

EFFECTIVE KINETIC ENERGY

I. Kinetic Energy, the Internucleon Interaction and the Binding and Excitation Energies of Light Nuclei

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Abstract: In order to find a simple shell model Hamiltonian, which at the same time gave the experimental binding energy of the ground state of light nuclei, and their correct excitation energies, various assumptions were made concerning the effective kinetic energy of a nucleon in the nucleus. The potentials possessed large repulsive matrix elements in p -states, which together with the reduction in kinetic energy resulted in the lowering of the deformed excited states to the experimental region. The binding energies of the spherical nuclei were also given correctly.

1. Introduction

Now that forms for the internucleon interaction have been derived from free nucleon scattering data it is customary to perform calculations in nuclear structure with these potentials. Such calculations are usually carried out with harmonic oscillator wavefunctions adapted to allow for the hard nucleon core. Considerable evidence has been collected that certain linear combinations of these states corresponding to the normal vibrations of a many-body oscillator configuration, and given by its SU3 symmetry, approximate well to the correct eigenstates. However if these wavefunctions are used in shell model calculations two connected discrepancies with experiment are found. Firstly, insufficient binding energy is obtained in the ground states of light nuclei, and secondly, the excitation energies of states of both even and odd parity, with the one exception of the dipole state, are predicted to occur at a much higher energy than is experimentally found.

In what follows we are led to adapting the usual concept of the effective kinetic energy of a nucleon in the nucleus by two considerations. Usually in

the independent particle model the nucleons have been considered to move freely within the nuclear radius, independently of one another. We now investigate the consequences of supposing that a viscous drag is experienced by a nucleon due to the pion sea it moves in. This viscous drag we shall see later is connected with many-body forces.

Secondly, we have a piece of evidence that we should modify the usual Hamiltonian, directly derived from the experimental data. In ^{1, 2} certain formulae were derived for the binding energy of the ground state, and for the single particle energies, that is the energies required to remove a single 1s or 1p particle, from the ground state of Oxygen 16. These quantities obey the equation

$$(1)$$

where E is the ground state energy, E_{1s} and E_{1p} are the single particle energies, and T_{1s} and T_{1p} are the kinetic energies of a particle in the 1s and 1p shells, respectively. A similar equation holds for Helium 4 which is

$$(2)$$

These equations hold for any two-body force in the p-shell and are not dependent on additional assumptions. Recalling the experimental values for the binding energies and single particle values we obtain for the kinetic energies of the nucleons in the two nuclei, the following relations. For Oxygen 16 we find

$$T_{1s} + 6 T_{1p} = 74 \text{ MeV}, \quad (3)$$

and for Helium 4

$$T_{1s} = 12 \text{ MeV}. \quad (4)$$

Now it is usual to derive the oscillator constant $\hbar \omega$, for the nucleus concerned, from the experimental root mean square radius. Experimental values for these quantities are given by Hofstadter³ derived from high-energy electron scattering. From these data we may derive that the oscillator constants for Helium 4 and Oxygen 16 are 24.7 and 13.8 MeV, respectively. We see immediately that if these values were used to calculate the kinetic energies in the usual way, that is $T_{1s} = \frac{3}{4} \hbar \omega$ and $T_{1p} = \frac{5}{4} \hbar \omega$, equations (3) and (4) are overestimated.

There are several ways out of this dilemma. Firstly, the r. m. s. estimates for the oscillator constants may be too large, and we must reduce them for shell model calculations. This, however, seems unlikely. Secondly, some phenomenological factor has reduced the size of T_{1s} and T_{1p} . Calculation shows that reduction by the factor $\frac{2}{3}$ would satisfy equations (3) and (4) for the r. m. s. values of the oscillator constants.

