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APPLICATION OF THE BRILLOUIN-WIGNER PERTURBATION METHOD IN MANY-BOSON THEORY

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Abstract: The perturbation method of Brillouin and Wigner and its rearrangements are considered in many-boson problem. It is applied to a boson system of Bogoljubov.

1. Introduction

In the many-body theory the Brillouin-Wigner perturbation method is rejected as inconvenient because of non correct dependence of the energy equation terms of the number of particles. The problem is in the form of summation. A correct rearrangement of terms in the sums would bring this equation in a proper structure. We will present here two such rearrangements and show their applicability and value in the many-body problems. We will restrict ourself to infinite boson systems.

The first rearrangement is the one due to Feenberg[1, 2]. We will derive it in a somewhat different way. The second rearrangement is a next step on the Feenberg's line.

We will apply the new formula to Bogoljubov's boson systems. The complete Bogoljubov's spectrum will be reproduced (essentialy) and some necessary conditions of applicability of the Bogoljubov's approximation will be found. The conditions will be analysed for the case of a square-wall two body potential in momentum space.

The first part contains the Brillouin-Wigner equation and related problems, the second the Feenberg's rearrangement, the third the application to a boson system in Bogoljubov's approximation, the fourth the new rearrangement and the last are some necessary conditions of applicability of the Bogoljubov's approximation and analysis.

2. *The Brillouin-Wigner equation and related problems*

Let the Hamiltonian be

$$H = H_0 + V \tag{1}$$

and let the solution of the equation

$$H_0 \mid \varphi \rangle = \epsilon \mid \varphi \rangle \tag{2}$$

be known.

The Brillouin-Wigner (BW) solution of the equation

$$H \mid \psi \rangle = E \mid \psi \rangle \tag{3}$$

for the energies is

$$E = \epsilon_l + V_{ll} + \sum_n{}' \frac{\mid V_{ln} \mid^2}{E - \epsilon_n} + \sum_{\substack{n\\n'}}{}' \frac{V_{ln} V_{nn'} V_{n'l}}{(E - \epsilon_n)(E - \epsilon_{n'})} + \dots, \tag{4}$$

where is

$$V_{ij} = \langle \varphi_i \mid V \mid \varphi_j \rangle . \tag{5}$$

The usual way of solving the BW Equ. is iteration. In that case one gets only one solution.

In a case of infinite boson system with constant density ($N \to \infty$, $\Omega \to \infty$, $N/\Omega = \rho =$ const.) the Equ. (4) for the ground state becomes

$$E_0 = \frac{1}{2} N_\infty \rho V + \frac{N_\infty \rho}{2\Omega_\infty} \sum_k{}' \frac{V_k^2}{E_0 - 2\varepsilon_k} + \frac{\rho^2}{2\Omega_\infty} \sum_k{}' \frac{V_k^2 V_{2k}}{(E_0 - 2\varepsilon_k)^2} +$$
$$+ \frac{\rho^2 N_\infty}{\Omega_\infty} \sum_k{}' \frac{V^3_k}{(E_0 - 2\varepsilon_k)^2} + \frac{N^2_\infty \rho^2}{4\Omega_\infty} \sum_k{}' \frac{V^2_k V_0}{(E_0 - 2\varepsilon_k)^2} + \dots \tag{6}$$

The apostrophe at the sums means that the index $k = 0$ is omitted, V_k is the Fourier transform of $V(r)$ and ε_k is the free particle energy. Because $E_0 \sim N_\infty$ each term of the series is negligible with respect to the first one. It follows conclusion that the sum of these infinitely small terms, when one devides the equation by N, is generally finite and nonzero. This is the reason why the BW equation for the energy is considered as unsuitable for the many body theory[3-5].

3. *Feenberg's rearrangement*

Feenberg considered the convergence of the BW energy equation[1] and rearranged the terms in such a way to avoid repetition of matrix elements. The equation which he obtained reads

$$E = \mathcal{E}_l + V_{ll} + \sum_{n}^{\odot} \frac{|V_{ln}|^2}{E - E_n} + \sum_{\substack{n\\ n'}}^{\odot} \frac{V_{ln} V_{nn'} V_{n'l}}{(E - \overset{F}{E}_n)(E - \overset{F}{E}_{n'})} + \ldots, \tag{7}$$

where

$$\overset{F}{E}_n = \mathcal{E}_n + V_{nn} + \sum_{n'}^{\odot} \frac{|V_{nn'}|^2}{E - \overset{F}{E}_{n'}} + \sum_{\substack{n'\\ n''}}^{\odot} \frac{V_{nn'} V_{n'n''} V_{n''n}}{(E - \overset{F}{E}_{n'})(E - \overset{F}{E}_{n''})} \ldots \ . \tag{8}$$

The mark $\odot$ means that all indices are different.

In order to see how the BW energy equation terms have been collected into the Feenberg's equation we will derive this equation directly by partial summations.

Let us write first the BW equation (4) in a semigraphical form:

$$E = \mathcal{E}_l + l \,\vdots\, l + l \,\vdots\, n \,\vdots\, l + l \,\vdots\, n \,\vdots\, n' \,\vdots\, l + \ldots . \tag{9}$$

The vertical dashed lines represent interaction and letters between two lines the states of the H_0, in which one makes summation, with energy denominators.

Now, let us take all terms which containe the matrix element $l \,\vdots\, n \,\vdots\, l$. They are the third term of Equ. (9), the part of the fourth for $n' = n$, the part of the fifth for $n'' = n$ and so on. Explicitely

$$l \,\vdots\, n \,\vdots\, l + l \,\vdots\, n \,\vdots\, n \,\vdots\, l + l \,\vdots\, n \,\vdots\, n' \,\vdots\, n \,\vdots\, l + \ldots \qquad = G_1. \tag{10}$$

Taking out the common factor, one gets:

$$G_1 = l \,\vdots\, n \,\vdots\, l \,\{1 + \frac{1}{E - \mathcal{E}_n}\, n \,\vdots\, n + \frac{1}{E - \mathcal{E}_n}\, n \,\vdots\, n' \,\vdots\, n + \ldots\}. \tag{11}$$

The equation (9) then reads

$$E = \mathcal{E}_l + l \,\vdots\, l + G_1 + \underset{(n' \neq n)}{l \,\vdots\, n \,\vdots\, n' \,\vdots\, l} + \underset{(n'' \neq n)}{l \,\vdots\, n \,\vdots\, n' \,\vdots\, n'' \,\vdots\, l} + \ldots . \tag{12}$$

What we have done with respect to the matrix element $l \,\vdots\, n \,\vdots\, l$ we may do with $\underset{(n' \neq n)}{l \,\vdots\, n \,\vdots\, n' \,\vdots\, l}$ and with other similar terms.

