarily without violating the orthogonality condition $\langle \vec{k} \mid 0 \rangle = 0$. We assume that the function $u_1(k, r)$ may be approximated by the partial-wave radial wave function of a free particle

$$u_1(k, r) = j_1(kr) . \tag{6}$$

We describe the deuteron bound state using the asymptotic wave function

$$\phi_0(\vec{r}) = \left(\frac{\alpha}{2\pi}\right)^{1/2} \frac{e^{-\alpha r}}{r} ; \qquad \alpha = 0.23\ f^{-1} . \tag{7}$$

Taking into account expressions (4), (5), (6) and (7), we evaluate the following transition matrix element

$$\langle \vec{k} \mid V_1 \mid 0 \rangle = -i \sqrt{8\pi\alpha}\, Z e^2 \frac{\vec{\rho} \cdot \vec{k}}{\rho^3 (k^2 + \alpha^2)^2} . \tag{8}$$

The perturbed wave function in the first-order perturbation theory is given by

$$\Phi(\vec{r}) = \phi_0(\vec{r}) + \frac{1}{(2\pi)^{3/2}} \int F(\vec{k})\, \phi_{\vec{k}}(\vec{r})\, d\vec{k}, \tag{9}$$

where

$$F(\vec{k}) = \frac{1}{i\hbar} \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{t_0} e^{i\omega(t - t_0)} \langle k \mid V_1(t) \mid 0 \rangle\, dt , \tag{10}$$

$$\hbar\,\omega = E_{\vec{\varkappa}} - E_0 . \tag{11}$$

We readily find

$$F(\vec{k}) = -\frac{\sqrt{\alpha}}{\pi}\,\frac{Z e^2}{m v^2}\,\frac{\vec{G}\cdot\vec{k}}{k^2+\alpha^2}, \tag{12}$$

where

$$\vec{G}(k) = \frac{v^2}{\omega}\int_{-\infty}^{t_0} e^{i\omega(t-t_0)}\,\frac{\vec{\rho}}{\rho^3}\,d\tau , \tag{13}$$

$$\omega = \frac{\hbar}{m}(k^2+\alpha^2) . \tag{14}$$

We have introduced the relative velocity v between the deuteron and the particle Z. The nucleon mass has been denoted by m.

The wave function assumes a simple form in the momentum representation. It can be found immediately from expression (9), which can be put in the form

$$\Phi(\vec{r}) = \frac{1}{(2\pi)^{3/2}}\int \Phi(\vec{k})\,e^{i\vec{k}\vec{r}}\,d\vec{k}, \tag{15}$$

where

$$\Phi(\vec{k}) = \phi_0(k) + F(\vec{k}), \tag{16}$$

$$\phi_0(k) = \frac{\sqrt{\alpha}}{\pi}\,\frac{1}{k^2+\alpha^2} . \tag{17}$$

Taking into account expression (12), one obtains

$$\Phi(\vec{k}) = \phi_0(k)\left[1 - \frac{Z e^2}{m v^2}\vec{k}\cdot\vec{G}(k)\right], \tag{18}$$

where $\hbar\vec{k}$ is the relative momentum of the proton-neutron internal motion, i. e.,

$$\vec{k} = \frac{1}{2}(\vec{k}_p - \vec{k}_n) . \tag{19}$$

We proceed with the evaluation of the function $\vec{G}(k)$. We restrict ourselves to the central (head-on) collision for which the direction of the vector $\vec{\rho}$ remains constant during the time

$$\vec{\rho}(t) = \rho(t) \cdot \vec{n}. \tag{20}$$

Here $\vec{n}$ represents the unit vector in the direction of the deuteron velocity. The function (20) now assumes the following form

$$\Phi(\vec{k}) = \phi_0(k)\,[1 - \frac{Z e^2}{m v^2} G(k)\, \vec{k} \cdot \vec{n}], \tag{21}$$

where

$$G(k) = \frac{v^2}{\omega} \int_{-\infty}^{t_0} e^{i\omega(t-t_0)} \frac{dt}{\rho^2}. \tag{22}$$