2. A form for the effective kinetic energy operator

A reduced mass is by no means new in nuclear matter calculations; here, however, we need an enhanced mass and one moreover which may be shown to vary from nucleus to nucleus. A straightforward enhanced mass was tried but failed to lower the excited states sufficiently. Abandoning this, we try the following form, chosen because of its simplicity.

$$T = \sum_i \frac{\hbar}{4m} \left(p_i^2 e^{-\frac{r_i^2}{\alpha^2}} + e^{-\frac{r_i^2}{\alpha^2}} p_i^2 \right). \quad (5)$$

The expectation value of this operator in oscillator states is readily obtainable. We find

$$T_{1s} = \frac{3\hbar\omega}{4} \frac{(1 + \frac{2}{x^2})}{(1 + \frac{1}{x^2})^{5/2}}, \quad T_{1p} = \frac{5\hbar\omega}{4} \frac{(1 + \frac{2}{x^2})}{(1 + \frac{1}{x^2})^{7/2}}, \quad (6)$$

where $x = \frac{\alpha}{b}$ and b is given by $b^2 = \frac{\hbar}{m\omega}$. If the expressions for the kinetic energy are to be reduced as already stated, it appears that α varies from nucleus to nucleus in such a way as to reduce the kinetic energy by the observed amount. We also consider the case where α is a constant, 2.6 fm, for all nuclei equal to, or heavier than, Helium 4. This would seem reasonable when considered in conjunction with the »constant density« properties of such nuclei. The triton and the deuteron being more diffuse possess larger values of α . We find that the value for the deuteron is in the order of 14 fm. As a nucleon leaves the nucleus the value of α will become infinite and the kinetic energy will reduce to its usual form.

Objections may be raised that expression (5) for the effective kinetic energy is not translationally invariant. We should, however, remember that only states in which the centre of mass of the nucleus is stationary are made use of in calculations, the spurious ones being omitted.

We take as our two-body interaction the sum of two Gaussian potentials with Majorana exchange. More complicated exchange mixtures are unnecessary if we only consider wavefunctions of [44 . . . 4] multiplet symmetry. Our total Hamiltonian then takes the form

$$\begin{aligned}
 H = & \sum_i \frac{1}{4m} \left(e^{-\frac{r_i^2}{\alpha^2}} p_i^2 + p_i^2 e^{-\frac{r_i^2}{\alpha^2}} \right) + \\
 & + \sum_{i < j} S_1 e^{-\frac{r_{ij}^2}{\mu_1^2}} (1 - m_1 + m_1 P_{ij}) + \sum_{i < j} S_2 e^{-\frac{r_{ij}^2}{\mu_2^2}} (1 - m_2 + m_2 P_{ij}).
 \end{aligned} \tag{7}$$

We fix μ_1 at 1.4 fm, the one-pion exchange radius, and vary μ_2 at distances less than this. In addition to α , we are left with $S_1 S_2 m_1 m_2$ to determine. We choose to fit the binding energy of Helium 4 and the binding energy per particle of nuclear matter, together with their stability equations, following a procedure used by Brink and Boeker⁴). We do this for the following cases

- α equals infinity, that is the unadapted expression for the kinetic energy, and
- $\alpha = 2.6$ fm for all nuclei heavier than Helium 4.

Table 1.

The parameters of the potentials A_i

	μ_2 fm	S_1 MeV	$1 - 2 m_1$	S_2 MeV	$1 - 2 m_2$	$K \cdot \text{MeV}$
A_0	1.0	-158	0.1111	191	0.9541	73.5
A_1	0.9	-140	0.1209	197	1.408	95
A_2	0.8	-127	0.5601	221	4.385	316
A_3	0.7	-119	0.328	270	5.060	216
A_4	0.6	-112	-0.0101	362	2.864	18
A_5	0.48	-108	0.4384	599	13.56	252
A_6	0.38	-104	0.789	1251	32.15	361
A_7	0.28	-102	1.120	2486	87.80	595
A_8	0.14	-101	1.000	18630	346.7	704
A_9	0.07	-100	0.1811	144700	513.0	266

Table 2.

