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THE DISPERSION OF AN AXICON-SCANNED FABRY-PEROT SPECTROMETER

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Abstract: A correlation is found between the linear dispersion of an Axicon-scanned Fabry-Perot (A-F-P) spectrometer and a Fabry-Perot (F-P) interferometer. The position of the axicon, as the scanning element, is related to the parameters: the focal length of the imaging lens, the wavelength, the spacing of the F-P plates, the refraction index and the vertex angle of the axicon. A functional relation is established between the wavelength and the displacement of the axicon, from the focal plane stop. The linear dispersion of the A-F-P spectrometer is derived and shown that, at a given wavelength, it increases linearly with increasing focal length of the imaging lens as the square root of the spacing of the F-P plates. The dispersion also increases through the cot function if the vertex angle and the refractive index of the axicon decrease.

1. Introduction

Any dispersing system must have a high resolution luminosity product for an analysis of the profile of a faint and narrow spectral line. Such a requirement is fulfilled by the Fabry-Perot (F-P) etalon[1]. The scanning F-P spectrometer[2] has been known since 1948 when Jaquinot and Dufour demonstrated the method of pressure scanning. The F-P photoelectric spectrometer, with an axicon[3] as a scanning element, was first described by Katzenstein[4]. Such a system has all the advantages of a F-P etalon in terms of high luminosity and resolving power, as well as the high sensitivity and time resolution of photoelectric detectors.

If an etalon is illuminated with a broad monochromatic source a system of rings is formed in the focal plane of an imaging lens placed behind it. For the use of the F-P etalon as a spectrometer the light beam must be restricted in angular width by a diaphragm placed in the focal plane of the collimating lens, since convergence or divergence of incident rays cause instrumental line broadening. In general case of a heterochromatic source a single order must be isolated by a restriction of the spectral range of the incident light. This can be done by an external monochromator which may be an interference filter. In the case of a passband monochromator wider that the free spectral range, an overlapping of the orders may be expected, because a wavelength leaving a fringe on its outer rim will enter at once on the inner rim of its next order. This leads to a decrease in contrast of the etalon and therefore affects the ratio of the scanned spectral line to the continuum radiation.

In many cases the line spectrum does not depend on the wavelength itself but only on the difference from the centre wavelength. In studying Stark broadening of spectral lines whose wavelengths are known or for the use of laser light scattering methods, for example, one does not need any absolute wavelength scale calibration. In such cases it is sufficient to know the linear dispersion of the spectrometer in order to analyse the profile of the spectrum. Up to now the linear dispersion of an Axicon-scanned Fabry-Perot (A-F-P) spectrometer is not analytically known. The purpose of this paper is to derive the dispersion for this spectrometer.

2. *Dispersion of F-P interferometer*

In a Fabry-Perot interferometer the intensity maxima of the concentric Haidinger fringe pattern are related to the emergence angle of light, φ, by the relation

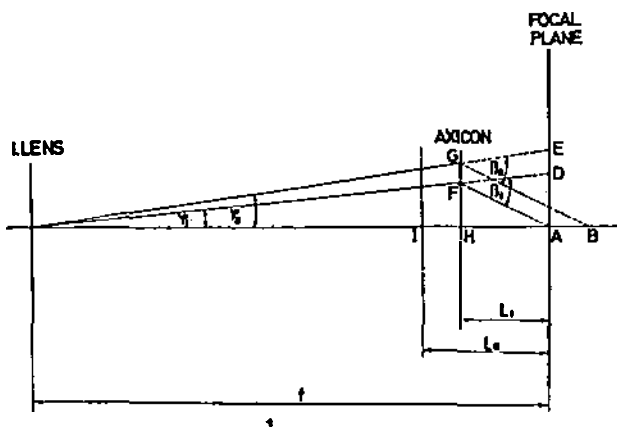


Fig. 1.

