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CORRELATIONS IN NUCLEI

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Abstract: The way in which the correlations in nuclei appear in the hyperspherical expansion method is discussed. Comparisons to other methods are given. The cases of the trinucleon system and the α particle are investigated.

1. Introduction

The definition of the correlations in nuclei is a difficult task. A conference[1] has been devoted recently to this subject.

We intend to investigate various aspects of this problem and try to clarify what is hidden behind this concept.

One does not speak of correlation in a two body problem because the two body interaction is equivalent to be one body potential around the centre of mass.

Roughly one can say[2] that the correlations are that part of the motion of a system of N particles interacting through two body forces which cannot be taken into account by a superposition of single particle wave functions like a Slater determinant for a system of fermions.

This definition clearly refers to a model in which the particles are thought to be mainly independent of each other and can be described by the product of individual waves solution of a hamiltonian in which the particles are put into a common well.

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An illustration of this kind of model is given by the Hartree-Fock method in which a set of single particles radial functions is derived by a self consistent approach from a hamiltonian including two body forces by a procedure of minimization with respect to the total energy.

The correlations are therefore induced by the residual interaction, difference between the actual potential $\sum_{i,j>i} V(\vec{x_i} - \vec{x_j})$ and the self consistent potential $\sum_i U(\vec{x_i} - \vec{X})$.

From this point of view a two body harmonic oscillator potential does not introduce any correlation because the total interaction

$$\sum_{i,j>i} (\vec{x_i} - \vec{x_j})^2 = N \sum_i (\vec{x_i} - \vec{X})^2$$

is completely equivalent to a sum of oscillator central potential in which the position $\vec{x_i}$ of each particle refers to the center of mass $\vec{X}$. There is therefore no residual interaction to generate correlations.

If one refers now to the hyperspherical formalism, in which one introduces the collective variable $r = [2 \sum_{i,j>i} (\vec{x_i} - \vec{X})^2]^{1/2}$, a two body harmonic oscillator interaction generates a central harmonic oscillator potential into the 3 N dimensional space. The Schrödinger equation reduces itself in this case to a single radial differential equation, like for a central potential into a 3 dimensional space. When the two body potential is not an oscillator the total interaction $\sum V(\vec{x_i} - \vec{x_j})$ is no longer central in the 3 N dimensional space and exhibits a deformation generating coupled differential equations between the various partial waves of the hyperspherical harmonic expansion (H. E.). This means that the wave function itself is deformed in the 3 N dimensional space. In this case the correlations are induced by the part of the potential which is not »central« in the hyperspherical expansion of the interaction. It is worthwhile to point out that these two cases are not equivalent: a sum of one body potential $\sum U(\vec{x_i} - \vec{X})$ is not central in the 3 N space. It therefore generates correlations in the H E method and not in the independent particle (I.P) model if one excludes the one arising from the centre of mass itself.

This illustrates clearly that when one speaks about correlations one has to specify the model one refers to.

Generally one can say that for any model the correlations are that part of the motion which cannot be taken into account by a single state of the system and requires an expansion of the wave function in terms of the specific complete basis of the model.

It really discloses the weakness of the model used for describing the motion we are interested in. One has now to distinguish between short range and long range correlations.

The short range correlations are those arising from the short range behavior of the two body interaction, specially the repulsive core in a nuclear potential.

The long range correlations refer to the asymptotic behavior of the wave function outside the interaction.

We first intend to discuss the last case for bound states only.

2. *Long range correlations*

Outside the interaction the wave function is solution of the free equation

$$\left\{ \sum_{i=1}^{A} \nabla_i^2 - \chi^2 \right\} \psi^{(\infty)} = 0, \tag{1}$$

where the total binding energy is $E_B = -\frac{\hbar^2}{2m}\chi^2$. In an I.P model the asymptotic wave $\psi_{\text{IP}}^{(\infty)}$ is the product of an angular function of the solid angles ω_i of the A particles, including eventually the spin and isospin variables, by a properly symmetrized function of the radial coordinates r_i of the particles

$$\psi_{\text{IP}}^{(\infty)} \sim A\, F(\omega_1, .., \omega_N) \prod_{i}^{A} r_i^{-1}\, e^{-\chi_i r_i}, \tag{2}$$

where A is (if needed) a symmetrisation operator.