The parameters of the potentials B_i

	μ_2 fm	S_1 MeV	$1 - 2 m_1$	S_2 MeV	$1 - 2 m_2$	$K \cdot \text{MeV}$
B_0	1.0	-124.1	-0.9061	134	-1.387	-323
B_1	0.9	-109.9	-0.5878	135.4	-0.1483	149
B_2	0.8	-102.9	-1.068	144.5	-2.987	-51
B_3	0.7	-97.5	-1.24	192.7	-5.7	10
B_4	0.6	-92.55	-0.7543	254	-1.57	157
B_5	0.48	-88.6	-0.900	428.6	-4.331	150
B_6	0.4	-87.2	-0.9253	698.7	-7.718	100
B_7	0.28	-84.8	-1.474	1732	-59.61	7
B_8	0.14	-84.49	-1.623	13140	-295	-225
B_9	0.07	-83.75	-1.053	102200	-377	71

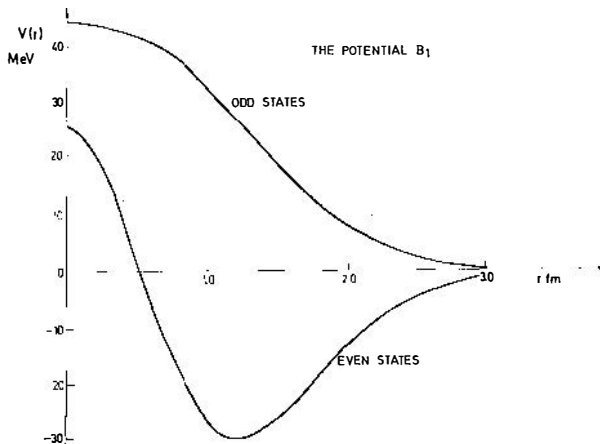
Varying μ_2 we obtain a set of potentials in each case. We shall call these A_i and B_i , respectively. Other values for α were tried but proved less profitable.

Before going further, let us examine the effect of a constant value for α on the kinetic energy as we go to heavy nuclei. The kinetic energy for Helium 4, Oxygen 16, Calcium 40 and a hypothetical nucleus with 164 protons and 164 neutrons (Appendix 1), are reduced respectively by 14%, 45%, 75% and 92%. Nuclear matter under these conditions will be left with very little free kinetic energy. This somewhat drastic conclusion may however be not so surprising when one considers the low temperature aspects of medium and heavy nuclei⁹.

A quantity which is of great importance in assessing the physical significance of each potential is the compressibility of nuclear matter defined by

Brueckner as $K = -k_F^2 \frac{\partial^2 (\frac{E}{A})}{\partial k_F^2}$. He quotes experimental values of 100 to 150 MeV. In Table 1 and Table 2 we list the parameters of potentials A_i and B_i .

We see from the tables that for potentials B_i the compressibility K oscillates becoming negative at minima. At maxima, however, K is at its experimental value. The potentials with negative K are of course unphysical. Looking for the most realistic potential we would expect the exchange properties of the longer range Gaussian to resemble the one pion exchange potential,



which has exchanges parameters $(1-2 m_1)$ equal to -0.6 in states of [44 . . . 4] symmetry. We see that B_1 has this behaviour, as well as the experimental value for K . Having obtained the parameters of the potentials, their expectation value between the oscillator wave functions of nuclei may be found. Thus V_0 and V_1 are the matrix elements between $1s$ and $1p$ states etc. Calculation shows that the matrix element V_1 for potential B_1 is large and repulsive.

3. The binding and excitation energy of light nuclei
with maximum symmetry

In ^{1, 2)} formulae were derived for the binding energies and excitation energies of light nuclei. For Oxygen 16 we have

$$O^{16} \text{ g. s.} \quad +E = 3 T_{1s} + 12 T_{1p} + 42 V_0 + 60 V_1 + 18 V_2,$$

$$\text{Triplet} \quad (21) \quad \Delta E = T_{sd} - T_{1p} - \frac{3}{2} V_0 - \frac{5}{2} V_1 + \frac{3}{2} V_2, \tag{8}$$

$$2p \ 2h \quad (42) \quad \Delta E = 2 T_{sd} - 2 T_{1p} - \frac{21}{8} V_0 - 5 V_1 + \frac{9}{4} V_2,$$

$$4p \ 4h \quad (84) \quad \Delta E = 4 T_{sd} - 4 T_{1p} - \frac{15}{4} V_0 - 10 V_1 + \frac{3}{2} V_2.$$

Examination of the above expressions reveals that a reduction in the value of the kinetic energy together with an increase in the repulsion in *p*-states would not alter the binding energy in the ground state overmuch, and would also lower the excitation energies.