$$m\lambda = 2\mu t \cos\varphi, \tag{1}$$

where $2\mu t \cos\varphi$ is the optical retardation between successive emergent rays, λ is the wavelength, m is the order of the fringe corresponding to φ, t is the separation between the reflecting surfaces and μ is the refractive index of the medium between the plates. This interference pattern will be sharpest at infinity or in the focal plane of an imaging lens. The radii of fringe circles depend upon φ and the focal length f of the imaging lens. From Fig. 1 it can be seen that AD and AE represent the radii of the successive fringes and in the first approximation[5]

$$\cos\varphi = \frac{f^2}{\sqrt{f^2 + R^2}} = 1 - \frac{R^2}{2f^2}, \tag{2}$$

where R is the radius of the corresponding circle. Thus using Equ. (1) we can write Equ. (2) in the form

$$m\lambda = 2\mu t\left(1 - \frac{R^2}{2f^2}\right). \tag{3}$$

Differentiating Equ. (3) and for $\mathrm{d}m = -1$ we obtain

$$\Delta R = \frac{\lambda \cdot f^2}{2\mu t R}, \tag{4}$$

which means that for larger radii consecutive circles are close together. If m, μ, f and t are considered as constants, it is easy to obtain the following expression by differentiating Equ. (3).

$$\pm\, m\Delta\lambda = \mp \frac{2\mu t R}{f^2}\Delta R. \tag{5}$$

This relation shows that increasing λ produces a shrinking of the ring system, whereas decreasing λ causes a swelling of the rings. The linear dispersion can be derived from Equ. (3) by differentiating it and using the relation (1) for small angle

$$\left|\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right| = \frac{2 \cdot f^2}{D \cdot \lambda}, \tag{6}$$

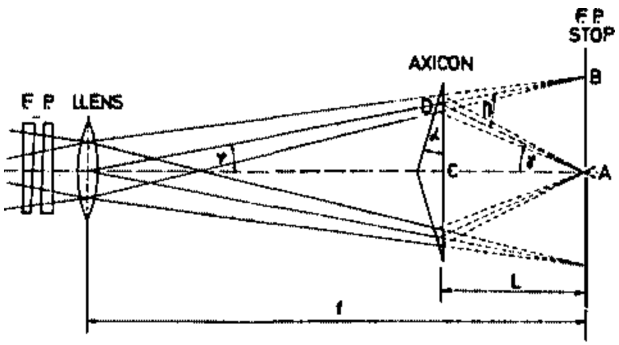
where D is the diameter of the corresponding fringe. The relation (6) is valid only without an axicon.

3. Dispersion of A-F-P spectrometer

If incident rays on an axicon are coplanar with the plane containing the optical axis, use can be made of the same approximation for an angle of deviation β as for thin prism)

$$\beta = (n - 1)\, \alpha, \tag{7}$$

where n is the refractive index of the axicon and α is the vertex angle. For small apertures of the F-P plates and relatively large focal length of the imaging lens the most of incident rays are coplanar with the plane in which lies the optical axis. In this case beams of rays, which form the fringe pattern at positions of the axicon and the focal plane, may be replaced by their bisectors approximately. If incidence angle φ (Fig. 1) is very small the angle of deviation β of thin axicon is close to the minimum deviation angle and equation (7) is valid. For a focal length, say $f = 40$ cm, of the imaging lens it can be seen that the radii of successive fringes are $R_1 \ll f$ and $R_2 \ll f$ and consequently φ is very small. Therefore the angle of deflection β produced by the axicon should be independent of the angle of incidence φ. Thus according to equation (7) $\beta_1 = \beta_2 = \beta$. Hence a thin axicon has the property of changing the inclination of rays with respect to the optic axis without affecting their convergence. To shrink a fringe from a radius R to a point, the axicon must be displaced (starting from the focal plane) from position O to a distance L. Since



AB=R CD=R'

Fig. 2.

$R < L$ an increase in the value of the dispersion should be expected. Inserting the axicon between the imaging lens and the focal plane gives the same result conerning the dispersion as reducing the diameter of the fringe by a factor of $\Delta R/\Delta L$. This factor C can be expressed (Fig. 1) as $C = \dfrac{DE}{AB}$ and since $AB = IH$

$$C = \frac{\Delta R}{\Delta L} = \frac{\Delta D}{2 \Delta L} = \frac{D}{2 L}. \tag{8}$$

Such a »new« diameter D^0 can be written as $D^0 = C \cdot D$. On substituting this diameter into Equ. (6) we obtain

$$\left| \frac{\mathrm{d} L}{\mathrm{d} \lambda} \right| = K_0^2 \frac{f^2}{L \cdot \lambda}, \tag{9}$$

where $K_0 = C^{-1} = 2\Delta L/\Delta D$ can be determined experimentally by scanning over two successive orders. On the other hand, the linear relation existing between the displacement L of the axicon, from the focal plane stop, and the diameter of fringe D has been shown[6] to be

$$L = K \cdot D, \tag{10}$$

where K is related to K_0 as $K = \frac{1}{2} K_0$.