The trajectory for the central Coulomb collision is represented by a straight line with the time dependence of an interparticle distance ρ expressed by the equations

$$\rho = \frac{a}{2}(1 + \text{ch}\,\tau), \qquad a = 2\frac{Z e^2}{\mu v^2} = \frac{Z e^2}{E}. \qquad v t = \frac{a}{2}(\tau + \text{sh}\,\tau), \tag{23}$$

Here τ is a parameter, v stands for the relative asymptotic velocity of the two particles, and the parameter a is the classical distance of closest approach. The reduced mass $\mu = m_d m_Z/(m_d + m_Z) \simeq m_d = 2m$ for $m_Z \gg m_d$, so that the parameter a is essentially present in the second term of the deuteron wave function (21). In order to justify the application of the perturbation theory, this term should be small. Consequently, the classical distance of closest approach should be small, i. e., small compared with the nuclear radius R

$$a \ll R \quad \text{or} \quad E \gg \frac{Z e^2}{R}. \tag{24}$$

The second inequality states that the relative energy should be higher than the Coulomb barrier of the nucleus. Under the conditions (24) the relative motion of the two particles is practically a uniform motion with constant velocity v, i. e.,

$$\rho = -v\,t, \qquad t < t_0 < 0\,, \tag{25}$$

where the collision time t_0 is related to the nuclear radius by

$$R = -v\,t_0\,. \tag{26}$$

After substituting ρ from Equ. (25) into Equ. (22), one readily finds

$$G(k) = A(k) - \mathrm{i}\,B(k)\,,$$

where

$$\begin{aligned} A(k) &= \frac{1}{x} - f(x)\,, \\ B(k) &= g(x)\,, \\ x &= \frac{\hbar R}{m\,v}(k^2 + \alpha^2)\,, \\ f(x) &= \int_0^\infty \frac{\sin u}{u + x}\,\mathrm{d}u\,, \\ g(x) &= \int_0^\infty \frac{\cos u}{u + x}\,\mathrm{d}u\,. \end{aligned} \tag{27}$$

The two transcendental function $f(x)$ and $g(x)$ are known as auxiliary functions in the theory of sine and cosine integral functions[11]. Both of them are positive and monotonically decreasing functions of the argument. The combination $1/x - f(x)$ has the same properties, and we have the inequalities

$$A(k) > 0, \qquad B(k) > 0\,. \tag{28}$$

The functions $A(k)$ and $B(k)$ determine the average projection of the relative momentum $\vec{k}$ and the relative coordinates $\vec{r}$ along the direction $\vec{n}$. We find

$$\langle \Phi \mid \vec{k} \cdot \vec{n} \mid \Phi \rangle = -\frac{8}{3\pi} \frac{Z e^2 \alpha}{m v^2} \int_0^\infty \frac{A(k)\, k^4}{(k^2 + \alpha^2)^2} \, \mathrm{d} k \, , \tag{29}$$

$$\langle \Phi \mid \vec{r} \cdot \vec{n} \mid \Phi \rangle = -\frac{16}{3\pi} \frac{Z e^2 \alpha}{m v^2} \int_0^\infty \frac{B(k)\, k^4}{(k^2 + \alpha^2)^3} \, \mathrm{d} k \, . \tag{30}$$

The components of $\vec{k}$ and $\vec{r}$ perpendicular to $\vec{n}$ have the zero average values.

The norm of the wave function (21) can be expressed as

$$\langle \Phi \mid \Phi \rangle = 1 + \frac{4\alpha}{3\pi} \left(\frac{Z e^2}{m v^2}\right)^2 \int_0^\infty \frac{A^2 + B^2}{(k^2 + \alpha^2)^2} k^4 \, \mathrm{d} k \, . \tag{31}$$

The second term should be negligible compared to unity in order to ensure the applicability of the perturbation theory. It can be used to estimate the reliability of the results obtained for particular values of Z, v and R.