Table 3.

The binding energies of light nuclei for potentials *A*₂ and *B*₁

	⁴ Be	¹² C	¹⁶ O	⁴⁰ Ca
<i>A</i> ₂	24	60	125	417 MeV
<i>B</i> ₁	34	67	140	411
Exp	60	100	142	420

Table 4.

The energies of excited states in Oxygen 16

	(21) 1 <i>p</i> 1 <i>h</i>	(42) 2 <i>p</i> 2 <i>h</i>	(84) 4 <i>p</i> 4 <i>h</i>
<i>A</i> ₂	162	30.3	52.2 MeV
<i>B</i> ₁	6.2	10.3	15.9
Exp	7.2	~ 9	~ 9

The SU3 designation (21) corresponds to the odd parity triplet 2⁻ 1⁻ 3⁻ at 8.87, 7.12 and 6.14 averaged to 7.2. The 2*p* 2*h* and 4*p* 4*h* states are more difficult to locate, and the composition of the low-lying 0⁺ and 2⁺ states is the subject of much discussion. The lowest occurring 0⁺ state is at 6.06 MeV.

Looking at the above tables we see that the energies predicted for a conventional force A_2 are all much too large, but excellent agreement is found for force B_1 .

4. The dipole states in Oxygen 16

The dipole resonance states in Oxygen 16 have been perhaps more widely studied than any other configurations. Experimentally, Tanner, Thomas and Earle⁶⁾ have obtained a spectrum over a wide energy range using the reaction $N^{15}(p, \gamma) O^{16}$. They obtained peaks at 24.5, 22, 19.5 and 17.2 MeV with other minor resonances. Let us now see how the potentials under study match up to this data. To do this, we must adapt our theory as so far presented. This is because the dipole states have [44431] symmetry. We consider a Hamiltonian with a full exchange mixture and also a spin orbit term. The latter will have zero matrix elements in states of [44...4] symmetry and therefore will not affect our discussions of ${}^4\text{He}$ or nuclear matter. Rather than working with the parameters WMB and H we choose linear combination which are the scattering amplitudes. For instance

$${}^{13}A_i = W_i + M_i - B_i - H_i, \quad i = 1, 2.$$

In our previous work $A_i^{13} = {}^{31}A_i = 1$ and ${}^{11}A_i = 33 A_i = (1 - 2 m_i)$. For configurations of higher symmetry we shall now adopt the following values for our scattering amplitudes. For both Gaussians we retain ${}^{31}A_i$ at unity but reduce ${}^{13}A_i$ to 0.6. This is usual to account for deuteron data. We keep ${}^{11}A_i$ equal to ${}^{33}A_i$ and both equal to 0.8 ($1 - 2 m_i$). Thus for potential B_1 ${}^{11}A_1 = {}^{33}A_1 = -0.4662$ and ${}^{11}A_2 = {}^{33}A_2 = -0.1186$, and for potential B_5 ${}^{11}A_1 = {}^{33}A_1 = -0.7200$ and ${}^{11}A_2 = {}^{33}A_2 = -0.3465$. We now discuss the possible singly excited 1^- states. There are five such states with $T = 1$ all with [44431] symmetry. These are $|1\rangle$ $|2\rangle$ $|3\rangle$ $|4\rangle$ $|5\rangle$, (10) $L = 1$ $s = 0$, (10) $L = 1$ $s = 1$, (21) $L = 1$ $s = 0$, (21) $L = 1$ $s = 1$, (21) $L = 2$ $s = 1$. Expressions for the interaction energy of a particle with full exchange mixture have been derived⁷⁾ for all the above states. For instance $|1\rangle$ the interaction energy is given by

$$\frac{1}{8} \cdot \{ ({}^{31}A + 3 {}^{13}A) (-5 V_0 + V_2) - ({}^{11}A + 3 {}^{33}A) V_1 \}. \quad (9)$$

These expressions are listed in Appendix 5. The diagonal matrix elements of the spin orbit coupling vanish for these configurations.

Using these techniques the diagonal matrix elements of the five 1^- states may be derived.