The same result as given by relation (9) can be obtained by substituting D from (10) into Equ. (3) and by differentiating this expression. Using the geometry and the properties of thin axicon the dispersion can be derived as follows. From Fig. 2 it can be written

$$\tan \gamma = \frac{D'}{2L}, \tag{11}$$

where D' is the diameter of the fringe at the axicon position. Similarly, from the diameters of the same fringe at positions of the axicon and focal plane, it can be written

$$\frac{D}{f} = \frac{D'}{f - L}. \tag{12}$$

Solving this equation for D', on substituting into (11) we have

$$\tan \gamma = \frac{f - L}{f} \cdot \frac{D}{2L}. \tag{13}$$

The deviation angle β and incidence angle φ are related by γ as

$$\gamma = \beta - \varphi. \tag{14}$$

Using the expression $\tan \gamma = D/2f$ and the known relation for $\tan(\beta - \varphi)$, on substituting (7) into (14) and solving (13) for L, we obtain

$$L = \frac{D}{2\left[1 + \left(\frac{D}{2f}\right)^2\right]} \left[\frac{D}{2f} + \cot(n - 1)\,\alpha\right]. \tag{15}$$

Since $D \ll 2f$ it is possible to neglect the terms $D/2f$ and $(D/2f)^2$ in comparison with 1 and $\cot(n - 1)\,\alpha$ respectively. Even for an axicon of light flint at an angle of about 20° the term $\cot(n-1)\,\alpha$ is greater than $D/2f$ (for $f = 40$ cm) by more than two orders of magnitude. This approximation gives

$$L = \frac{D}{2} \cot(n-1)\,\alpha. \tag{16}$$

The diameter of fringe of the F-P etalon, in the focal plane of the imaging lens, is given[7] by

$$D_p = 2f \sqrt{\frac{\lambda}{t}(p - 1 + \varepsilon)}, \tag{17}$$

where p is the order of the fringe starting from the centre ($\varphi = 0$) and $\varepsilon = D_0^2/(D_1^2 - D_0^2)$. Substituting (17) into (16) we have

$$L_p = f \sqrt{\frac{\lambda}{t}(p - 1 + \varepsilon)} \cot(n-1)\,\alpha. \tag{18}$$

Having the relation (18) for L_p it is useful, for practical purposes, to find a relation between the displacement L of the axicon and the corresponding change of the wavelength within an interval of the free spectral range. Using relations

$$m\lambda = 2\mu t\left(1 - \frac{D^2}{8f^2}\right) \text{ and } K_0 = \frac{2L}{D}$$

one obtains

$$m\lambda = 2\mu t\left(1 - \frac{L^2}{2K_0^2 f^2}\right), \tag{19}$$

and on differentiating

$$\mathrm{d}\lambda = -\frac{2\mu t}{mK_0^2 f^2} L\,\mathrm{d}L = 2A_0 L\,\mathrm{d}L, \tag{20}$$

where $A_0 = -\dfrac{2\mu t}{mK_0^2 f^2}$. Then from (20) one has

$$\lambda = A_0 L^2 + B, \tag{21}$$

where B is the integration constant. Using the boundary condition $\lambda = \lambda_0$ when $L = L_1$, so that $B = \lambda_0 - A_0 L_1^2$. Thus the solution (21) can be written as

$$\lambda = \lambda_0 + A_0 (L^2 - L_1^2). \tag{22}$$

The constant A_0 is inconvenient for practical calculations (because of the order of the fringe m). Therefore, it is better to determine this constant by the additional condition $\lambda = \lambda_0 + \Delta\lambda_0$, ($\Delta\lambda_0 = \frac{\lambda_0^2}{2\mu t}$ is the free spectral range) when $L = L_2$, where L_2 is the position of the axicon for the peak of the next fringe Then

$$A = \frac{\Delta\lambda_0}{L_2^2 - L_1^2} \tag{23}$$

and Equ. (21) can be written

$$\lambda = \lambda_0 + A\,(L^2 - L_1^2). \tag{24}$$

The Equs. (23) and (24) give the required relation between λ and L. The values L_1 and L_2 can be determined experimentally. However, sometimes it may be difficult to scan two successive fringes. In that case, using the Equ. (18) for L_p, one can find the values of A as follows

$$A = \frac{\Delta\lambda_0}{L_2^2 - L_1^2} = \frac{\Delta\lambda_0}{f^2\cot^2(n-1)\,\alpha\left[\frac{\lambda}{t}(1+\varepsilon-\varepsilon\right]} = \frac{\lambda_0}{2\mu f^2}\tan^2(n-1)\,\alpha. \tag{25}$$