By denoting corresponding contributions with G_2, G_3, ..., we obtain

$$E = \epsilon_l + l \,\vdots\, l + G_1 + G_2 + \dots . \tag{13}$$

The structure of the wiggle bracket of G_1 is similar to that of E. Instead of the fixed index l comes the fixed index n with the factor $1/(E - \epsilon_n)$ and instead of ϵ_l comes 1. There is a restriction on indices in E to be different from l. We may achieve the same with respect to n in G_1 by selecting corresponding terms

$$G_1 = \underline{l \,\vdots\, n \,\vdots\, l}\{1 + q_n + q_n^2 + \dots\},$$

where

$$q_n = \frac{1}{E - \epsilon_n}(n \,\vdots\, n + n \,\vdots\, n' \,\vdots\, n + n \,\vdots\, n' \,\vdots\, n'' \,\vdots\, n + \dots), \tag{13 a}$$
$$n', n'', \dots \neq n.$$

By a similar procedure we find

$$G_2 = \underline{l \,\vdots\, n \,\vdots\, n' \,\vdots\, l}\{1 + q_n + q_n^2 + \dots\}\{1 + q_n' + q_n'^2 + \dots\},$$

$$G_3 = \underline{l \,\vdots\, n \,\vdots\, n' \,\vdots\, n'' \,\vdots\, l}\{1 + q_n + q_n^2 + \dots\}\{1 + q_n' + q_n'^2 + \dots\} \cdot$$
$$\cdot \{1 + q_n'' + q_n''^2 + \dots\}$$

$\vdots$

The line below dashed lines denotes that state indices are different. We will call these graphs connected. The expressions in the wiggle brackets are infinite geometrical series. Thus we may write

$$G_1 = \underline{l \,\vdots\, n \,\vdots\, l}\ \frac{1}{1 - q_n},\quad G_2 = \underline{l \,\vdots\, n \,\vdots\, n' \,\vdots\, l}\ \frac{1}{1 - q_n} \cdot \frac{1}{1 - q_n'}, \dots .$$

After substitution of G's in (13) the explicit analytic expression of E reads

$$E = \epsilon_l + V_{ll} + \sum_{n}^{\odot} \frac{|V_{ln}|^2}{E - [n]} + \sum_{\substack{n \\ n'}}^{\odot} \frac{V_{ln} V_{nn'} V_{n'l}}{(E - [n])(E - [n'])} + \dots, \tag{14}$$

where

$$[m] = \epsilon_m + m \,\vdots\, m \,\vdots\, + m \,\vdots\, n' \,\vdots\, m + m \,\vdots\, n' \,\vdots\, n'' \,\vdots\, m + \dots .$$

We see that the series $[m]$ is the same as the right side of the equation (9). Therefore we may apply exactly the same procedure. The iteration is evident. At the end we get the Feenberg's equation (7).

4. Application of Feenberg's formula

We will apply the Feenberg's formula to a boson system in the Bogoljubov's approximation[6-10]. We will evaluate the ground state energy and the elementary excitation spectrum.

The Hamiltonian in the second quantization form reads:

$$H_0 = \sum_k \varepsilon_k a_k^* a_k , \tag{15}$$

$$V = \frac{1}{2\Omega} \sum_{k_1, k_2, q} V_q a_{k_1+q}^* a_{k_2-q}^* a_{k_2} a_{k_1} . \tag{16}$$

The operators a_k and a_k^* satisfy the comutation rules

$$[a_k, a_k'] = [a_k^*, a_k'^*] = 0, \qquad [a_k, a_k'^*] = \delta_{k, k'} . \tag{17}$$

In order to get the ground state energy one has to take $l = 0$ in the Feenberg's formula (7). The ground state vector of the unperturbed system is

$$|0\rangle = |N;\, 0\rangle , \tag{18}$$

that means all practicles ocupy the zero momentum state.

An energy of the Bogoljubov elementary excitation spectrum is a function only of one Fourier component of the two-body interaction. The corresponding fact one finds also in the ground state energy. This corresponds to transitions of the system, due to interaction, from the ground free particle state to excited states with two particles of opposite momenta, scattering processes in the excited states, transitions from a excited state to a higher state with two more particles of the same momenta and so on, and reverse processes to the ground state. Therefore, let us look at those terms in the Feenberg's equation (7) and let them sum up.

All numerators under the sums in the Feenberg's formula contain products of mixture of the Fourier components except the numerator of the first sum. This can be verified easily. The first and the last matrix elements of each of these numerators give a product of different Fourier components (see *Appendix* or the Equ. (5) and restriction on different states). Thus follows that only the first sum has to be retained. The Feenberg's equation in the Bogoljubov's approximation with respect to numerators for the ground state energy, therefore, reads

$$E = \epsilon_0 + V_{00} + \sum_n^{\odot} \frac{|V_{0n}|^2}{E - \overset{F}{E_n}}. \tag{19}$$

So far we have concentrated our attention to the denominator of Equ. (19). But because the structure of $\overset{F}{E_n}$ in the denominator of Equ. (19) with respect to numerators is the same as of E itself, the conclusion is valid also for $\overset{F}{E_n}$. There is only additional restriction on the first sum. Because the initial state is now n, the transition state n' should be such that matrix elements $V_{nn'}$ give the Fourier component as V_{0n}. The state $|n'\rangle$ is $|N-4;\ 2_k 2_{-k}\rangle$. Analogous conclusions follow for the denominator of $\overset{F}{E_n}$ and other denominators.

The explicit expressions of E in the fourth, fifth and sixth successive approximations read

$$E = \frac{1}{2} N_\infty\, V_0 \rho + \frac{1}{2} \sum_k{}' \frac{\rho^2 V_k^2}{-2\varepsilon_k - 2\rho V_k - \rho^2 V_k^2/(-2\varepsilon_k)},$$

$$E = \frac{1}{2} N_\infty\, V_0 \rho + \frac{1}{2} \sum_k{}' \frac{\rho^2 V_k^2}{-2\varepsilon_k - 2\rho V_k - \frac{1}{-2\varepsilon_k - 2\rho V_k}\rho^2 V_k^2},$$

$$E = \frac{1}{2} N_\infty\, V_0 \rho + \frac{1}{2} \sum_k{}' \frac{\rho^2 V_k^2}{-2\varepsilon_k - 2\rho V_k - \frac{\rho^2 V_k^2}{-2\varepsilon_k - 2\rho V_k - \frac{\rho^2 V_k^2}{-2\varepsilon_k}}} \cdots . \tag{20}$$

It is assumed that the number of excited particles n_k is such that is

$$\lim_{\substack{N \to \infty \\ \Omega \to \infty}} \frac{N - n_k}{\Omega} = \rho .$$

The evaluation of the matrix elements is given in the *Appendix.*

The expressions under the sums in (20) lead to a continuous fraction. Denoting

$$\rho^2 V_k^2 = f(k) = f,$$

and

$$-2\,\varepsilon_k - 2\,\rho\,V_k = \beta(k) = \beta\,,$$

E becomes

$$E = \frac{1}{2} N_\infty \, V_0 \rho + \frac{1}{2} \sum_k{}' \cfrac{f}{\beta - \cfrac{f}{\beta - \cfrac{f}{\beta - \cfrac{f}{\beta - \cfrac{f}{\beta - \cdots}}}}}. \tag{21}$$

