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INFLUENCE OF MAGNETIC DIFFUSIVITY ON SHOCK WAVE FRONT THICKNESS

V. ČADEŽ* and S. SELAK

Geomagnetic Institute, Grocka

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Abstract: A formula for estimation of a shock wave front thickness is derived from magnetohydrodynamic equations in a dissipative medium, with a finite magnetic diffusivity. It follows that the front thickness can become significant even in media with high electrical conductivity providing that the shock speed is low enough. In the case of strong shock the effects of magnetic diffusivity are reduced and became negligible.

1. Introduction

A shock wave propagating through a dissipative medium will not cause a discontinuous jump of physical quantities on its front. Instead, they will change gradually over a certain distance δ from the values they have in undisturbed fluid to those existing behind the wave front.

Evidently, the distance δ determines, what we call, the front thickness and depends on the intensity of diffusion processes involved. Namely, slow diffusions will make it shorter, while the fast ones will tend to increase it.

This paper presents an estimation of the wave front thickness resulting from the magnetic diffusion as the only dissipative process taken into consideration.

We start from fully ionized and homogeneous plasma with finite electrical conductivity σ, in presence of a magnetic field B_0.

* Institut of Physics, Beograd and Geomagnetic institute, Grocka, Yugoslavia

Let the direction of this field be given by y-axis, as shown on Fig. 1. The plasma assumed to obey the perfect gas law, has density ϱ_0, temperature T_0 and pressure p_0, which are all constant in time and space.

Suppose now, there is a shock wave (Fig. 1, shaded area) propagating through such a medium with the velocity V, parallel to x-axis and causing an increase of physical quantities with respect to their values in the undisturbed region.

As an example, Fig. 2 presents a schematic distribution of magnetic field B in unperturbed plasma and throughout the region of the shock wave. Magnetic diffusion is responsibile for a slow transition from B_0 to B_1 which mostly occurs within the interval δ or the wave front.

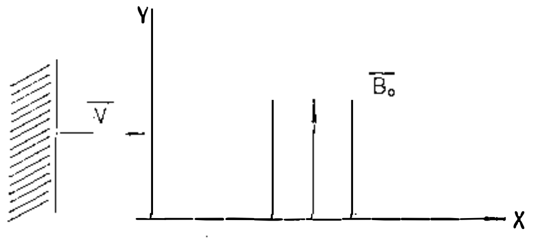

Fig. 1. Initial geometry. Shock wave propagating along x-axis with velocity V in magnetic field B_0.

2. *Basic equations*

We start from standard magnetohydrodynamic equations, which in this, one dimensional problem, have the following form

— continuity equation

$$\frac{\partial \varrho}{\partial t} + \frac{\partial}{\partial x}(\varrho u) = 0, \tag{1}$$

— x-component of momentum equation

$$\varrho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x}\right) = -\frac{\partial}{\partial x}\left(p + \frac{B^2}{2\mu}\right), \tag{2}$$

— energy conservation equation

$$\frac{\partial}{\partial t}\left(\frac{\varrho u^2}{2} + \frac{B^2}{2\mu} + \varrho e\right) = -\frac{\partial}{\partial x}\left[\left(\frac{u^2}{2} + e\right) u\varrho + up + \frac{1}{\mu}\left(u B^2 - \frac{1}{\mu\sigma} B \frac{\partial B}{\partial x}\right)\right], \tag{3}$$

— x-component of magnetic induction equation

$$\frac{\partial B}{\partial t} = \frac{\partial}{\partial x}\left(\frac{1}{\mu\sigma}\frac{\partial B}{\partial x} - uB\right), \tag{4}$$

— and equation of state

$$p = (\gamma - 1)\, e\, \varrho, \tag{5}$$

where u is x-component of the fluid velocity, e is internal energy per unit mass of the fluid and $\gamma = \frac{C_p}{C_v}$ ratio of specific heats.

It is convenient to introduce new variables ξ and η

$$\xi = x - vt, \; \eta = y, \tag{6}$$

where v is the velocity of the shock wave and is taken constant.