The coordinate representation of the deuteron wave function is defined by the Fourier transform, Equ. (15). After some mathematical manipulations the wave function can be expressed in the following way

$$\Phi(r) = \phi_0(r) \left[1 - \frac{Z e^2 \alpha}{m v^2} H(r\alpha) \frac{\vec{r} \cdot \vec{n}}{r}\right] , \tag{32}$$

where

$$\vec{r} = \vec{r}_p - \vec{r}_n \, , \tag{33}$$

$$H(y) = \frac{y}{2\gamma} \mathrm{i} + \left(1 + \frac{1}{y} - \frac{\mathrm{d}}{\mathrm{d} y}\right) h(y) \, , \tag{34}$$

$$h(y) = \int_0^\infty \mathrm{e}^{-\mathrm{i}\gamma x} \left[1 - \mathrm{e}^{y(1 - \sqrt{1+x})}\right] \frac{\mathrm{d} x}{x} \, , \tag{35}$$

$$\gamma = \frac{\hbar \alpha^2 R}{m v} \, . \tag{36}$$

Both the real and the imaginary part of the complex function $H(y)$ are positive, monotonically increasing functions of y,

$$\mathrm{Re}\, H(y) \geq 0, \qquad \mathrm{Im}\, H(y) \geq 0. \tag{37}$$

The sign of equality holds for $y = 0$.

3. *Interpretation and discussion of the results*

We may give a clear physical interpretation of the theoretical results obtained in Chapter 2. Let us first consider the unperturbed deuteron wave function of the s-wave type, Equ. (7). This wave function, viewed from the coordinate system in which the deuteron centre of mass is at rest at the origin, represents two clouds: the proton cloud in the proton coordinates given by the vector $\vec{r}/2$ and the neutron cloud with the vector $-\vec{r}/2$. Both clouds occupy the same part of the space around the origin with the same probability distribution. The cumulative effect of the external Coulomb field results in the deuteron wave function $\Phi(r)$, Equ. (32) at time t_0 when the nuclear collision takes place. This wave function evidently exhibits the distortion along the direction $\vec{n}$, i. e., along the direction of motion of the deuteron. There is no distortion in the plane perpendicular to the direction $\vec{n}$. The real part of the function H gives rise to the relative displacement of the two probability clouds, that of the proton and that of the neutron, along the direction $\vec{n}$. The mean value of this displacement is given by the value of $\langle | \vec{r} \cdot \vec{n} | \rangle$, Equ. (30). The imaginary part of the function H implies the relative motion of the two clouds. The mean relative momentum is given by $\langle | \vec{k} \cdot \vec{n} | \rangle$, Equ. (29).

Because of the inequalities (28), according to which both functions $A(k)$ and $B(k)$ are positive, the two values $\langle | \vec{r} \cdot \vec{n} | \rangle$ and $\langle | \vec{k} \cdot \vec{n} | \rangle$ are negative (for $Z > 0$). Physically, the proton cloud is pushed back, in the direction opposite to the direction of the deuteron motion. The neutron cloud is consequently shifted ahead with respect to the deuteron centre of mass. The two clouds possess a relative mean velocity in the direction $\vec{n}$, defined by the value $\langle | \vec{k} \cdot \vec{n} | \rangle$, i. e., they move in the opposite direction with respect to each other.

Table 1

The average relative displacement of the proton and the neutron centre of mass in the distorted deuteron is characterized by the quantity $\langle | \vec{r}\,\vec{n} | \rangle$. The average relative momentum is given by the quantity $\langle | \vec{k}\,\vec{n} | \rangle$. The slight deviation from unity for the form $\langle | \rangle$ illustrates the applicability of the perturbation theory.