The function which is represented by the continuous fraction one can easily find. Let us write

$$y = \cfrac{f}{\beta - \left|\, \cfrac{f}{\beta - \cfrac{f}{\beta - \cfrac{f}{\cdots}}} \right.}$$

then we have

$$y = \frac{f}{\beta - y}.$$

From here we find

$$y = \sqrt{\varepsilon_k^2 + 2\,\rho\,\varepsilon_k V_k} - \varepsilon_k - \rho\,V_k\,. \tag{23}$$

After substitution of (23) in (22) we get the ground state energy

$$E_0 = \frac{1}{2} N_\infty \, V_0 \rho + \frac{1}{2} \sum_k{}' \left\{ \sqrt{\varepsilon_k^2 + 2\,\rho\,\varepsilon_k V_k} - \varepsilon_k - \rho\,V_k \right\}. \tag{24}$$

As one can see, this is the ground state energy of the Bogoljubov theory $N_0 \rightarrow N_\infty$ [6].

The BW formula with iteration can be applied to any initial unperturbed state. In general the evaluated energies correspond to higher energy states of the total system. The simplest elementary excitation of the unperturbed system are single particle excitations of the energy ε_k and the momentum $\hbar \vec{k}$. Then states are

$$|1\rangle = |N-1;\ 1_k\rangle, \qquad k \neq 0\,.$$

We will now evaluate the energies according to (7) with respect to these states in the Bogoljubov's approximation. Applying the same reasoning as in the ground state energy case, we have to consider only

$$E_1 = \mathcal{E}_1 + V_{11} + \sum_n{}^{\odot} \frac{|V_{1n}|^2}{E - E_n^F}\,.$$

There are two possible states of $|n\rangle$

$$|n\rangle_1 = |N-3;\ 2_k\ 1_{-k}\rangle$$

$$|n\rangle_2 = |N-3;\ 1_k\ 1_k{}'\ 1_{-k}{}'\rangle, \quad \vec{k} \neq \pm \vec{k}'.$$

For each case one has to take in the denominator only those terms which don't give mixing products. One finds that these terms give the state vectors: $|N-3;\ 2_k\ 1_{-k}\rangle$, $|N-5;\ 3_k\ 2_{-k}\rangle, \ldots\ |N-2\,n_k-1;\ (n+1)_k\ n_{-k}\rangle, \ldots$ for the state $|n\rangle_1$, and $|N-3;\ 1_k\ 1_k{}'\ 1_{-k}{}'\rangle$, $|N-5;\ 1_k\ 2_k{}'\ 2_{-k}{}'\rangle, \ldots\ |N-2\,n_k{}'-1;\ 1_k\ n_k{}'\ n_{-k}{}'\rangle;\ \ldots$ for the state $|n\rangle_2$.

The successive approximations of the fourth, fifth and sixth order of the energy now read

$$E_1^4 = E_1^1 + \frac{1}{2}\sum_k{}' \frac{\rho^2 V_k^2}{-2\,\varepsilon_k - 2\,\rho\,V_k - \dfrac{\rho^2 V_k^2}{-2\,\varepsilon_k}} + \frac{\rho^2 V_k^2}{-2\,\varepsilon_k - 2\,\rho\,V_k - \dfrac{\rho^2 V_k^2}{-2\,\varepsilon_k}}\,,$$

$$E_1^5 = E_1^1 + \frac{1}{2}\sum_k{}' \frac{\rho^2 V_k^2}{-2\,\varepsilon_k - 2\,\rho\,V_k - \dfrac{\rho^2 V_k^2}{-2\,\varepsilon_k - 2\,\rho\,V_k}} +$$

$$+ \frac{\rho^2 V_k^2}{-2\,\varepsilon_k - 2\,\rho\,V_k - \dfrac{\rho^2 V_k^2}{-2\,\varepsilon_k - 2\,\rho\,V_k^2}}\,,$$

$$E_1^6 = E_1^1 + \frac{1}{2}\sum_k{}' \frac{\rho^2 V_k^2}{-2\,\varepsilon_k - 2\,\rho\,V_k - \dfrac{\rho^2 V_k^2}{-2\,\varepsilon_k - 2\,\rho\,V_k - \dfrac{\rho^2 V_k^2}{-2\,\varepsilon_k}}} +$$

$$+\frac{\rho^2 V_k^2}{-2\varepsilon_k-2\rho V_k-\frac{\rho^2 V_k^2}{-2\varepsilon_k-2\rho V_k-\frac{\rho^2 V_k^2}{-2\varepsilon_k}}} \tag{25}$$

with

$$E_1^1=\varepsilon_k+\rho V_k+\frac{1}{2}N_\infty V_0\rho .$$

Similarly to the ground state energy case, it now follows:

$$E_1=E_1^1+\frac{1}{2}\sum_k{}' \frac{\rho^2 V_k^2}{-2\varepsilon_k-2\rho V_k-\frac{\rho^2 V_k^2}{-2\varepsilon_k-2\rho V_k-|}}+$$

$$+\frac{\rho^2 V_k^2}{-2\varepsilon_k-2\rho V_k-\frac{\rho^2 V_k^2}{-2\varepsilon_k-2\rho V_k-\frac{\rho^2 V_k^2}{-2\varepsilon_k-2\rho V_k-|}}} \tag{26}$$

From here we find

$$E_1=E_0+E_k, \tag{27}$$

where E_0 is the ground state energy (24) and

$$E_k=E_1-E_0=\sqrt{\varepsilon_k^2+2\varepsilon_k\rho V_k} \tag{28}$$

are energies of the elementary excitations.

We again see that they are in accordance with Bogoljubov's theory ($N_0 \to N_\infty$) They are the energies of bogolons.

5. *Further improvement of BW formula*

In spite of a great value of the Feenberg's formula, its general dependence of the number of particles is not correct.

Let us iterate the Feenberg's equation (7) for $l=0$ and let us write down it explicitely up to the second order with respect to numerator and denominator

$$E = \epsilon_0 + V_{00} +$$

$$+\sum_n^{\odot} \frac{V_{0n} V_{n0}}{(\epsilon_0 - \epsilon_n) + (V_{00} - V_{nn}) + \sum_n^{\odot} \frac{|V_{0\bar{n}}|^2}{\epsilon_0 - \epsilon_{\bar{n}}} - \sum_{n'}^{\odot} \frac{|V_{nn'}|^2}{\epsilon_0 - \epsilon_{n'}} + \ldots} + \ldots \quad (29)$$

Because the matrix element V_{0n} is proportional to $V_k N_\infty / \Omega$, the third term in Equ. (29) will be correct, that means proportional to N_∞, if the denominator is independent of N_∞. The first two brackets in the denominator do not depend of N_∞. However, the third bracket has a term which is proportional to N_∞. Therefore, with this term the Feenberg's equation begins to be incorrect.