The system of differential equations (1)—(5) can further be written in terms of variables ξ and η as follows

$$\frac{\mathrm{d}}{\mathrm{d}\xi}\left[\varrho\,(u - v)\right] = 0, \tag{7}$$

$$\varrho\,(u - v)\frac{\mathrm{d}u}{\mathrm{d}\xi} = -\frac{\mathrm{d}}{\mathrm{d}\xi}\left(p + \frac{B^2}{2\mu}\right), \tag{8}$$

$$\varrho\,(u - v)\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\frac{u^2 + A^2}{2} + e\right) = \frac{\mathrm{d}}{\mathrm{d}\xi}\left[\frac{\varrho A^2}{\mu\sigma}\frac{\mathrm{d}}{\mathrm{d}\xi}\ln B - \varrho\, u\,(C^2 + \frac{A^2}{2}\right], \tag{9}$$

$$v\frac{\mathrm{d}B}{\mathrm{d}\xi} = \frac{\mathrm{d}}{\mathrm{d}\xi}\left(uB - \frac{\lambda}{\mu\sigma}\frac{\mathrm{d}B}{\mathrm{d}\,\xi}\right). \tag{10}$$

Here $A = B/(\mu\,\sigma)^{\frac{1}{2}}$ is the Alfven speed and $C = (p/\varrho)^{\frac{1}{2}}$ is the isothermal sound speed.

We state the initial and boundary conditions. First, the center of the shock wave front was initially ($t = 0$) at $x = 0$, so that variable ξ, given by (6), represents the distance, relative to the front. Then we take the fluid to be unperturbed at large and positive ξ (far ahead the shock)

$$u = 0, \; \varrho = \varrho_0, \; p = p_0, \; B = B_0, \frac{\mathrm{d}B}{\mathrm{d}\xi} = 0. \tag{11}$$

Finally, at large but negative ξ (far behind the front and inside the shock) we again have constant values for all physical quantities

$$u = u_1, \; \varrho = \varrho_1, \; p = p_1, \; B = B_1, \frac{\mathrm{d}B}{\mathrm{d}\xi} = 0. \tag{12}$$

We make an assumption that magnetic diffusion does not affect significantly characteristics of the fluid inside the shock wave, which means that one may use standard shock equations for a perfectly conducting plasma to connect (11) and (12). These equations are

$$\varrho_1 (v - u_1) = \varrho_0 v, \tag{13}$$

$$v \varrho_0 u_1 = p_1 - p_0 + \frac{B_1^2}{2\mu} - \frac{B_0^2}{2\mu}, \tag{14}$$

$$v \varrho_0 \left(\frac{u_1^2 + A_1^2}{2} + e_1 \right) = v \varrho_0 \left(\frac{A_0^2}{2} + e_0 \right) + u_1 \varrho_1 \left(C_1^2 + \frac{A_1^2}{2} \right), \tag{15}$$

$$B_1 (v - u_1) = B_0 v. \tag{16}$$

3. *Estimation of front thickness.*

Equation (10) can be integrated and, considering conditions (11), one obtains

$$\frac{1}{\mu\sigma} \frac{dB}{d\xi} = -(B - B_0) v + u B. \tag{17}$$

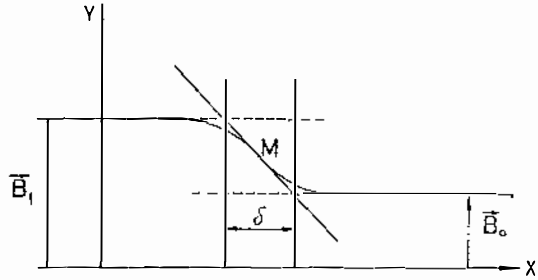

Fig. 2. Distribution of magnetic field in transition region.

To estimate the shock wave front thickness δ, we consider the equation (17) locally in the region where $B = \overline{B}$ and $u = \overline{u}$ (where $\overline{B} = (B_0 + B_1)/2$ and $\overline{u} = u_1/2$ are the corresponding arithmetic means), and which is represented by the vicinity of the point M on Fig. 2. Thus, we get

$$(v - \overline{u}) \overline{B} = v B_0 - \frac{1}{\mu\sigma} \frac{B_0 - B_1}{\delta}, \tag{18}$$

where $\left(\frac{dB}{d\xi}\right)$ is approximately equal to $(B_0 - B_1)/\delta$.