E [MeV] \ Z, A	10 20	20 40	30 64	40 90	50 112	60 142	70 168	80 196	90 228	[fermi] [fermi]$^{-1}$
20	0,146	0.229	0,288	0.336	0.385	0.419	0.456	0.489	0.516	— $\langle \vert \vec{r}\,\vec{n} \vert \rangle$
	0.015	0.020	0.023	0.025	0.027	0.028	0.029	0.030	0.030	— $\langle \vert \vec{k}\,\vec{n} \vert \rangle$
	1.006	1.012	1.017	1.022	1.027	1.031	1.036	1.040	1.044	$\langle \vert \rangle$
30	0.119	0.189	0.240	0.283	0.326	0.357	0.390	0.419	0.443	— $\langle \vert \vec{r}\,\vec{n} \vert \rangle$
	0.014	0.019	0.022	0.024	0.026	0.027	0.028	0.029	0.030	— $\langle \vert \vec{k}\,\vec{n} \vert \rangle$
	1.004	1.009	1.013	1.017	1.022	1.025	1.029	1.032	1.036	$\langle \vert \rangle$
40	0.102	0.164	0.210	0.248	0.287	0.315	0.345	0.372	0.395	— $\langle \vert \vec{r}\,\vec{n} \vert \rangle$
	0.014	0.019	0.021	0.023	0.026	0.027	0.028	0.029	0.030	— $\langle \vert \vec{k}\,\vec{n} \vert \rangle$
	1.004	1.008	1.011	1.014	1.018	1.021	1.024	1.028	1.030	$\langle \vert \rangle$
50	0.091	0.146	0.188	0.223	0.259	0.285	0.313	0.338	0.359	— $\langle \vert \vec{r}\,\vec{n} \vert \rangle$
	0.013	0.018	0.021	0.023	0.025	0.026	0.027	0.029	0.029	— $\langle \vert \vec{k}\,\vec{n} \vert \rangle$
	1.003	1.006	1.010	1.012	1.016	1.018	1.022	1.024	1.027	$\langle \vert \rangle$
60	0.082	0.133	0.172	0.204	0.237	0.262	0.288	0.311	0.331	— $\langle \vert \vec{r}\,\vec{n} \vert \rangle$
	0.013	0.017	0.020	0.022	0.024	0.026	0.027	0.028	0.029	— $\langle \vert \vec{k}\,\vec{n} \vert \rangle$
	1.002	1.006	1.008	1.011	1.014	1.016	1.019	1.022	0.024	$\langle \vert \rangle$

To gain an insight into the magnitude of the effect considered here, in Table 1 we give the values of $\langle | \vec{r} \cdot \vec{n} | \rangle$ and $\langle | \vec{k} \cdot \vec{n} | \rangle$ over the range of values of the deuteron energy and the set of nuclei covering the periodic system. In the calculations the motion of the heavy nucleus was neglected. For the nuclear radius we used the formula $R = 1.25\ A^{1/3}$. The characteristic energy dependence is evident. The slight deviation from the proportionality with the nuclear charge Z is due to the increase of the nuclear radius. The norm is added in the table to show that the applicability of the perturbation theory cannot be questioned over the whole range in the table. This is particularly true for smaller Z values. Even for extremely large Z values and small energies where the approximation, Equ. (25), of the uniform motion fails, the correction of the norm is less than 5 %. The neglected second-order perturbation term may be expected to be of the same order of magnitude. The uncertainty of the result is, however, much larger due to the plane wave approximation, Equ. (6), for the p-wave component of the scattering state (5). Because of that, the assumption (7) for the deuteron bound-state wave function, which was used instead of a more appropriate Hulthén wave function[12], is immaterial.

The dipole type factors in Equs. (21) and (32), which modify the bound state wave function, show different behaviours in momentum and coordinate representations. This is associated with the character of the functions $G\,(x)$ and $H\,(y)$. The first function is a rapidly decreasing function, while the second is an increasing function of its argument. Asymptotically we have the behaviours

$$G\,(x)_{x \to \infty} \to (1 - i)\frac{1}{x^2}\,,$$

$$H\,(y)_{y \to \infty} \to \ln\frac{y}{2\gamma} + i\,\frac{y}{2\gamma}\,. \tag{38}$$

The deformation of the deuteron strongly affects smaller values of the deuteron internal momentum, although this effect disappears for $k = 0$ (because of the factor $\vec{k} \cdot \vec{n}$).