We can see also that the numerators begin to be incorrect from the fourth order assuming that the denominators are independent of N_∞.

It is illustrative, and we will use it later on, to look at the dependence of N_∞ of the successive approximations in Feenberg's equation (7). They are

$$E_0^0 = \epsilon_0 ,$$

$$E_0^1 = \epsilon_0 + V_{00} ,$$

$$E_0 = \epsilon_0 + V_{00} + \sum_n^{\odot} \frac{|V_{0n}|^2}{E_0^0 - E_n^0}$$

$$E_0^3 = \epsilon_0 + V_{00} + \sum_n^{\odot} \frac{|V_{0n}|^2}{E_0^1 - E_n^1} + \sum_{\substack{n \\ n'}}^{\odot} \frac{V_{0n} V_{nn'} V_{n'0}}{(E_0^1 - E_n^0)(E_0^0 - E_{n'}^0)} \quad (30)$$

In these equations we have dropped F from E_n's. These approximations have correct dependence of N_∞ up to the E_0^3 (let us mention that the corresponding BW successive approximations have correct dependence up to the E_0^2).

We will now correct these approximations up to fifth order (E_0^5).

The successive approximations of the fourth and fifth order let us write in the form:

$$E_0^4 = E_0^1 + \sum_n^{\odot} \frac{|V_{0n}|^2}{E_0^2 - E_n^2} + \sum_{n,n'}^{\odot} \frac{V_{0n} V_{nn'} V_{n'0}}{(E_0^1 - E_n^1)(E_0^1 - E_{n'}^1)} +$$

$$+ \sum_{\substack{n,n' \\ n''}}^{*} \frac{V_{0n} V_{nn'} V_{n'n''} V_{n''0}}{(E_0^0 - E_n^0)(E_0^0 - E_{n'}^0)(E_0^0 - E_{n''}^0)} + \sum_{\substack{n,n' \\ n''}}^{**} \frac{V_{0n} V_{nn'} V_{n'n''} V_{n''0}}{(E_0^0 - E_n^0)(E_0^0 - E_{n'}^0)(E_0^0 - E_{n''}^0)}$$

and

$$E_0^5 = E_0^1 + \sum_n^{\odot} \frac{|V_{0n}|^2}{E_0^3 - E_n^3} + \sum_{n,n'}^{\odot} \frac{V_{0n} V_{nn'} V_{n'0}}{(E_0^2 - E_n^2)(E_0^2 - E_{n'}^2)} +$$

$$+ \sum_{\substack{n,n' \\ n''}}^{*} \frac{V_{0n} V_{nn'} V_{n'n''} V_{n''0}}{(E_0^1 - E_n^1)(E_0^1 - E_{n'}^1)(E_0^1 - E_{n''}^1)} + \sum_{\substack{n,n' \\ n''}}^{**} \frac{V_{0n} V_{nn'} V_{n'n''} V_{n''0}}{(E_0^1 - E_n^1)(E_0^1 - E_{n'}^1)(E_0^1 - E_{n''}^1)} +$$

$$+ \sum_{\substack{n,n' \\ n'',n'''}}^{*} \frac{V_{0n} V_{nn'} V_{n'n''} V_{n''n'''} V_{n'''0}}{(E_0^0 - E_n^0)(E_0^0 - E_{n'}^0)(E_0^0 - E_{n''}^0)(E_0^0 - E_{n'''}^0)} +$$

$$+ \sum_{\substack{n,n' \\ n'',n'''}}^{**} \frac{V_{0n} V_{nn'} V_{n'n''} V_{n''n'''} V_{n'''0}}{(E_0^0 - E_n^0)(E_0^0 - E_{n'}^0)(E_0^0 - E_{n''}^0)(E_0^0 - E_{n'''}^0)} . \tag{31}$$

The marks * and ** denote the sums with correct and incorrect numerators, respectively.

The expression E_0^4 doesn't have the correct dependence on N_∞ because of the denominator of the first sum and the numerator of the last sum. One can expect that correction of the denominator of the first sum will eliminate the last sum.

In order to see what has been done, let us write explicitely this denominator and the states which appear in it

$$E_0^2 - E_n^2 = (\mathfrak{E}_0 - \mathfrak{E}_n) + (V_{00} - V_{nn}) + \left(\sum_{\bar{n}}^{\odot} \frac{|V_{0\bar{n}}|^2}{E_0^0 - E_{\bar{n}}^0} - \sum_{n'}^{\odot} \frac{|V_{nn'}|^2}{E_0^0 - E_{n'}^0} \right)$$

$$n = 1_k\, 1_{-k}, \quad \bar{n} = 1_{k'}\, 1_{-k'},$$

$$n' = \left\{ \begin{matrix} 2_k\, 2_{-k} & 1_k\, 1_k'\, 1_{-k-k'} & \\ 1_k'\, 1_{-k}' & 1_k\, 1_{-k}'\, 1_{k'-k} & 1_{-k}\, 1_{-k}'\, 1_{k+k}' \\ 2_k\, 1_{-2k}' & 1_{-k}\, 1_k'\, 1_{k-k}' & 1_k\, 1_{-k}\, 1_k'\, 1_{-k}' \\ 2_{-k}\, 1_{2k} & & \end{matrix} \right\} \tag{32}$$

For simplicity, only the number and state of particles out of the condensate have been expressed. The first two brackets do not depend of N_∞. After substitution of explicit values of the matrix elements the term which dependens of N_∞ reads

$$\sum_{\substack{\bar{n} \\ (\neq n)}}^{\odot} \frac{|V_{0\bar{n}}|^2}{E_0^0 - E_{\bar{n}}^0} - \sum_{\substack{n' \\ (=a)}}^{\odot} \frac{|V_{nn'}|^2}{E_0^0 - E_{n'}^0} = \frac{N(N-1)}{2\,\Omega^2} \sum_k' \frac{V_k'^2}{-2\,\varepsilon_k} -$$

$$- \frac{(N-2)(N-3)}{2\,\Omega^2} \sum_{k'} \frac{V_k'^2}{-2\,\varepsilon_k - 2\,\varepsilon_k'}\,; \quad a = 1_k\, 1_{-k}\, 1_k'\, 1_{-k}'. \tag{33}$$

Other terms either go to zero on the volume limit, or are functions of the density ρ.

Let us notice that on the left side of Equ. (33) the energy indices do not follow transitions given by numerators in both terms. We have

	the first sum	the second sum
numerator	$0 \to \bar{n} \to 0$	$n \to n' \to n$.
denominator	$0 \to \bar{n}$	$0 \to n'$

Inspection of the right side of Equ. (33) shows that just it causes appearance of the N_∞ (corresponding terms can not be canceled). Therefore, it has to be corrected.

