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WAVE PROPAGATION IN PLASMA HAVING SMALL TEMPERATURE GRADIENT

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Abstract: The wave propagation through a hot and non-uniform plasma has been studied from the dispersion relation. It is found that the temperature gradient can affect the wave propagation and it helps the damping or instability of the longitudinal wave.

1. Introduction

When any disturbance propagates through a non-uniform plasma, it may split up in two different modes, one of which undergoes damping and the other is amplified. Gold[1] has shown that in the presence of streams (static magnetic field may or may not be present) the electromagnetic instability occurs. Sturrock[2], Sudan[3] and many others have discussed the stream instability. Clemmow and Daugherty[4] have discussed the same considering relativistic effect. Paul and Bandyopadhaya (unpublished) have shown that the particle density gradient of the medium may lead to the damping or instability of the longitudinal wave, provided streaming or thermal motion is present. In the present paper we have investigated the possibility of spatial instability of the waves in the absence of any stream. It is found here that the temperature gradient of the medium can affect such instability of the longitudinal wave.

2. *Assumptions and basic equations*

We consider a fully ionised plasma of homogeneous density. The temperature of the medium is non-homogeneous, it varies slowly with space. A static and spatially uniform magnetic field $\vec{H}_0$ is present in the medium. We assume that:

— initially the medium was in equilibrium condition and the equilibrium condition is disturbed when perturbation was made,

— gravitational force, viscous force and the force due to collision are negligibly small compared with other forces present in the medium,

— the motions of the ions and the electrons are non-relativistic, and

— the thermal change being of diabatic nature, the perturbed pressure and density are related as

$$p = m v^2 n, \tag{1}$$

where p is the perturbed pressure, n is the perturbed density, m is the mass of the particles and V is the thermal velocity.

With the above assumptions, the equation of momentum transfer and mass conservation for the ions and electrons are

$$\frac{\delta \vec{v}_s}{\delta t} + \left\{ \frac{1}{m_s (N_s + n_s)} \right\} \operatorname{grad} (P_s + p_s) - \frac{1}{m_s N_s} \operatorname{grad} (P_s) - \\ - \frac{q_s}{m_s} \left\{ \vec{E} + \frac{1}{c} [\vec{v}_s, \vec{H} + \vec{H}_0] \right\} = 0, \tag{2}$$

and

$$\frac{\delta n_s}{\delta t} + \operatorname{div.} \{(N_s + n_s) \cdot \vec{v}_s\} = 0, \tag{3}$$

where, $s = 1$ for ions, $s = 2$ for electrons; $q_s =$ charge density;

$$q_1 = e, \quad q_2 = -e; \qquad P_s + p_s = m_s V_s^2 (N_s + n_s),$$

and $\vec{v}$, n, p are perturbed quantities.

The above equations will be supplemented by usual Maxwell's equations (written in C.G.S. units). It should be noted that the equations satisfy the equilibrium condition (Zheleznyakov[5]).

3. *Simplification of the basic equations*

We shall linearise the above equations by the help of following assumptions:

— the perturbed quantities are proportional to exp $(\mathrm{i} k z - \mathrm{i} \omega t)$, where k is the wave number and ω is the wave frequency,

— the perturbed density n_s is much smaller than the unperturbed density N_s,

— the phase velocity ω/k is much greater than the macroscopic velocity $\vec{v}_s$ of the particles,

— the temperature gradient has only component in the direction of wave propagation, i.e.

$$\operatorname{grad}(T) = \frac{\mathrm{d}T}{\mathrm{d}z} = \frac{T}{h}, \tag{4}$$

where h is the characteristic length of variation of temperature.

With the above assumptions the Equs. (1) and (2) are linearised as

$$\mathrm{i}\,\omega\, v_s - \frac{1}{N_s} \operatorname{grad}(n_s V_s^2) + \frac{q_s}{m_s} \vec{E} + [\vec{v_s}, \vec{\Omega}_{vs}] = 0, \tag{5}$$

and

$$n_s = \frac{k}{\omega} N_s v_s z, \tag{6}$$

where

$$\Omega_{vs} = \frac{e \vec{H_0}}{m_s c}.$$

Using equation (6), equation (5) is further simplified as

$$\omega^2 v_s + k^2 V_s^2 \hat{I}_z v_{sz} - \mathrm{i} k v_{sz} \operatorname{grad}(V_s^2) + \mathrm{i} \frac{e}{m} \omega \vec{E} + \mathrm{i}\,\omega\,[\vec{v_s}, \vec{\Omega}_{vs}] = 0, \tag{7}$$

where $\hat{I}_z$ is the unit vector along z-direction.