Of our five states only $|1\rangle$ should contain dipole strength. This is because the dipole operator transforms as (10) and cannot alter the spin from that of the ground state. If a matrix of the interaction Hamiltonian was diagonalised then the dipole strength would be shared between these five states.

Table 5.

Diagonal matrix elements for the singly excited $T = 1 \ 1^-$ states in Oxygen 16

	$ 1\rangle$	$ 2\rangle$	$ 3\rangle$	$ 4\rangle$	$ 5\rangle$
B_1	23	20	16.2	15.2	15.2 MeV
B_2	19	17	15.3	14.8	14.8
Exp.	24.5	22	19.5	17.2	13

Experimentally, the strength occurs almost equally divided between the highest two, indicating strong mixing between the states $|1\rangle$ and $|2\rangle$ due to the spin orbit coupling.

5. A simulated hard core

The potential B_1 though it gave good agreement with the calculation of binding energies and excited energy levels, possessed a soft core some 30 MeV in height. For realistic calculations a hard core should be introduced. This may be achieved in this framework where oscillator wavefunctions are employed, by the inclusion of a correlation effect by means of a delta function at the origin, in the effective Hamiltonian.

The additive term to equation (7) is

$$\sum_{i < j} A \delta (r_i - r_j) . \quad (10)$$

The constants of this modified expression are fitted as before with the help of the additional experimental equation comprising the p particle separation energies of Oxygen 16. Thus for each pair of values for α_{16} and α_4 a set of potentials may be generated. The input parameters for various potentials are given in Table 6.

Table 6.

$\hbar\omega_{16}$ MeV	$\hbar\omega_4$ MeV	α_{16} fermi	α_4 fermi	Potential
13.8	24.7	∞	∞	A_i^*
		2.6	2.6	B_i^*
		2.6	2	C_i^*
		2.6	2.3	D_i^*
		2.8	2.6	E_i^*
		3.2	1.3	F_i^*

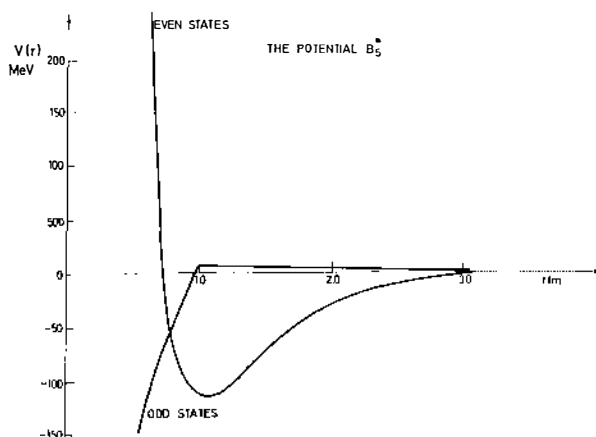
The calculated values of the force parameters are listed in Appendix 3. Typical values are those of A_5^* , B_5^* and C_5^* (see Table 7).

Table 7.

	S_1 MeV	S_2 MeV	$1-2m_1$	$1-2m_2$	A MeV fm ³
A_5^*	-235	104000	0.006	0.698	$-1.37 \cdot 10^{-35}$
B_5^*	-231	113000	-0.046	0.696	$-1.56 \cdot 10^{-35}$
C_5^*	-369	223000	0.096	0.698	$-2.98 \cdot 10^{-35}$

We notice there is a large repulsion of the order of 100,000 MeV near the origin. This is to be expected from experiment. However, there is also a large attraction counterbalancing it at the origin, resulting in a finite matrix element when the potential is integrated over an s -state. The long range part of the potentials are either weakly attractive or weakly repulsive in states of odd angular momentum, where we would expect a „ $(1-2m_1)$ value” of -0.6 if the long range gaussian is to simulate the one pion exchange potential. In this sense therefore they are not so realistic as those previously treated with a soft core.