4. *Application to scattering processes*

The results from the preceding Chapters find application in scattering processes or reactions induced by deuterons. A particularly suitable process is the deuteron stripping reaction in which one of the deuteron constituents is captured by the nucleus. In the early studies of this reaction[1-3] a qualitative argument was put forward to explain the influence of the polarization of the deuteron in the Coulomb field. The argument was that the deformed deuteron favoured the nuclear (d, p) reaction over the (d, n) reaction. The obvious interpretation is that the proton, being pushed away from the nucleus, has less chance to be captured by the nucleus than the neutron. As far as the energies below the Coulomb barrier are concerned, this picture may have significance. In the region of higher energies, i. e., above the Coulomb barrier, the application of the same argument may be questioned. In this region the proton easily penetrates the Coulomb barrier and the distortion of the deuteron is relatively small. We are not going to discuss here to what extent the spatial distortion of the deuteron wave function may influence the strong interaction of the proton and the neutron with the nucleus. Rather we shall take into account the momentum distortion of the wave function.

The proton gains an additional momentum in the direction opposite to the direction of its translatory motion which it possesses as part of the deuteron. This fact indicates that the forward scattering of the proton becomes less probable. However, if an analogous argument is applied, neutron forward

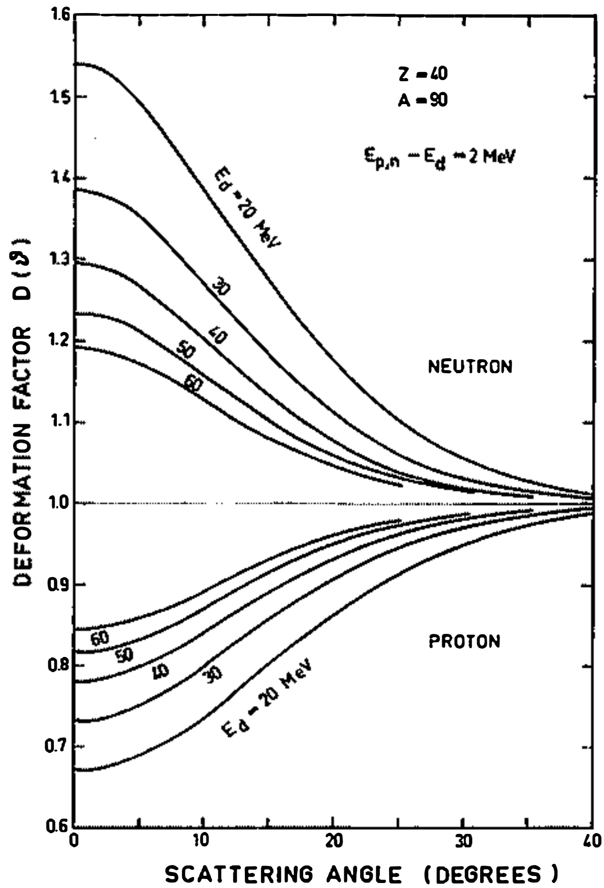

Fig. 1. A typical angular dependence of the modification factor $D(\theta)$ for (d, n) and (d, p) nuclear stripping reactions.

scattering is favoured. On the other hand, forward scattering has the dominant role in direct reactions, which makes it desirable to study the influence of the deuteron polarization effect on the cross section of the reaction. To this purpose we shall use the simple plane-wave theory for the deuteron stripping reaction according to which the differential cross section of a (d, p) reaction is given by the formula[4,5]

$$\frac{d\sigma_p}{d\Omega} \sim |\Phi(\vec{k}_p - \frac{\vec{k}_d}{2})|^2 \sum_l |M_l(\vec{k}_p - \vec{k}_d)|^2. \qquad (39)$$