We redefine E_n^2 by $E_n^2 \to \varepsilon_n^2$, where

$$\varepsilon_n^2 = \epsilon_n + V_{nn} + \sum_{\substack{n' \\ (\neq a)}}^{\odot} \frac{|V_{nn'}|^2}{E_0^0 - E_{n'}^0} + \sum_{\substack{n' \\ (=a)}}^{\odot} \frac{|V_{nn'}|^2}{E_n^0 - E_{n'}^0}\,, \tag{34}$$

and write

$$E_n^2 = \varepsilon_n^2 + D_n^2\,,$$

with

$$D_n^2 = \sum_{\substack{n' \\ (=a)}}^{\odot} \frac{|V_{nn'}|^2}{E_0^0 - E_{n'}^0} - \sum_{\substack{n' \\ (=a)}}^{\odot} \frac{|V_{nn'}|^2}{E_n^0 - E_{n'}^0}.$$

Expanding the sum with respect to D_n^2 we get the new E_0^4

$$E_0^4 = E_0^1 + \sum_{n}^{\odot} \frac{|V_{0n}|^2}{E_0^2 - \varepsilon_n^2} + \sum_{n,n'}^{\odot} \frac{V_{0n} V_{nn'} V_{n'0}}{(E_0^1 - \varepsilon_n^1)(E_0^1 - \varepsilon_{n'}^1)} +$$

$$+ \sum_{\substack{n,n' \\ n''}}^{*} \frac{V_{0n} V_{nn'} V_{nn''} V_{n''0}}{(E_0^0 - \varepsilon_n^0)(E_0^0 - \varepsilon_{n'}^0)(E_0^0 - \varepsilon_{n''}^0)} + \Bigg(\sum_{\substack{n,n' \\ n''}}^{**} \frac{V_{0n} V_{nn'}}{(E_0^0 - \varepsilon_n^0)(E_0^0 - \varepsilon_{n'}^0)} \cdot$$

$$\cdot \frac{V_{n'n''} V_{n''0}}{(E_0^0 - \varepsilon_{n''}^0)} - \sum_{\substack{n,n' \\ (=a)}}^{\odot} \frac{|V_{0n}|^2 |V_{nn'}|^2}{(E_0^0 - \varepsilon_n^0)(E_0^0 - \varepsilon_{n'}^0)(E_0^0 - \varepsilon_{n'}^0)} \Bigg). \tag{35}$$

For a symmetry reason we have written also $E_n^1 = \varepsilon_n^1$ and $E_n^0 = \varepsilon_n^0$.

The sums in the bracket don't have correct dependence on N_∞. We expect this whole expression to vanish. It is indeed so. Both terms are equal to

$$\frac{1}{4\,\Omega^4} N(N-1)(N-2)(N-3) \sum_{k,k'}^{\odot} \frac{V_k^2 V_{k'}^2}{-2\,\varepsilon_k(-2\,\varepsilon_k - 2\,\varepsilon_{k'})(-2\,\varepsilon_{k'})}.$$

So, the whole bracket is equal to zero.

By this way we have shown how one can get the correct successive approximation of the fourth order.

We may apply the same procedure to the Feenberg's approximation of the fifth order. The Feenberg's energies which do not have correct forms are: E_n^2, $E^2{}_n'$ and E_n^3. The first two are corrected by the fourth order approximation. The last one we correct similarly:

$$E_n^3 = \varepsilon_n^3 + D_n^3,$$

where

$$\varepsilon_n^3 = \mathcal{E}_n + V_{nn} + \sum_{\substack{n' \\ (=a)}}^{\odot} \frac{|V_{nn'}|^2}{E_0^1 - E_{n'}^1} + \sum_{\substack{n' \\ (\neq a)}}^{\odot} \frac{|V_{nn'}|^2}{E_n^1 - E_{n'}^1} +$$

$$+\sum_{\substack{n',n''\\(\neq a)}}^{\odot} \frac{V_{nn'}V_{nn''}V_{n''n}}{(E_0^0-E_{n'}^0)(E_0^0-E_{n''}^0)}+\sum_{\substack{n',n''\\(\neq a)}}^{\odot} \frac{V_{nn'}V_{n'n''}V_{n''n}}{(E_n^0-E_{n'}^0)(E_n^0-E_{n''}^0)}$$

and

$$D_n^3=\sum_{\substack{n'\\(=a)}}^{\odot} \frac{|V_{nn'}|^2}{E_0^1-E_{n'}^1}-\sum_{\substack{n'\\(=a)}}^{\odot} \frac{|V_{nn'}|^2}{E_n^1-E_{n'}^1}+$$

$$+\sum_{\substack{n',n''\\(=a)}}^{\odot} \frac{V_{nn'}V_{n'n''}V_{n''n'}}{(E_0^0-E_{n'}^0)(E_0^0-E_{n''}^0)}-\sum_{\substack{n',n''\\(=a)}}^{\odot} \frac{V_{nn'}V_{n'n''}V_{n''n}}{(E_n^0-E_{n'}^0)(E_n^0+E_{n''}^0)}.$$

The mark a denotes the states which lead to a total term with N_∞ as a factor.

The new E_0^5 now reads

$E_0^5=$ [Equ. (31) without sums with * * and with $E_m \to \varepsilon_m$] +

$$+\left\{\left(\sum_{\substack{n,n'\\n''}}^{**} \frac{V_{0n}V_{nn'}V_{n'n''}V_{n''0}}{(E_0^1-\varepsilon_n^1)(E_0^1-\varepsilon_{n'}^1)(E_0^1-\varepsilon_{n''}^1)}-\right.\right.$$

$$\left.-\sum_{\substack{n\\n'(=a)}}^{\odot} \frac{|V_{0n}|^2|V_{nn'}|^2}{(E_0^1-\varepsilon_n^1)(E_0^1-\varepsilon_{n'}^1)(E_n^1-\varepsilon_{n'}^1)}\right)+$$

$$+\left(\sum_{\substack{n,n'\\n'',n'''}}^{**} \frac{V_{0n}V_{nn'}V_{n'n''}V_{n''n'''}V_{n'''0}}{(E_0^0-\varepsilon_n^0)(E_0^0-\varepsilon_{n'}^0)(E_0^0-\varepsilon_{n''}^0)(E_0^0-\varepsilon_{n'''}^0)}-\right. \tag{36}$$

$$-\sum_{\substack{n\\n',n''(=a)}}^{\odot} \frac{|V_{0n}|^2V_{nn'}V_{n'n''}V_{n''n}}{(E_0^0-\varepsilon_n^0)(E_0^0-\varepsilon_{n'}^0)(E_0^0-\varepsilon_{n''}^0)(E_n^0-\varepsilon_{n''}^0)}-$$