Thus the final relation becomes

$$\lambda = \lambda_0\left[1 + \frac{\tan^2(n-1)\,\alpha}{2\mu t}\cdot(L^2 - L_1^2)\right], \tag{26}$$

where $L_1 < L < L_2$.

The derived relation (26) enables us to evaluate any wavelength, within the free spectral range, as a function of the displacement of the axicon from the focal plane stop. For a given Axicon-scanned Fabry-Perot spectrometer the parameters n, μ, α, f and L_1 are constants and therefore the wavelength λ depends on the square value of L.

Since the magnification factor for the dispersion of an axicon system is given by $K_0 = 2\Delta L/\Delta D$, using relations (17) and (18), one can write

$$\begin{aligned} \Delta L &= L_{p+1} - L_p = f\sqrt{\frac{\lambda}{t}}\cot(n-1)\,\alpha\left[\sqrt{p+\varepsilon} - \sqrt{p-1+\varepsilon}\right], \\ \Delta D &= D_{p+1} - D_p = 2f\sqrt{\frac{\lambda}{t}}\left[\sqrt{p+\varepsilon} - \sqrt{p-1+\varepsilon}\right], \end{aligned} \tag{27}$$

and hence:

$$K_0 = \cot(n-1)\,\alpha. \tag{28}$$

Therefore according to the relation (28) the magnification factor of the dispersion depends only on n and α. Thus the relationship between the linear dispersion of an F-P interferometer and a system with an axicon can be written as

$$\left| \frac{\mathrm{d}L}{\mathrm{d}\lambda} \right| = K_0 \left| \frac{\mathrm{d}R}{\mathrm{d}\lambda} \right|, \tag{29}$$

or after substitution of (6) into (29)

$$\left| \frac{\mathrm{d}L}{\mathrm{d}\lambda} \right| = K_0 \frac{2 \cdot f^2}{D \cdot \lambda}. \tag{30}$$

Substituting (17) and (28) into (30) one finally obtains

$$\left| \frac{\mathrm{d}L}{\mathrm{d}\lambda} \right| = f \sqrt{\frac{t}{\lambda^3 (p - 1 + \varepsilon)}} \cot (n - 1)\, \alpha. \tag{31}$$

If f, t and λ are expressed in millimeters the right-hand side of Equ. (31) should be multiplied by a factor of 10^{-7} in order to express $\mathrm{d}L/\mathrm{d}\lambda$ in mm/Å.

The relation (31) shows that the linear dispersion of an A-F-P spectrometer increases linearly, at a given wavelength, with increasing focal length of the imaging lens, and as the square root of the spacing of the F-P plates. The dispersion also increases through the cot function if the vertex angle of the axicon and refractive index n decrease.

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DISPERZIJA AKSIKON FABRY-PEROT SPEKTROMETRA

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Sadržaj

Nadena je korelacija izmedu linearne disperzije aksikon Fabry-Perot spektrometra i Fabry-Perot interferometra. Položaj aksikona, kao elementa za skaniranje, je funkcija: fokalne dužine sočiva, talasne dužine, rastojanja izmedu ploča Fabry-Perot etalona, indeksa refrakcije i ugla aksikona. Relacija izmedu talasne dužine i pomeranja aksikona od fokalne ravni dobivena je analitički.

Izvedena je linearna disperzija Aksikon Fabry-Perot spektrometra. Pokazano je da ova disperzija raste, za datu talasnu dužinu linearno s povećanjem fokalne dužine sočiva i drugim korenom rastojanja izmedu ploča Fabry-Perot etalona. Disperzija ovog spektrometra takoder raste sa smanjenjem indeksa refrakcije i ugla aksikona kao kotangensna funkcija.