FIZIKA 5 (1973) 61 – 65

EFFECTIVE KINETIC ENERGY

II. Four-Body Forces and their Contribution to the Binding of ^{4}He and ^{8}Be

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Received 25 September 1972

Abstract: In order to investigate the four particle clustering tendency in α particle nuclei, an effective four-body potential taken in the form of a generalised Gaussian was added to the conventional two-body potential. The strength of this four-body potential was fitted to the binding energy of Helium 4, and its contribution to the binding of Beryllium 8 shown to be of the order of 5 MeV or some two per cent of the total potential energy.

1. Introduction

In previous papers[1 2] the contribution of three-body forces to light nuclei has been investigated and shown to be negligible, by considering a simplified three-body interaction taken in the form of a three-body Gaussian. Physically, however, since the triton is diffuse, but the α particle tightly bound, we would expect possibly a larger contribution from four-body forces.

In this paper the range of the four-body Gaussian is taken from the size of the α particle, and its strength from the discrepancy between the two-body contribution estimated from the matrix elements of Elliott et al.[3], and the experimental binding energy. Since the α particle's excited states are at an extremely high excitation energy, of the order of 20 MeV, only α particle configurations, where the four nucleons are not in relative motion are considered to add to the binding.

2. Helium 4

The α particle may be considered to be the first properly bound nucleus. Being a closed shell nucleus it is extremely stable, and α clusters are experimentally found within the radius of light nuclei, many low lying levels of which can be thought of as being formed by relative oscillations of α and other clusters, one with another.

The α particle is made up of two deuterons, the wave function so symmetrised that the final total spin is zero. For a two-body interaction there are six pairs of nucleons to consider, the two protons interacting in a singlet state as do the two neutrons, and the pairs of a proton and a neutron in a triplet and a singlet deuteron state. If therefore we wish to take for our two-body interaction the matrix elements of Elliott et al.[3] we must average over spins.

In the previous paper[4] the phenomenon of collision damping in nuclei was investigated, and the dynamics of the process bound up in an effective kinetic energy. This effective kinetic energy is reduced from the usual form by a reducing exponential simulating the damping phenomena. In this paper the strength of the four-body interaction is calculated and the contribution to the binding of Beryllium 8 deduced, for a mean free path parameter of infinite length (that is the conventional kinetic energy) and for a value of 2.6 fm (the best and constant value decided upon in[4]).

The expression for the binding energy of the particle is thus

$$E_4 = 3\,(3/4\,\hbar\,\omega_4)\,G_4 + 6\,V_0 + V_{000}, \tag{1}$$

where G_4 is the damping factor to the kinetic energy, V_0 is the strength of the two-body interaction in an s-state (averaged over spins), and V_{000} is the four-body interaction in the α-particle (to be considered in the next section), and $\hbar\omega_4$ is the oscillator constant for Helium 4 taken to be equal to 24. 7 MeV (deduced from the r. m. s. radius).

3. A phenomenological four-body interaction

Inside a nucleus there is a picture of the composite nucleons moving in a sea of pions. Thus when from time to time four nucleons find themselves close together a rapid mutual exchange of mesons takes place resulting in a four-body force. The theoretical form of this force is rather complicated, so we choose instead to work with a simple Gaussian potential which will simulate the four nucleon clustering cause by the genuine four-body force.

Let us consider the position vectors of four nucleons $\underline{r}_1, \underline{r}_2, \underline{r}_3$ and $\underline{r}_4$. We choose the following relative scheme

$$\begin{aligned} \underline{R} &= \frac{1}{2}(\underline{r}_1 + \underline{r}_2 + \underline{r}_3 + \underline{r}_4), \\ \underline{S} &= \frac{1}{2}(\underline{r}_1 + \underline{r}_2 - \underline{r}_3 - \underline{r}_4), \\ \underline{r} &= \frac{1}{\sqrt{2}}(\underline{r}_1 - \underline{r}_2), \\ \underline{t} &= \frac{1}{\sqrt{2}}(\underline{r}_3 - \underline{r}_4), \end{aligned} \tag{2}$$

where $\underline{S}$, $\underline{r}$ and $\underline{t}$ are relative coordinates.

$$S_4 \exp - \frac{\sum\limits_{i<j<k<l = 1,2,3,4} (ri - rj)^2}{\mu_4^2}, \tag{3}$$

where μ_4 is the range of the four-body interaction. This in terms of the new coordinates may be written

$$S_4 \exp - 4 \frac{(S^2 + r^2 + t^2)}{\mu_4^2}.$$

The matrix elements of this potential may readily be found in four nucleon states and may be expressed in terms of strength functions Vsrt. We only consider configurations in which the a particles are not excited to contribute to the binding energy, in other words only contributions with the strength V_{000}, all the others being taken to be zero.