$$-\sum_{\substack{n\\n',n''(=a)}}^{\odot} \frac{|V_{0n}|^2V_{nn'}V_{n'n''}V_{n''n}}{(E_0^0-\varepsilon_n^0)(E_0^0-\varepsilon_{n'}^0)(E_n^0-\varepsilon_{n'}^0)(E_0^0-\varepsilon_{n''}^0)}-$$

$$-\sum_{\substack{n,n'\\\bar{n}(=a)}}^{\odot} \frac{V_{0n}V_{nn'}V_{n'0}|V_{n\bar{n}}|^2}{(E_0^0-\varepsilon_n^0)(E_0^0-\varepsilon_{n'}^0)(E_0^0-\varepsilon_{\bar{n}}^0)(E_n^0-\varepsilon_{\bar{n}}^0)}-$$

$$\left.\left.-\sum_{\substack{n,n'\\\bar{n}'(=a)}}^{\odot} \frac{V_{0n}V_{nn'}V_{n'0}|V_{n'\bar{n}'}|^2}{(E_0^0-\varepsilon_n^0)(E_0^0-\varepsilon_{n'}^0)(E_0^0-\varepsilon_{\bar{n}'}^0)(E_n^0-\varepsilon_{\bar{n}'}^0)}\right)\right\}.$$

The difference $E_0^3 - \varepsilon_n^3$ is independent of N_∞. We expect again that the wiggle bracket is equal to zero. Explicit evaluation of the matrix element and substitution into the wiggle bracket shows that is so. We do not repeat this calculation here but rather indicate[11].

Therefore, equation (36) without the wiggle bracket is the correct fifth order successive approximation.

Continuation of this procedure gives a new rearrangement of the BW formula.

The new formula, unfortunately, is still not the final solution. One can show that the sixth order approximation has some wrong terms.

This formula can also be obtained by partial summation. We don't present it here because of further steps which seem to require a somewhat different initial basis.

The result which we have got in this section one may use in finding some necessary conditions of validity of the boson theory from the Chapter 4. We will do it in the next section.

6. *Some criteria of applicability of Bogoljubov approximation*

The Bogoljubov's energies (24) and (28) contain all powers of V. They are certain selection from the total series An establishment of validity of this selection is not simple. Here we will make use of Equ. (35) from the previous section and determine and analyse conditions of applicability of the fourth order Bogoljubov's successive approximation.

After substitution of values of the matrix elements, Equ. (35) reads

$$
\begin{aligned}
E_0^4 = {} & \frac{1}{2} N_\infty V_0 \rho + \frac{N_\infty \rho}{2\Omega_\infty} \sum_k{}' \frac{V_k^2}{-2\varepsilon_k - 2\rho V_k - \dfrac{\rho^2 V_k^2}{-2\varepsilon_k} - \dfrac{2\rho}{\Omega_\infty} \sum\limits_{\substack{k' \\ (\neq k)}} \dfrac{V_k'^2}{2\varepsilon_k'}} + \\
& + \frac{N_\infty \rho}{2\Omega_\infty^2} \sum_{k,k'}^{\odot} \frac{V_k V_k' V_{k+k'}}{(2\varepsilon_k + 2\rho V_k)(2\varepsilon_k' + 2\rho V_k')} - \\
& - \frac{N_\infty \rho}{2\Omega_\infty^3} \sum_{k,k',k''}^{\odot} \frac{V_k V_k'' V_{k+k''} V_{k'+k''}}{8\varepsilon_k \varepsilon_k' \varepsilon_k''} - \\
& - \frac{N_\infty \rho^2}{\Omega_\infty^2} \sum_{k,k'}^{\odot} \frac{V_k V_k' (V_k' + V_{k+k'})(V_k + V_{k+k'})}{4\varepsilon_k \varepsilon_k' (\varepsilon_k + \varepsilon_k' + \varepsilon_{k+k'})} -
\end{aligned}
\tag{37}
$$

(The mark $\odot$ includes $k, k', \ldots \neq 0$).

This equation becomes the Bogoljubov's fourth order successive approximation, if

$$\left| -2\,\varepsilon_k - 2\,\rho\, V_k + \frac{\rho^2 V_k^2}{2\,\varepsilon_k} \right| \gg \left| \frac{\rho}{\Omega_\infty} \sum_{\substack{k' \\ (\neq k)}} \frac{V_{k'}^2}{\varepsilon_k'} \right| \tag{38}$$

and

$$\left| \frac{V_0}{2} + \frac{1}{2\Omega_\infty} \sum_k \frac{V_k^2}{-2\,\varepsilon_k - 2\,\rho\, V_k - \frac{\rho^2 V_k^2}{-2\,\varepsilon_k}} \right| \gg$$

$$\gg \left| \frac{1}{8\,\Omega_\infty^2} \sum_{k,k'}^{\odot} \frac{V_k V_k' V_{k+k}'}{(\varepsilon_k + \rho V_k)(\varepsilon_k' + \rho V_k')} - \right.$$

$$- \frac{1}{16\,\Omega_\infty^3} \sum_{k,k',k''}^{\odot} \frac{V_k V_k' V_{k+k}'' V_{k'+k}''}{\varepsilon_k\, \varepsilon_k'\, \varepsilon_k''} - $$

$$\left. - \frac{\rho}{4\,\Omega_\infty^2} \sum_{k,k'}^{\odot} \frac{V_k V_k' (V_{k+k}' + V_k)(V_k' + V_{k+k}')}{\varepsilon_k\, \varepsilon_k' (\varepsilon_k + \varepsilon_k' + \varepsilon_{k+k}')} \right| . \tag{39}$$

In order to see the meaning of these conditions more simply we select

$$V_k = \begin{cases} V_0 & k < k_g \\ 0 & k > k_g . \end{cases} \tag{40}$$

Reciprocal value of the k_g determines range of the interaction. The condition (38) then becomes

$$\left| -\left(\frac{\hbar^2 k^2}{m}\right)^2 - \frac{\hbar^2 k^2}{m} \cdot 2\,\rho\, V_0 + \rho^2 V_0^2 \right| \gg \left| \frac{\hbar^2 k^2}{m}\, \rho\, V_0^2 \frac{m\, k_g}{\hbar^2 \pi^2} \right| . \tag{41}$$

The second condition is still complicated. We simplify it by taking only the first term on the right side and leaving out ρV_k and $\rho V_k'$ from its denominator. The condition becomes more stronge and now reads

$$\left| 1 - \frac{V_0\, m}{2\,\pi^2 \hbar^2} \left\{ k_g + b\, C_1 \ln \left| \frac{k_g - C_2}{k_g + C_2} \right| - b\, C_3 \operatorname{arctg} \frac{k_g}{b\, C_4} \right\} \right| \gg$$

$$\gg V_0^2\, k_g^2 \left(\frac{m}{\hbar^2}\right)^2 C_5, \tag{42}$$

with

$$C_1 = \frac{3 - 2\sqrt{2}}{4\sqrt{2(\sqrt{2}-1)}} \quad C_2 = \sqrt{\sqrt{2}-1}, \quad C_3 = \frac{3 + 2\sqrt{2}}{4\sqrt{2(\sqrt{2}+1)}},$$

$$C_4 = \sqrt{\sqrt{2}+1}, \quad C_5 = \frac{9 + 2\pi^2}{2^6 \cdot 3 \cdot \pi^4}, \quad b = \sqrt{\rho V_0 \frac{m}{\hbar^2}},$$

where integrations have been performed.

