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MULTIPLE PARTICLE PRODUCTION IN THE STRAIGHT-LINE PATH APPROXIMATION

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Abstract: Some results of the investigation of processes of multiple production in the framework of field-theory models, using the Feynman-Bogoljubov functional integral method, are given. By the use of the straight-line path approximation it is shown that the N meson production amplitude is factorized and a multiplicity distribution of the Poisson type is obtained. An interesting relation between the multiplicity of secondary particles, the slope of the diffractional peak, and the total cross section is obtained. By a simple assumption about the isospin structure the model is used to fit the charged particle multiplicity distribution for both $\pi^- p$ and $\pi^- n$ scattering at 40 GeV.

1. *Introduction*

In the present paper processes of multiple particle production are investigated in the framework of field-theory models, using the Feynman-Bogoljubov functional integration method. The straight-line path approximation is used as the main

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approximation which makes it possible to extract the neccssary information from expressions containing functional integrals. As is known, this approximation has been suggested and developed by the Dubna group (Tavkhelidze, Barbashov, Matveev et al.[1]) for high energies and fixed momentum transfers, and represents a generalization of the approximation $k_i k_j = 0$ formulated by Fradkin[2] and Barbashov[3] in their papers on the infrared asymptotics of the Green functions.

The concept of the straight-line path approximation is the following. At high energies and fixed momentum transfers the main contribution to the amplitude of the process written in the form of a Feynman integral over the particle paths is assumed to come from trajectories which are nearly straight lines having the same direction as the momentum vectors of the leading particles before and after interaction.

Such paths were taken into account in expressions containing functional integrals because of the procedure of averaging over the functional variable

$$\int [\delta \nu]\, e^{F(\nu)} \rightarrow e^{\int [\delta \nu] F(\nu)}, \tag{1}$$

where $\int [\delta \nu]$ means the integration over the Gaussian measure.

We note that the averaging according to the rule

$$\int [\delta \nu]\, e^{F(\nu)} \rightarrow e^{F(\nu = 0)} \tag{2}$$

would mean to take into account the classical particle trajectory.

Calculations performed in the straight-line path approximation at high energies and fixed momentum transfers as well as corrections to this approximation[4] show that the application of this method is well justified, at least in models which make use of a nonsingular effective quasipotential.

This statement is in agreement with the situation in quantum mechanics and confirms the importance of the hypothesis about the smoothness of the local quasipotential suggested by Blokhintsev and Logunov and co-workers[5] and investigated in Refs.[6].

It is important to note that the straight-line path approximation as applied to the study of processes of multiple particle production leads to a partial neglect of the nucleon recoil when emitting secondary particles. In this sense the suggested model is close to multiple particle production models constructed by analogy to the »bremsstrahlung« model in electrodynamics.

These problems were also studied by Heisenberg[7], who considered a special field-theory model using the Bloch-Nordsik method, as well as by Lewis, Oppenheimer, and Wouthuysen, who suggested the »shaking« model[8]. Among more recent studies we should note a series of papers on the »bremsstrahlung« model by Kastrup[9].

2. *The main results of the model*

We perfomed a detailed study of field-theory models with the following interaction Lagrangians

$$L_{\rm int} = g : \psi\psi^*\Phi: \quad , \tag{A}$$

$$L_{\rm int} = g : \psi \; \mathrm{i} \overleftrightarrow{\partial}_\alpha \psi A_\alpha : + g^2 : A_\alpha^2 \psi^* \psi: \quad . \tag{B}$$

In applying the straight-line path approximation we first considered the possibility of obtaining the eikonal representation by summing directly a certain class of Feynman graphs. As is well known, the summing of the s-channel ladder graphs with a multiparticle exchange yields an eikonal formula for the scattering amplitude[10]. It is also important to note that by taking into account radiative corrections in our consideration, we obtained a smooth or non-singular effective quasipotential[11].

Inelastic process amplitudes describing the production of a certain number of quanta of the field A in the collision of two nucleons may be found by means of the generating function $f(p_1, p_2; q_1, q_2 \mid A^{\rm ext})$, (we are considering model B)[1]. The quantity $f(p_1, p_2; q_1, q_2 \mid A^{\rm ext})$ denotes the scattering amplitude for two nucleons in the presence of the external field. As an example, we give expressions for the generating function in the presence of the external field A^{ext}. In the framework of the straight-line path approximation it has the following form:

$$\begin{aligned} & if(p_1, p_2; q_1, q_2 \mid A^{\rm ex}) = g^2 \int \mathrm{d}^4 y \, \mathrm{e}^{iy(p_1-q_1)} \, \mathrm{d}^4 x \, \mathrm{e}^{ix(p_1-q_1)} \\ & \Delta(x-y; p_1, q_1 : p_2, q_2) \exp \Big\{ ig \int \mathrm{d}^4 l \, A_\alpha^{\rm ext}(l) \\ & \big| j_\alpha^{(1)}(l; p_1, q_1) \, \mathrm{e}^{ilx} + j_\alpha^{(2)}(l; p_2, q_2) \, \mathrm{e}^{ily} \big| \Big\} = \\ & = \int_0^1 \mathrm{d}\lambda \exp \Big\{ \frac{ig^2}{2} \int \mathrm{d}^4 k \, D_{\gamma\xi}(k) \, \big| \, \mathrm{e}^{ik(x-y)} \, 2\lambda j_\gamma^{(1)}(k; p_1, q_1) \\ & j_\xi^{(2)}(-k; p_2, q_2) + \sum_{i=1}^{2} j_\gamma^{(i)}(k; p_i, q_i) \, j_\xi^{(i)}(-k; p_i, q_i) \big| \Big\} \end{aligned} \tag{3}$$

where j are the nucleon currents averaged over the functional variables

$$\Delta(x) = \int d^4k\, e^{ikx} D_{\alpha\beta}(k)(k + p_1 + q_1)_\alpha(-k + p_2 + q_2)_\beta. \tag{4}$$

The production amplitude for N quanta of the field A is determined by means of variational derivatives with respect to the field A^{ext}. If we require the components of the produced mesons to obey the following conditions in the c. m. s.

$$\frac{1}{\sqrt{s}} \sum_{i=1}^{N} k_{0i} \ll 1, \quad \left| \sum_{i=i}^{N} \vec{k}_{i\perp} \right| \ll |\vec{p}_{l\perp} - q_{l\perp}|, \quad l = 1, 2, \tag{5}$$

then the N meson production amplitude is factorized

$$f_{i,el}(N) = f(q_1, q_2; p_1, p_2) \times \prod_{i=1}^{N} g E_\alpha^*(k_i) \left| j_\alpha^{(1)}(k_i, p_1, q_1) + j_\alpha^{(2)}(k_i, p_2, q_2) \right|,$$

$$j_\alpha^{(l)}(k; p_l, q_l) = \left(\frac{2p_{l\alpha} + k_\alpha}{2p_l k - \mu^2} - \frac{2q_{l\alpha} + k_\alpha}{2q_l k + \mu^2 u} \right), \quad (l = 1, 2). \tag{6}$$

This fact may be considered as the first important result for the models studied.

In the straight-line path approximation the differential cross section for N meson production in the collission of two nucleons is also factorized.

$$(d\sigma)_{n_1, n_2} \to \frac{1}{2s} \frac{d^4\Delta}{(2\pi)^4} |f_{el}|^2 W_{n_1}(p_1, \Delta) W_{n_2}(p_2, -\Delta), \tag{7}$$

$$s \to \infty, \quad t = \Delta^2 - \text{fixed},$$

where

$$W_{n_1}(p_1, \Delta) = \frac{2\pi}{n_1!} \int \frac{d\vec{q}_1}{2q_{10}} \delta\left(p_1 - q_1 - \sum_{i=1}^{n_1} k_i + \Delta\right)$$
$$\prod_{i=1}^{n_1} \frac{d\vec{k}_i}{2k_{0i}} \frac{(-g^2)}{(2\pi)^3} |j_\alpha(k_i, p_1, q_1)|^2. \tag{8}$$

The differential cross section for production of $N = n_1 + n_2$ secondary particles the momentum components of which satisfy the conditions (5) in the straight-line path approximation has the form[1, 12]

$$\left(\frac{d\sigma}{dt}\right)_{n_1, n_2} \to \left|\frac{d}{dt}\right|_0 \widetilde{W}_{n_1}(s, t)\, \widetilde{W}_{n_2}(s, t), \tag{9}$$

where $\widetilde{W}_n$ are functions of the Poisson type

$$\widetilde{W}_n(s, t) = \frac{1}{n!} e^{At}[\bar{n}(s, t)]^n. \tag{10}$$

The average multiplicity is determined as

$$\bar{n}(s, t) = + \frac{g^2}{(2\pi)^3} \int \frac{d\vec{k}}{2k_0} \, |j^{(l)}(k, p_l, q_l)|^2, \tag{11}$$

and e^{At} is the radiative correction factor.