The contribution to the binding of nuclear matter by the various parts of the potentials may be evaluated. In general we find that in even states the



large repulsion by the inner gaussian keeps nuclear matter from collapsing. A measure of this is the compressibility. This may be found for any potential and is found not to oscillate as it did for the potentials A_i and B_i . Typical values are 215 for B_1^* and 214 for B_3^* . The experimental value should be about 150 MeV. The combination of the reduced kinetic energy and the correlation effects of the delta function consequently give rise to stable conditions in nuclear matter. The values of the compressibilities are listed in Appendix 4.

6. The binding energy of Oxygen 16 for the hard core potentials

The potential energy of Oxygen 16 may easily be written down in terms of the two-body matrix elements of the potentials in relative $1s$, $1p$ and $(2s\ 1d)$ states, V_0 , V_1 and V_2

$$V_{16} = 42 V_0 + 60 V_1 + 18 V_2 . \quad (11)$$

Adding the kinetic energies¹⁾ the binding energy may be deduced for any potential once the matrix elements have been calculated. These matrix elements have been called strength functions because they measure the strength of the interaction in a given state. Tables of the strength functions V_0 , V_1 and the binding energies are presented in Appendix 4.

There are essentially two parameters to each set of potentials, the mean free path for Oxygen 16 α_{16} and for Helium 4 α_4 . We may discuss how the variation of these parameters effect the experimentally determinable quantities, the binding energy of Oxygen 16 and the compressibility of nuclear matter, -142 MeV and 150 MeV, respectively. Consulting Appendix 4 we see that perhaps the most satisfactory potential is B_5^* which yields -138 MeV binding and 216 MeV for the compressibility. For the soft core potential where no delta function was included at the origin, considerable instability occurred in both the binding energy of Oxygen 16 and K, so it was possible to pick out a best value of μ_2 . With the inclusion of the delta function the oscillations in K disappeared and any of the values of μ_2 appeared equally satisfactory; however, a general improvement appeared as μ_2 became very small.

The matrix elements may be compared with those deduced by Elliott et al.⁹⁾ from the free internucleon scattering phase shifts. These are given in singlet and triplet states.

Table 8.
Matrix elements from phase shifts

$(\hbar\omega_0 = 13.8\ \text{MeV})$		
	singlet	triplet
V_0	-6.63	$-7.92\ \text{MeV}$
V_1	1.75	-0.22

These must be averaged over spins if they are to be compared with the matrix elements deduced here and in Ref.¹⁾. We notice the p -state interaction is large and repulsive in singlet states and small and attractive in triplet states. For potential B_1^* $V_0 = -8.30$ and $V_1 = 1.745$. The equivalent expres-

sions for potential B_5^* are $V_0 = -8.55$, $V_1 = 2.37$ MeV. We see that in relative s -states B_1^* and B_5^* are slightly more attractive than Elliott's. In p -states they approximate well to the singlet p -state free nucleon value. It would appear that inside nuclei the scattering is confined to singlet singlet interaction strength when the relative angular momentum is in a p -state.

7. Conclusions

The actual significance of lowering the kinetic energy is the building in of an interaction into the Hamiltonian in a convenient manner. We have seen how the concept of collision damping has been introduced into the framework of the shell model. These collisions will cause certain redistributions of the nucleons in a given configuration as manifested by the much larger exchange forces present in potentials B than in potentials A . This results in larger p -state interactions which contribute towards the lowering of the excited configurations.

Variation of the mean free path parameter α from nucleus to nucleus has been examined and the potentials where this mean free path parameter is constant at 2.6 fm were found to be the most realistic ones. However, for potentials B_i with a soft core, only certain ranges of the inner gaussian gave realistic values for the compressibility of nuclear matter, B_1 being the most satisfactory. For the hard core potentials no negative values for K were found, the presence of the delta function resulting in stable conditions in nuclear matter. However, the exchange parameters in a $1p$ -state are more nearly the same as a one pion exchange potential for force B_1 than for any of the hard core potentials. Both B_1 and most of the hard core potentials gave a good value for the binding energy of Oxygen 16.

Comparison with the matrix elements of Elliott et al.⁹⁾ shows in general good agreement in even states, but the potentials discussed here are far more repulsive in odd states than theirs. Comparison with the size of the matrix element in singlet p -states leads us to suspect that the interaction inside nuclei in p -states is confined to this singlet singlet coupling strength.