The curves on left and right side of the relation (41) are represented on

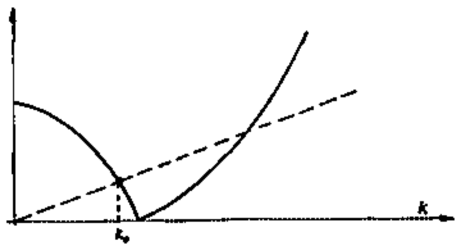


Fig. 1.

From here follows

$$k_g \ll k_0. \tag{43}$$

The k_0 is given by

$$k_0^2 = -\rho V_0 \frac{m}{\hbar^2} (1 + k_g V_0 \frac{m}{\hbar^2} \frac{1}{2\pi^2}) +$$

$$+ \frac{m}{\hbar^2} \rho V_0 \sqrt{(1 + k_g V_0 \frac{m}{\hbar^2} \frac{1}{2\pi^2})^2 + 1}. \tag{44}$$

Explicite restriction on k_g from (43) we give for

$$\rho V_0^3 \gg (\frac{\hbar^2}{m})^3 \pi^4 \quad \text{and} \quad \rho V_0^3 \ll (\frac{\hbar^2}{m})^3 \pi^4. \tag{45}$$

In these cases one finds

$$k_g \ll \pi^{\frac{2}{3}} \rho^{\frac{1}{3}}, \qquad k_g \ll \{ \rho V_0 \frac{m}{\hbar^2} (-1 + \sqrt{2}) \}^{\frac{1}{2}} \tag{46}$$

respectively.

Similar analysis of the relation (42) gives

$$k_g \ll \frac{\hbar^2}{V_0 m} \cdot 37, \qquad k_g \ll \{\rho V_0 \frac{m}{\hbar^2} \cdot (-1 + \sqrt{2})\}^{\frac{1}{2}} \tag{47}$$

also for the cases (45).

The first condition of (47) is stronger than the corresponding one from (46). Thus, we have

$$k_g \ll \frac{\hbar^2}{V_0 m} \cdot 37, \qquad \rho V_0^3 \gg (\frac{\hbar^2}{m})^3 \pi^4$$

and (48)

$$k_g \ll \{\rho V_0 \frac{m}{\hbar^2} \cdot (-1 + \sqrt{2})\}^{\frac{1}{2}}, \qquad \rho V_0^3 \ll (\frac{\hbar^2}{m})^3 \pi^4.$$

We see that strenght of interaction, range, mass of particles and density are involved in the conditions. They seem reasonable.

From (48) follows that for a given interaction there is a restriction on the density. Below certain density the conditions are no more satisfied. This can be understood as a consequence of the fact that in low-density limit with finite k_g and V_0 the system behaves more like free particles.

7. Comparison with Rayleigh-Schrödinger method

In order to illustrate the character of the partial summations performed in the equation of Feenberg, we give here the terms of the Rayleigh-Schrödinger expansion contained in

$$E_0^4 = \epsilon_0 + V_{00} + \sum_n^{\odot} \frac{V_{0n} V_{n0}}{\epsilon_0 - \epsilon_n + V_{00} - V_{nn} + \sum_{\bar{n}}^{\odot} \frac{|V_{0\bar{n}}|^2}{\epsilon_0 - \epsilon_{\bar{n}}} - \sum_{\bar{n}'}^{\odot} \frac{|V_{nn'}|^2}{\epsilon_0 - \epsilon_{n'}}},$$

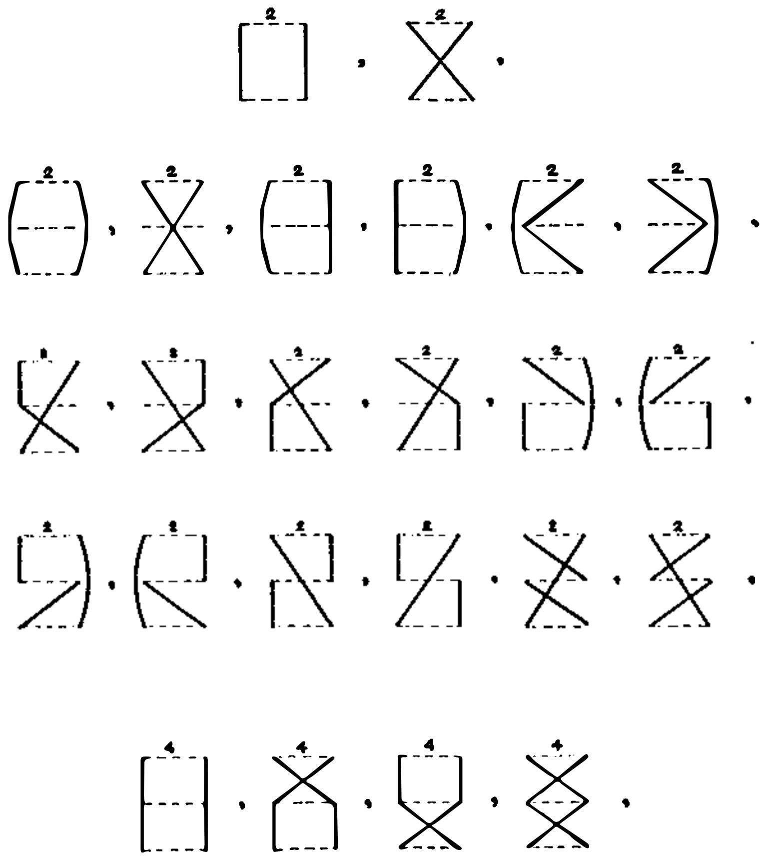
up to the third powers of V (in the Bogoljubov's approximation).

The E_0^4 written in powers of V reads:

$$E_0^4 = \epsilon_0 + V_{00} + \sum_n^{\odot} \frac{V_{0n} V_{n0}}{\epsilon_0 - \epsilon_n} + \sum_n^{\odot} \frac{V_{0n} V_{nn} V_{n0}}{(\epsilon_0 - \epsilon_n)^2} -$$

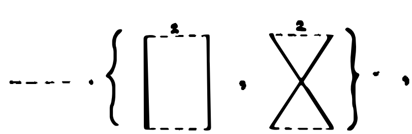
$$- V_{00} \sum_n^{\odot} \frac{V_{0n} V_{n0}}{(\epsilon_0 - \epsilon_n)^2}$$

The corresponding Goldstone diagrams are:
, connected,



(we do not draw the lines with $k = 0$),

and



disconnected.
The lines on the diagrams have to be denoted in all possible ways. The figure above a diagram tells the number of these possibilites.