The second factor is an oscillating factor describing the capture of the neutron by the nucleus; it gives the characteristic oscillatory shape of the angular distribution. However, this factor is not the object of our considerations. The first factor contains the deuteron wave function in the momentum representation, where $\vec{k}$ was substituted by $k_p - \frac{\vec{k}_d}{2}$. This factor is affected by the Coulomb field. Introducing the wave function given by Equ. (21), we find the cross section modified by a factor denoted by D_p

$$\frac{d\sigma_p}{d\Omega} = D_p(\vec{k}_p - \frac{\vec{k}_d}{2}) \frac{d\sigma_0}{d\Omega}, \qquad (40)$$

where

$$D_p(\vec{k}) = |1 - \frac{Ze^2}{mv^2} G(k) \vec{k} \cdot \vec{n}|^2, \qquad (41)$$

and $\frac{d\sigma_0}{d\Omega}$ is the differential cross section obtained from the undisturbed deuteron wave function $\phi_0(k)$. In the case of the (d, n) reaction the factor $D_n(\vec{k}_n - \frac{\vec{k}_d}{2})$ should be introduced; it is defined by

$$D_n(\vec{k}) = D_p(-\vec{k}). \qquad (42)$$

A typical dependence of the factor $D(\theta)$ on the scattering angle is shown in Fig. 1. The deviation from unity is displayed for scattering angles $\theta < 40°$. The decrease of the effect with energy increase is clearly seen. These two characteristic features are present over the whole periodic system of nuclei.

Table 2

The modification factor for the (d, n) and (d, p) stripping reaction cross section at zero scattering angle has been tabulated for variable incident deuteron energy and the set of atomic nuclei over the periodic system. The values ± 2 MeV for the Q value of the reaction have been assumed.

E_d [MeV] \ Z / A	10 / 20	20 / 40	30 / 64	40 / 90	50 / 112	60 / 142	70 / 168	80 / 196	90 / 228	$E_{n,p}$ [MeV]	
20	1.397	1.590	1.717	1.817	1.932	1.997	2.079	2.159	2.201	18	neutron
	1.290	1.412	1.484	1.538	1.602	1.633	1.675	1.710	1.732	22	
30	1.285	1.415	1.496	1.559	1.632	1.670	1.720	1.761	1.791	28	
	1.214	1.300	1.350	1.386	1.430	1.450	1.478	1.500	1.514	32	
40	1.216	1.309	1.364	1.407	1.456	1.480	1.513	1.540	1.558	38	
	1.167	1.232	1.269	1.295	1.326	1.340	1.360	1.375	1.385	42	
50	1.170	1.240	1.281	1.311	1.346	1.363	1.386	1.404	1.416	48	
	1.136	1.187	1.215	1.234	1.258	1.268	1.283	1.294	1.301	52	
60	1.139	1.194	1.225	1.247	1.274	1.286	1.303	1.316	1.324	58	
	1.113	1.155	1.177	1.192	1.211	1.218	1.229	1.238	1.243	62	
20	0.689	0.595	0.555	0.533	0.513	0.512	0.510	0.512	0.520	18	proton
	0.766	0.704	0.680	0.669	0.656	0.659	0.657	0.660	0.666	22	
30	0.764	0.689	0.654	0.634	0.613	0.609	0.601	0.598	0.600	28	
	0.818	0.766	0.744	0.732	0.718	0.717	0.713	0.712	0.714	32	
40	0.814	0.755	0.727	0.710	0.691	0.686	0.678	0.674	0.674	38	
	0.853	0.810	0.791	0.781	0.768	0.766	0.761	0.759	0.759	42	
50	0.850	0.801	0.778	0.765	0.748	0.744	0.737	0.733	0.732		
	0.878	0.842	0.826	0.817	0.806	0.804	0.799	0.796	0.796	52	
60	0.875	0.835	0.816	0.805	0.791	0.788	0.781	0.777	0.777	58	
	0.897	0.866	0.853	0.845	0.835	0.833	0.829	0.826	0.826	62	

Therefore, only the values $D(0)$ for the zero scattering angle have to be calculated for the evaluation of the effect. In Table 2 these values are listed for two values of the proton energy (two Q values, ± 2 MeV) so that a quick rough estimation of the effect in a particular stripping reaction can be made without performing numerical calculations.