In the ground state of Beryllium 8 which we take to be of the »44« symmetry of the radial coordinates, there are in all 70 four nucleon clusters. Five of these are of symmetry »4«, forty-five of symmetry »31« and twenty of symmetry »22«. We consider the five first mentioned in detail. Of these the first is one in which all four nucleons are in s-states, the next is a (s^3p) configuration, there is a (s^2p^2), a (sp^3) and a (p^4) configuration. The interaction in a particle states in which there is no relative motion in the S, r and t coordinates, though there may be centre of mass motion R, in each of these configurations may be worked out. The total interaction in the nine clusters is

$$3^{\frac{7}{32}} V_{000}, \tag{4}$$

where $V_{000} = S_4 \beta_4^{9/2}$ and

$$\beta_4 = \frac{\mu_4^2/b^2}{(4 + \mu_4^2/b^2)} \quad \text{and} \quad b^2 = \frac{\hbar}{m\hbar}. \tag{6}$$

The strength function V_{000} of course depends on the oscillator constant $\hbar\omega$ for the nucleus concerned and thus will vary continuously from nucleus to nucleus.

4. The calculation of the contributions of the four-body potential to ^4He and to ^8Be

The expression for the binding energy of the α particle was given in Section 2, Equ. (1). The damping factor G_4 to the kinetic energy will be given two values. Firstly unity, that is we have an unadapted expression for the kinetic energy, and secondly 0.86 corresponding to a mean free path of 2.6 fm[4].

The values for the matrix elements of the two-body potential V_0 may either be evaluated from current interaction potentials, or be taken as we do in this paper, direct from the lists of such matrix elements in the paper of Elliott, Jackson, Mavromatis, Sanderson and Singh[3]. When averaged over spins V_0 has the value of 11.8 MeV. Substituting the value of the experimental binding energy of the α particle, we can deduce values for V_{000} in this nucleus. For $G = 1$ we obtain

$$V_{000} = -\ 12.6 \text{ MeV},$$

and for $G = 0.86$

$$V_{000} = -\ 4.7 \text{ MeV}.$$

The four-body contribution to the α particle is some 10—15% of the total binding.

Before values for S_4 may be deduced from expressions (1), a value for the range of the four-body interaction must be decided upon. A working value may be obtained from considering the range to be equal to the side of a regular tetrahedron fitted inside a sphere of radius equal to the r. m. s. radius for the α particle. In this way we decide upon 3.2 fm for the range of the four-body interaction. We can then deduce values for S_4. In the two cases $S_4 = -\ 123.3$ MeV and $S_4 = -\ 46.0$ MeV, respectively. The interaction is attractive and thus contributes to the binding actively.

We may now easily obtain quantitave estimates for the four-body contribution to the binding of Beryllium 8, from Equs. (5) and (6). This contribution turns out to be -8 MeV for the conventional value of an infinite mean free path inside a nucleus, and -3 MeV for a mean free path of 2.6 fm. We have arrived at these values using $\hbar\omega_8 = 17.6$ MeV.

We may now deduce from this that if part of the effect of the core is simulated by an adaptation of the kinetic energy, less importance may be delegated to four-body forces, the effect of which is in any case shown to be of only secondary importance.

5. *Conclusions*

In Ref.[4] a two-body potential was fitted to Helium 4. The matrix elements of the potentials found in this way when compared with those of Elliott et al.[3] were shown to be a little too large. It would appear consequently that part of the interaction was missing from the Hamiltonian, and that this was of a many-body type.

In this paper an estimate of the contribution of clustering, four-body forces was calculated. The contribution towards the experimental binding energy of beryllium 8 of 60 MeV.was shown to be of the order of five MeV or some two per cent of the total potential energy.

References

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EFEKTIVNA KINETIČKA ENERGIJA

II. Sile četiri tijela i njihov doprinos vezanju ^4He i ^8Be

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Sadržaj

Da bi se istražila tendencija okupljanja nukleona u α-čestice u jezgrama, uobičajenom potencijalu za dva tijela dodan je efektivni potencijal za četiri tijela i to u obliku općeg Gaussovog potencijala.

Taj potencijal za četiri tijela primijenjen je na slučaj energije vezanja ^4He a pokazao je, da njegov doprinos energiji vezanja ^8Be iznosi nekoliko MeV-a ili oko 2% ukupne potencijalne energije.