For $t \ll m^2$, i. e., in the diffraction region

$$\bar{n}(s, t) = -Bt, \tag{12}$$

a linear dependence of the average multiplicity upon t is observed. In this case, in a certain domain of secondary particle momenta

$$B = A. \tag{13}$$

The total differential cross section obtained by summing over the number of all emitted mesons is found to be independent of t

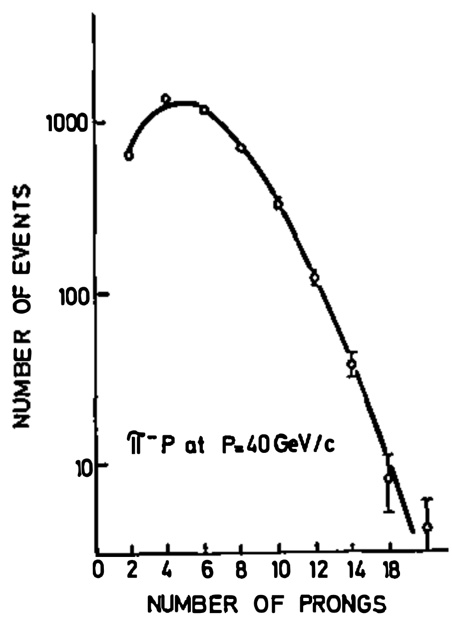

Fig. 1. Distribution of charged particle multiplicity in $\pi^- p$ at 40 GeV. The solid line is a fit by the present model.

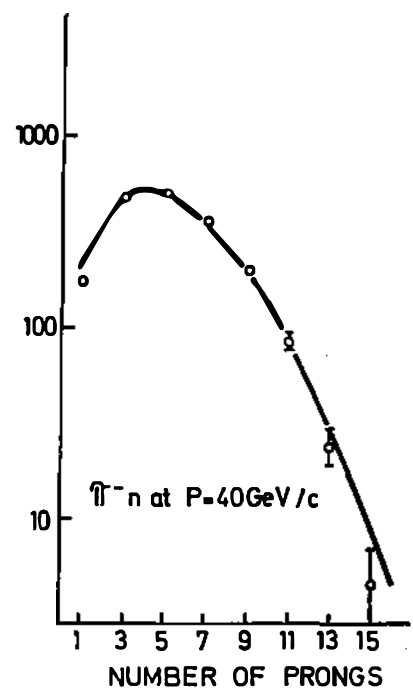

Fig. 2. Distribution of charged particle multiplicity in $\pi^- n$ at 40 GeV. The solid line is a fit by the present model.

$$\frac{d\sigma^{tot}}{dt} = \left(\frac{d\sigma^{el}}{dt}\right)_0 = \text{const.} \tag{14}$$

This is, in a certain sense, analogous to the point-like or automodel behaviour of the cross sections for deeply inelastic hadron-lepton processes[13].

These consequences of the model are in qualitative analogy with the predictions of the coherent state model[14], which is a realization of the concept that hadrons are complicated systems with many internal degrees of freedom.

3. On the relationship of multiplicity with the slope of the diffractional peak and the total cross section

We return to formula (14), which may be viewed as a consequence of the straight-line path approximation and the coherent state model. It is obvious that this relation is meaningful only for momentum transfers restricted by the domain of the diffractional peak.

The real content of the result (14) consists in the fact that the total differential cross section can change noticeably only with changing $\Delta t \sim t_{eff}$, which greatly exceeds the sizes of the diffractional domain.

To estimate t_{eff}, we may make use of the unitarity condition which yields

$$-t_{eff} \leq \frac{16}{\sigma^{tot}}. \tag{15}$$

This value of t_{eff} can be employed for estimating the average number of secondary particles $\bar{n}_{diffr}$. produced in diffractional collissions of hadrons at high energies

$$\bar{n}_{diffr} \cdot (s) = \frac{1}{\sigma^{tot}} \int_0^{t_{eff}} \frac{d\sigma^{tot}}{dt} A(s)\, t\, dt \leq \frac{8\pi A(s)}{\gamma_{tot}}. \tag{16}$$

Thus, the diffractional or peripheral part of the average multiplicity is defined by the parameters of the elastic zero-angle scattering amplitude. The conclusion about the behaviour of the total particle number $\bar{n}(s)$ can be drawn only under definite assumptions about the contribution of small distances to high-energy multiple production processes. In particular, if the assumption about the disappearance of »pionization« effects at high energies, i. e., the production of secondaries with limited momenta in the c. m. s. of colliding hadrons, is used, then relation (16) will define the behaviour of the total average multiplicity[15]

$$\bar{n}(s) = \frac{8\pi A(s)}{\sigma^{tot}} + \tilde{\nu}, \tag{17}$$

where $\tilde{\nu}$ is the number of »leading« particles.

It is interesting to note that relation (17) correctly describes the behaviour of the average multiplicity with increasing energy. Indeed, the approximate constancy of the cross section and the logarithmic narrowing of the diffractional peak at energies presently available corresponds to the logarithmic increase of the average multiplicity with energy.

Using the well-known restriction on the asymptotic behaviour of the diffractional peak width in quantum field theory[16] from Equs. (17), we get in the general case

$$\bar{n}(s) \leq \frac{\text{const}}{\sigma^{tot}} \ln^2 s. \tag{18}$$

This relation is an interesting interpretation of the increase of the strong interaction radius.