We have investigated the consequences of reducing the effective kinetic energy in the shell model. Greatly improved results were obtained from the assumption that the kinetic energy of the nucleons is progressively reduced as we go to heavier nuclei. The binding of the ground states of spherical nuclei were shown to agree with experiment, however, the binding in the unfilled shells was underestimated showing the necessity for deformation effects. The excitation energies of even parity states in the spherical nucleus Oxygen 16 were shown to be brought down to the region where they are experimentally found. The dipole spectrum was also reproduced correctly.

This lowering of the excited even parity states without having to perform a Hartree-Fock calculation, was in fact the greatest contribution of this paper.

Acknowledgement

I should like to thank the Science Research Council for a research grant.

Appendix 1. Matrix elements of the effective kinetic energy.

We consider two cases, wavefunctions of a harmonic oscillator and of nuclear matter enclosed in a box of side L .

$$T = \sum \frac{1}{4m} \left(e^{-\frac{r_i^2}{\alpha^2}} p_i^2 + p_i^2 e^{-\frac{r_i^2}{\alpha^2}} \right), \quad (1)$$

$$T_{1s} = \frac{3}{4} \hbar \omega \frac{\left(1 + \frac{r_i^2}{x^2}\right)}{\left(1 + \frac{1}{x^2}\right)^{5/2}}, \quad T_{1p} = \frac{5}{4} \hbar \omega \frac{\left(1 + \frac{r_i^2}{x^2}\right)}{\left(1 + \frac{1}{x^2}\right)^{7/2}},$$

$$T_{2s} = \frac{7}{4} \hbar \omega \frac{\left(1 + \frac{20}{7x^2} + \frac{33}{14x^4} + \frac{3}{x^6}\right)}{\left(1 + \frac{1}{x^2}\right)^{9/2}}, \quad T_{1d} = \frac{7}{4} \hbar \omega \frac{\left(1 + \frac{2}{x^2}\right)}{\left(1 + \frac{1}{x^2}\right)^{9/2}},$$

$$T_{sd} = \frac{1}{6} (T_{1s} + 5T_{1d}),$$

where $x = \frac{\alpha}{b}$ and $b^2 = \frac{\hbar}{m\omega}$.

$$T_L = \frac{3}{10} \frac{\hbar^2 K_F^2}{m} x^3 \left\{ \text{Erf} \left(\frac{1}{x} \right) \right\}^3 + \frac{3}{2} \frac{\hbar^2}{m \alpha^2} x^2 e^{-\frac{1}{x^2}} \left\{ \text{Erf} \left(\frac{1}{x} \right) \right\}^2, \quad (2)$$

where $\text{Erf}(y) = \int_0^y e^{-z^2} dz$ and $x = \frac{2\alpha}{L}$.

Appendix 2. Strength functions for Oxygen 16 for states of [4444] symmetry

	V_0	V_1	V_2	K	E
A_0	-8.83	0.793	-1.754	73.5	-116.9
A_1	-8.78	0.764	-1.726	95	-113
A_2	-8.74	0.546	-1.708	316	-125.6
A_3	-8.71	0.863	-1.626	216	-104.3
A_4	-8.70	1.034	-1.433	18	-90.5
A_5	-8.55	0.888	-1.667	251	-97.8
A_8	-8.67	0.744	-1.594	704	-110.
B_0	-7.744	1.870	-1.543	-323	-107.5
B_1	-8.30	1.745	-1.519	149	-139.8
B_2	-8.03	1.975	-1.582	-51	-114.8
B_3	-7.70	1.847	-1.43	10	-106
B_5	-7.61	1.822	-1.487	150	-99.3
B_8	-7.66	1.721	-1.435	-225	-112.3
			Exp	150	-142