5. Conclusion

The internal wave function of the deuteron, deformed by the Coulomb field of the nucleus, is given in a simple close analytic form. The wave function shows the characteristic longitudinal polarization of the deuteron.

Whatever nuclear collision process involving deuterons may be concerned, it is always the deformed deuteron which encounters the nucleus. It is appropriate, therefore, to introduce the deformed deuteron wave function to describe the initial state of a collision process rather than use its spherically symmetric undeformed function. The whole problem can thus be treated in an approximate way which has the advantage of being simple and easy to handle. Many theories of nuclear processes show immediate applications of this method. We have obtained some results for the deuteron stripping process. We have also shown that the effect of the deuteron polarization in the Coulomb field is by no means a negligible effect. It has a remarkable influence on the value of the normalization constant in fitting the experimental results by means of formula (39). Not only is the overall normalization constant affected, but the relative amplitudes in the case of a mixture of angular momenta of the captured particle are also changed. This simply follows from the fact that this effect is concentrated on small angles, affecting strongly the $l = 0$ capture, for example, and practically leaving the next $l = 3$ capture untouched. An effect which can influence the ratio of the corresponding amplitudes by an amount of 30 %, cannot be overlooked in the investigations of nuclear structure.

We mentioned in Chapter 4 that part of the effect of the deuteron polarization on the cross section was taken into account in our considerations. This part comes from the proton momentum transfer $(k_p - \frac{\vec{k}_d}{2})$ (for (d, p) reactions) taken from the deuteron bound state.

The second effect connected with the spatial orientation of the deformed deuteron would require additional theoretical assumptions to describe the influence of the deuteron polarization on the nucleon capture. This effect cannot be introduced into formula (39) in a simple way. We may say that the expected enhancement of the (d, p) reaction over the (d, n) reaction can easily be comprehended classically but not quantum mechanically. We are therefore reluctant to make predictions concerning the influence of this second effect on the cross section, particularly on its angular dependence. It is quite possible that this effect is also concentrated on small angles with tendencies opposite to those resulting in the angular independent overall effect described in this paper. More detailed investigations of stripping reaction mechanisms would be required for the evaluation of the whole effect. That is, however, a separate theoretical problem, which is out of the scope of this paper.

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IZOBLIČENJE DEUTERONA COULOMBOVIM POLJEM

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Sadržaj

Proučava se deformacija interne deuteronske valne funkcije pri gibanju deuterona u Coulomb-ovom polju atomske jezgre. Pripadno međudjelovanje daje izraz (2) i tretira se kao perturbacija u kvantnomehaničkom sistemu proton—neutron, dok se translatorno gibanje tog sistema tretira klasično. Deformiranu valnu funkciju daje izraz (21) u impulsnoj ili izraz (32) u koordinatnoj reprezentaciji. Pripadne dvije karakteristične funkcije G (k) i H (y) definirane su integralnim reprezentacijama (27) i (35).

Kao mjera deformacije deuterona služe srednje vrijednosti projekcije internog impulsa i koordinate — izrazi (29) i (30) — na smjer gibanja deuterona. Očito je da dolazi do razvlačenja deuterona u smjeru njegovog gibanja, pri čemu proton zaostaje za neutronom.

Za dobivanje uvida u veličinu efekta, tablica 1 daje numeričke vrijednosti odgovarajućih veličina preko cijelog periodičnog sistema atomskih jezgara u zavisnosti o upadnoj energiji deuterona.

Izračunao se je uticaj na deuteronske stripping reakcije (d, p) i (d, n). Tipični korekcioni faktor kutne raspodjele izlaznih čestica prikazuje sl. 1 Odgovarajuće vrijednosti za kut raspršenja jednak nuli daje tablica 2.