Indeed, $A(s)$ is the »visible« hadron size, σ_{tot} defines the minimal distance R_0 for which the automodel behaviour holds. One can see from Equ. (17) that

$$A(s) \sim R^2 = \bar{n} R_0^2. \tag{19}$$

Thus, the strong interaction radius increases under the condition of the constant cross section, at the expense of the »swelling out« hadrons associated with the »clouds« of secondary particles.

4. *Phenomenology*

A unique possibility of examining various assumptions and models of multiple particle production has appeared in connection with putting into operation the Serpukhov accelerator. Recent experiments performed with a two-meter propane chamber irradiated with 40 GeV π^- mesons have shown that the distribution of charged particles over multiplicity is of crucial importance. Satisfactory agreement with experiment has been obtained up to an energy of 25 GeV by using the Poisson formula, the Wang model I, the Wang model II, the Chew-Pignotty model, the Czyzewski-Rybicki model, etc. (see, for example, Ref.[17]). However, at an energy of 40 GeV reasonable agreement has been obtained only by the Wang model I and the Czyzewski-Rybicki formula[18].

We attempted to give a concrete picture of multiple particle production in π^-p and π^-n collisions by taking into account the isotopic structure and the particle charge. The following assumptions were made:

— pairs $\pi^+\pi^-$ are produced in a statistically independent manner,

— the average number of these pairs is independent of the kind of colliding particles and is the same for π^-p and π^-n,

— the leading nucleon in the collision process can be dissociated into a meson and a nucleon with the local conservation of the isotopic spin.

On the basis of these assumptions the charged-particle multiplicity distributions have the form

$$W_{n_c}(\pi^-p) = P_{1/2\,(n_c-2)}(a) \tag{19}$$

for π^-p, and

$$W_{n_c}(\pi^-n) = (\alpha + \beta)\, P_{1/2\,(n_c-1}\ (a) + 2\beta\, P_{1/2\,(n_c-3)}(a) \tag{20}$$

for π^-n.

Here a is the average number of pairs $\pi^+\pi^-$ and P_n is the Poisson function; note also that $\alpha + 3\beta = 1$.

Table

Comparison between the predictions of the Wang model I and the present model for π^-p and π^-n scattering at 40 GeV.

Type of interaction	Number of events	$\bar{n}_c$	D	χ^2_{Wang}	Degrees of freedom
π^-p	4400	5.62±0.04	2.75	8	8
π^-n	1860	5.32±0.07	2.82	13	7
π^-n	Fit by the present model			8.5	7
π^-p				8	8

Experimental points and theoretical curves are plotted in the diagram in Figs. 1 and 2.

We have fitted this model in collaboration with Grishin and Jančo[19] and have obtained the following values for the parameters

$$\begin{aligned} a &= 1.81 \pm 0.02, \\ \alpha &= 0.46 \pm 0.05, \\ \beta &= 0.18 \pm 0.02. \end{aligned} \tag{21}$$

In the framework of this model we have succeeded in obtaining a good description of the data on $\pi^- p$ and $\pi^- n$ interactions. It is important to note (Table) that we have succeeded in reaching the same degree of agreement between theory and experiment for both $\pi^- p$ and $\pi^- n$, in contrast to the attempts of Wang.

Thus, the assumption about the identical average multiplicity of pairs of $\pi^+ \pi^-$ mesons produced in $\pi^- p$ and $\pi^- n$ collisions does not contradict experimental data.

Further extension of this model to the case involving strange particles and to the description of the π^0 meson distribution is in progress. For the latter case we have already obtained a prediction about the linear dependence $\bar{n}_{\pi^0}$ upon n_c *)

$$\bar{n}_{\pi^0} = A + Bn_c \tag{22}$$

which does not qualitatively contradict the experimental data[18] available at present.

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VIŠESTRUKA PRODUKCIJA ČESTICA U PRIBLIŽENJU PRAVOLINIJSKE PUTANJE

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Sadržaj

Članak sadrži rezultate istraživanja procesa višestruke produkcije u okviru modela teorije polja i uz upotrebu metode funkcionalne integracije Feynmana i Bogoljubova.

Upotrebom aproksimacije ravnolinijske putanje autori su pokazali da je amplituda stvaranja N mezona faktorizirana a raspodjela višestrukosti je Poissonovog tipa. Pokazana je zanimljiva veza između višestrukosti sekundarnih čestica, nagiba difrakcionog maksimuma i totalnog udarnog presjeka.

Uz jednostavne pretpostavke o izospinskoj strukturi model je primijenjen za parametrizaciju raspodjele višestrukosti nabijenih čestica istovremeno za $\pi^- p$ i $\pi^- n$ raspršenje kod 40 GeV.