Appendix 3. The Parameters of the Potentials

	μ_2 f	S_1 MeV	S_2 MeV	m_1	m_2	A MeV
A_1^*	0.7	-362	3800	0.444	0.232	-0.362 10^{-35}
A_2^*	0.6	-304	5890	0.461	0.219	-0.423 10^{-35}
A_3^*	0.5	-269	11300	0.476	0.210	-0.549 10^{-35}
A_4^*	0.4	-248	28400	0.488	0.204	-0.803 10^{-35}
A_5^*	0.3	-235	104000	0.497	0.201	-0.137 10^{-34}
A_6^*	0.2	-227	720000	0.503	0.200	-0.302 10^{-34}
A_7^*	0.1	-223	$2.18 \cdot 10^7$	0.507	0.200	-0.119 10^{-33}
B_1^*	0.7	-385	$4.35 \cdot 10^3$	0.458	0.240	-4.25 10^{-36}
B_2^*	0.6	-313	$6.60 \cdot 10^3$	0.478	0.225	-4.85 10^{-36}
B_3^*	0.5	-271	$1.25 \cdot 10^4$	0.497	0.214	-6.16 10^{-36}
B_4^*	0.4	-246	$3.11 \cdot 10^4$	0.511	0.206	-8.88 10^{-36}
B_5^*	0.3	-231	$1.13 \cdot 10^5$	0.523	0.202	-1.56 10^{-35}
B_6^*	0.2	-222	$7.81 \cdot 10^5$	0.531	0.200	-3.29 10^{-35}
B_7^*	0.1	-218	$2.36 \cdot 10^7$	0.536	0.200	-1.29 10^{-34}
C_5^*	0.3	-369	$2.23 \cdot 10^5$	0.452	0.201	-2.98 10^{-35}
D_5^*	0.3	-290	$1.59 \cdot 10^5$	0.485	0.201	-2.12 10^{-35}
E_5^*	0.3	-285	$1.53 \cdot 10^4$	0.492	0.201	-2.04 10^{-35}
F_5^*	0.3	-816	$5.69 \cdot 10^5$	0.392	0.201	-7.63 10^{-35}

Appendix 4. Matrix elements, and binding energies of Oxygen 16

	V_0	V_1	V_2	E	K MeV
A_5^*	-9.38	1.61	-2.32	-101	150
B_3^*	-8.55	2.24	-2.37	-138	216
C_5^*	-8.62	2.48	-2.87	-136	267
D_5^*	-8.52	2.27	-2.50	-137	236
E_5^*	-8.74	2.30	-2.50	-134	238
F_5^*	-9.78	3.74	-5.01	-118	449
			Exp.	-142	150

Appendix 5. Matrix elements of potentials with full exchange mixtures in some singly excited configurations in Oxygen 16

$$\langle 1|V|1\rangle = \frac{1}{8} \{ {}^{31}A + 3 {}^{13}A \} (-5V_0 + V_2) - ({}^{11}A + 3 {}^{33}A) V_1 \},$$

$$\langle 2|V|2\rangle = \frac{1}{8} \{ 2 {}^{31}A + 2 {}^{13}A \} (-5V_0 + V_2) - 4 {}^{33}A V_1 \},$$

$$\langle 3|V|3\rangle = \frac{1}{8} \{ {}^{31}A (-7V_0 + 3V_2) + {}^{13}A (-9V_0 - 3V_2) + {}^{11}A V_1 - 9 {}^{33}A V_1 \},$$

$$\langle 4|V|4\rangle = \langle 5|V|5\rangle = \frac{1}{8} \{ -({}^{31}A + {}^{13}A) 8V_0 - 2 {}^{11}A V_1 - 6 {}^{33}A V_1 \}.$$

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EFEKTIVNA KINETIČKA ENERGIJA

I. Kinetička energija, međunukleonska interakcija, vezanje i energije eksitacije laganih jezgara

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S a d r Ź a j

Da bi se našao Hamiltonian jednostavnog modela ljuske, pretpostavljene su različite efektivne kinetičke energije nukleona u jezgri. Iz Hamiltoniana slijede istodobno energije vezanja osnovnog stanja laganih jezgara, koje se podudaraju s eksperimentalnim vrijednostima, kao i njihove energije eksitacija.

Potencijali uključuju matrične elemente p -stanja s jakim repulzijom, što zajedno sa smanjenjem kinetičke energije vodi sniženju deformiranih eksitacijskih stanja u suglasnosti s eksperimentom.

Energije vezanja sfernih jezgara također se podudaraju.