From the equation (18) and by means of (13) and (16), one finally obtains the following expression for the wave front thickness.

$$\delta = \frac{4\eta}{u_1}, \tag{19}$$

where $\eta = {}^1/\mu\sigma$ is the magnetic field diffusivity.

From practical reasons, it is usually more convenient to keep the shock spred v rather than the speed of fluid u_1 in formula (19). To do this, one obtains the following equation in dimensionless form

$$M(1+\gamma)\,u^2 + [2 + (1+\gamma)\,\beta - M^2(3+\gamma)]\,u +$$
$$+ 2M(M^2 - 1 - 2\beta) = 0, \tag{20}$$

which results from the system (13)—(16). The dimensionless parameters in (20) are

$$M = v/C_0,\ \beta = A_0^2/C_0^2 \quad \text{and } U = u_1/C_0.$$

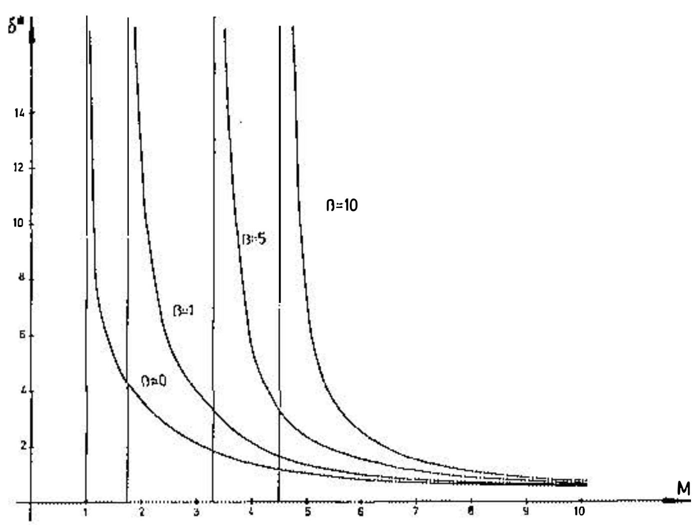

Fig. 3. Dimensionlees wave front thickness $\delta^\star$ as funkction of parameters $M = v/C_0$ and $\beta = A/C_0^2$.

Fig. 3 shows the resulting dimensionless front thickness $\delta^\star = \delta\, C_0/\eta$ as a function of M for various values of β. This picture shows that the effects of magnetic diffusivity become significant at lower values of the shock speed v thus making the front thickness $\delta^\star$ large. As the intensity of the shock wave is increasing, the thickness $\delta^\star$ gets shorter and less important. The vertical asymptotes are given by $M = (1 + 2\,\beta)^{1/2}$.

As an example, let us consider solar corona which is composed of a fully ionized gas of a typical temperature $T = 10^6$ K, fields of about 1 Gauss are reasonable there and shock wave speeds of $\sim$ 500 km/s were observed. The dimensionless front thickness in this case turns out to be $\delta^* = 6$ which further corresponds to δ of only 0.1 mm. The magnetic field diffusion has thus practically no effect on the structure of the observed shocks in the corona.

References

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UTICAJ MAGNETSKE DIFUZIJE NA ŠIRINU FRONTA UDARNOG TALASA

V. ČADEŽ i S. SELAK

Geomagnetski institut, Grocka

Sadržaj

Razmatra se prostiranje udarnih talasa u potpuno jonizovanoj plazmi, konačne električne provodljivosti i u konstantnom magnetnom polju, normalnom na pravac prostiranja talasa. Zbog magnetske difuzije, koja je posledica konačne električne provodljivosti sredine, dolazi do formiranja prelaznog sloja širine δ, koji predstavlja front udarnoga talasa. Fizičke veličine koje određuju plazmu, ne trpe dakle skokovitu promenu, nego se postepeno menjaju kroz front talasa.

Polazeći od poznatih MHD jednačina, dobivena je za razmatrani slučaj formula (19) za procenu širine fronta, a Sl. 3. grafički prikazuje dobiveni rezultat.