Again using (6), another equation is obtained from Maxwell's equation which is given by

$$\left(1 - \frac{\omega^2}{k^2 c^2}\right) k^2 \vec{E} - k^2 E_z \hat{I}_z - \frac{4\pi\,\mathrm{i}\,e\,\omega}{c^2} (N_1 \vec{v_1} - N_2 \vec{v_2}) = 0. \tag{8}$$

4. *Dispersion relation*

It is difficult to find separate dispersion relation for transverse and longitudinal wave because in (7) and (8) transverse and longitudinal waves are coupled to each other by the perpendicular components of static magnetic field. We, therefore, assume that the static magnetic field is parallel to the direction of wave propagation, i.e.,

$$\vec{H_0} = (0, 0, H_{0z}).$$

Thus $\vec{E}$ and $\vec{v}_s$ eliminant of the Equs. (7) and (8) give the dispersion relation, namely,

$$k^2 c^2 - \omega^2 + \omega \omega_1^2 \frac{\omega \pm \Omega_{v_1}}{\omega^2 - \Omega_{v_1}^2} + \omega \omega_2^2 \frac{\omega \mp \Omega_{v_2}}{\omega^2 - \Omega_{v_2}^2} = 0 \tag{9}$$

and

$$1 - \frac{\omega_1^2}{\omega^2 - k^2 V_1^2 + \mathrm{i}\, k \operatorname{grad} V_1^2} - \frac{\omega_2^2}{\omega^2 - k^2 V_2^2 + \mathrm{i}\, k \operatorname{grad} V_2^2} = 0, \tag{10}$$

for transverse and longitudinal wave, respectively, provided

$$\omega^2 - \Omega_{v_s}^2 \neq 0.$$

In the Equs. (9) and (10).

$$\omega_1^2 = \frac{4\pi N_1 e^2}{m_1}$$

and

$$\omega_2^2 = \frac{4\pi N_2 e^2}{m^2}.$$

5. *Discussions*

It is observed from the dispersion relation (9) that the temperature gradient has no effect on the transverse mode. Therefore, the equation (9) is not important for our present discussion.

From d.r. (10), we observe that the temperature gradient has an effect on longitudinal wave. To study the effect of temperature explicitly, the dispersion relation (10) is simplified with the assumptions

$$\omega_2^2 \gg \omega_1^2$$

since the general $N_2 \gg N_1$ and $m_1 \gg m_2$. Therefore, neglecting second term compared with the third term we get from (10) after writing $V_2 = (R' T)^{1/2}$

$$k = \mathrm{i} \frac{T'}{2T} \pm \frac{\omega}{2\sqrt{R' T}} \left\{ 4 \left(1 - \frac{\omega_2^2}{\omega^2} \right) - \frac{R' T'^2}{T \omega^2} \right\}, \tag{11}$$

where

$$R' = 3\chi/m_2,$$

$$T' = T/h,$$

and $\chi =$ Boltzman's constant.

From (11), it is observed that when

$$\omega < \omega_2 \{1 + (R' T'^2/4\, \omega_2^2 T)\}^{1/2}$$

the wave will not propagate at all. But for

$$\omega > \omega_2 \{1 + (R' T'^2/4\, \omega_2^2 T)\}^{1/2},$$

the wave will propagate producing two modes of propagation one of which is damped, other is amplified. These happen because the first and the second term of r.h.s. of (11) may be of same sign (both are + ve) or opposite in sign (+ ve and − ve). Again, we see that the damping factor, i.e., imaginary part of r.h.s. of (11) is inversely proportional to the characteristic length of variation of the temperature gradient. Therefore, the temperature gradient may affect the wave propagation and it has an important role in the process of damping or instability.

Phase velocity. The phase velocity of the longitudinal wave is

$$v_{ph} = \frac{\omega}{\text{real}\, k} = \frac{V_2}{\left\{\left(1 - \frac{\omega_2^2}{\omega^2} - \frac{R'^2\, T'^2}{V_2^2\, \omega^2}\right)\right\}^{1/2}}. \tag{12}$$

From (12), it is observed that phase velocity depends on the temperature and tem perature gradient of the medium. It is important to note that phase velocity of the longitudinal wave is increased due to presence of temperature gradient. From (12), we also observe that

$$v_{pk} \to \alpha$$

when

$$T' \to 2\omega \left\{\frac{T}{R'}\left(1 - \frac{\omega_2^2}{\omega^2}\right)\right\}^{1/2},$$

and

$$v_{ph} \to \frac{V_2}{\left\{1 - \frac{\omega_2^2}{\omega^2}\right\}^{1/2}},$$

when $T' \to 0$.

Applications. One can notice that temperature gradient can affect the wave provided the condition

$$T' > 2 \cdot 10^{-6}\, \omega \sqrt{T}, \qquad h < 10^6 \sqrt{T}/\omega \tag{A}$$

is satisfied. In fact, on the solar corona T' may be taken as 1 K per km when $T \sim 10^6$ K (Parker[6]). Therefore, the equation (A) suggests that for high frequency

($\sim$ 1 MHz), the temperature gradient has insignificant effect on the wave. In the ionosphere we may take $T' = 10\,\mathrm{K}$ per km where $T \sim 10^3\,\mathrm{K}$, (Parker[7]) thus the importance to the temperature should be given if $\omega \sim$ 10 which is remarkably smaller.

In general, longitudinal waves are density waves and density wave has small frequency. But yet it seems that the temperature gradient has little influence on the wave propagation through ionosphere or solar corona. However, the influence may be important in stellar interior near the surface or in the regions of stellar surface where the temperature gradient is large enough.

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