We see that connected and disconnected diagrams with no dispersion in different k-states are collected in the approximation of Bogoljubov. This is what we expected.

This comparison shows explicitly the advantage of summing these diagrams by the method of Brillouin—Wigner—Freenberg. For mostly used perturbation tecnique and related problems we refer to[10].

Appendix

The list of matrix elements which appear in the approximation of Bogoljubov.

$$\langle 0, N \mid V \mid N-2, 1_k 1_{-k} \rangle = \frac{1}{\Omega} \sqrt{N(N-1)}\, V_k,$$

$$\langle 1_k 1_{-k}, N-2 \mid V \mid N-4, 2_k 2_{-k} \rangle = \frac{2}{\Omega} \sqrt{(N-2)(N-3)}\, V_k,$$

$$\vdots$$

$$\langle (m-1)_k (m-1)_{-k},\ N-2m+2 \mid V \mid N-2m,\ m_k m_{-k} \rangle =$$

$$= \frac{1}{\Omega} \sqrt{(N-2m+2)(N-2m+1)}\, V_k; \quad m \geqslant 1,$$

$$\langle 0, N \mid V \mid N, 0 \rangle = \frac{1}{2\Omega} N(N-1)\, V_0 = V_{00},$$

$$\langle 1_k 1_{-k}, N-2 \mid V \mid N-2, 1_k 1_{-k} \rangle = V_{00} + \frac{2}{\Omega}(N-2)\, V_k + \frac{1}{\Omega} V_{2k},$$

$$\vdots\ ,$$

$$\langle m_k m_{-k}, N-2m \mid V \mid N-2m, m_k m_{-k} \rangle = V_{00} + \frac{2m}{\Omega}(N-2m)\, V_k +$$

$$+ \frac{1}{\Omega} m^2 V_{2k} + \frac{1}{\Omega} m(m-1)\, V_0; \quad m \geqslant 2,$$

$$\langle 1_k, N-1 \mid V \mid N-1, 1_k \rangle = \frac{1}{2\Omega}(N-1)(N-2)\, V_0 +$$

$$+ \frac{1}{\Omega}(N-1)(V_0 + V_k) = V_{nn},$$

$$\langle 2_k\, 1_{-k}, N-3 | V | N-3, 2_k\, 1_{-k}\rangle = V_{nn} + \frac{2}{\Omega}(N-4)\, V_k + \frac{2}{\Omega} V_{2k},$$

$$\langle 3_k\, 2_{-k}, N-5 | V | N-5, 3_k\, 2_{-k}\rangle = V_{nn} + \frac{4}{\Omega}(N-6)\, V_k + \frac{6}{\Omega} V_{2k},$$

$$\vdots$$

$$\langle m_k\, (m-1)_{-k}, N-2m+1 | V | N-2m+1, m_k\, (m-1)_{-k}\rangle =$$

$$= V_{nn} + 2\,(m-1)\,(N-2m)\, V_k + \frac{1}{\Omega}\, m\,(m-1)\, V_{2k},$$

$$\langle 1_k, N-1 | V | N-3, 2_k\, 1_{-k}\rangle = \frac{1}{\Omega}\sqrt{2\,(N-1)\,(N-2)}\, V_k,$$

$$\langle 2_k\, 1_{-k}, N-3 | V | N-5, 3_k\, 2_{-k}\rangle = \frac{1}{\Omega}\sqrt{3\cdot 2\,(N-3)\,(N-4)}\, V_k,$$

$$\vdots$$

$$\langle m_k'\, (m-1)_{-k}', N-2m+1 | V | N-2m-1, (m-1)_k\, m_{-k}\rangle =$$

$$= \frac{1}{\Omega}\sqrt{(m+1)\, m\,(N-2m+1)\,(N-2m)}\, V_k,$$

$$\langle 1_k\, 1_k'\, 1_{-k}', N-3 | V | N-3, 1_k\, 1_k'\, 1_{-k}'\rangle = V_{nn} + \frac{2}{\Omega}(N-3)\, V_k' -$$

$$-\frac{2}{\Omega} V_k + \frac{1}{\Omega}(V_{2k}' + V_{k-k}' + V_{k+k}'),$$

$$\langle 1_k\, 2_k'\, 2_{-k}', N-5 | V | N-5, 1_k\, 2_k'\, 2_{-k}'\rangle = V_{nn} + \frac{4}{\Omega}(N-5)\, V_k' -$$

$$-\frac{4}{\Omega} V_k + \frac{2}{\Omega}(V_{k-k}' + V_{k+k}') + \frac{4}{\Omega} V_{2k}',$$

$$\vdots\,,$$

$$\langle 1_k\, m_k'\, m_k', N-2m-1 | V | N-2m-1, 1_k\, m_k'\, m_{-k}'\rangle = V_{nn} +$$

$$+\frac{2m}{\Omega}(N-2m-1)\, V_k' - \frac{2m}{\Omega} V_k + \frac{m}{\Omega}(V_{k-k}' + V_{k+k}') + \frac{m^2}{\Omega} V_{2k}',$$

$$\langle 1_k, N-1 | V | N-3, 1_k\, 1_k'\, 1_{-k}'\rangle = \frac{1}{\Omega}\sqrt{(N-1)\,(N-2)}\, V_k',$$

$$\langle 1_k\, 1_{k'}\, 1_{-k'}, N-3 | V | N-5, 1_k\, 2_{k'}\, 2_{-k'} \rangle = \frac{2}{\Omega} \sqrt{(N-3)(N-4)}\, V_{k'},$$

$$\vdots$$

$$\langle 1_k\, (m-1)_{k'}\, (m-1)_{-k'}, N-2m+2 | V | N-2m, 1_k\, m_{k'}\, m_{-k'} \rangle =$$

$$= \frac{m}{\Omega} \sqrt{(N-2m+2)(N-2m+1)}\, V_{k'}.$$

The vector k and k' are everywhere different.

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PRIMJENA BRILLOUIN-WIGNEROVOG RAČUNA SMETNJE U TEORIJI MNOŠTVA BOZONA

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Sadržaj

Brillouin-Wignerov račun smetnje i njegova preuređenja razmatrana su u problemima mnoštva bozona. Pri tom je izvedena Feenbergova formula jednim novim postupkom.

Feenbergova formula primijenjena je na bozonske sisteme u Bogoljubovljevoj aproksimaciji. Potpuni Bogoljubovski spektar (u suštini) je reproduciran.

Nova formula upotrebljena je za nalaženje nekih nužnih uvjeta primjenljivosti Bogoljubovljeve aproksimacije. Ti uvjeti analizirani su na pravokutnoj barijeri u k-prostoru.

Na svršetku izvršena je i usporedba sa Rayleigh-Schrödingerovim računom smetnje.