The χ_i are completely determined by the energy $E_i = -\frac{\hbar^2}{2m}\chi_i^2$ of the individual states used in the construction of $\psi_{\text{I.P.}}$. The total binding energy is

$$E_B = -\frac{\hbar^2}{2m}\chi^2 = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\chi_i^2. \tag{3}$$

The asymptotic behaviour of $\psi_{\text{I.P.}}$ is therefore completely determined by the individual states involved.

The most general form of the tail of the wave function is

$$\int f\left[\frac{\chi_1}{\chi}, \ldots, \frac{\chi_N}{\chi}\right] (r_1 \ldots r_N)^{-1}\, e^{-(\chi_1 r_1 + \ldots + \chi_A r_A)}\, d\chi_1 \ldots d\chi_A \tag{4}$$

in which the χ_i fulfill the condition $\sum_{i=1}^{A} (\chi_i/\chi)^2 = 1$. f is a function which describes how the energy is shared among the particles.

The indidivual states are generally defined by a variational principle like in the Hartree-Fock method in which one minimizes with respect to the energy.

The shape of the tail is not sensitive to this procedure because the matrix element involving the interaction vanishes in this region.

Therefore even if the binding energy is well obtained nevertheless the asymptotic behaviour could be badly described.

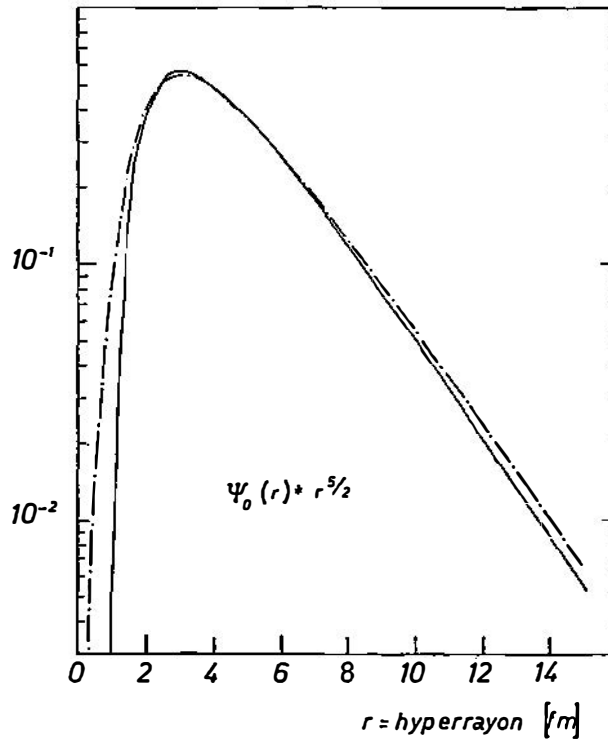

Fig. 1.

In the H.E method each partial radial wave has the same asymptotic behaviour $r^{-\frac{D-1}{2}} e^{-\sqrt{\frac{m}{\hbar^2} E_B} r}$ in a D dimensional space. $D = 3(A-1)$ or $D = 3A$ whether the coordinates refer or not to the center of mass. The wave function has the asymptotic behaviour

$$\psi^{(\infty)}_{\text{H.E}} \sim D(\Omega)\, r^{-\frac{D-1}{2}} e^{-\sqrt{\frac{m E_B}{\hbar^2}} r}, \tag{5}$$

which is completely determined by the binding energy $EB \cdot D(\Omega)$ is an angular function which contains all the correlations when we are dealing with the exact solution of the Schrödinger equation. This exact solution is reached when a number of equations, enough to insure the complete convergence of the method, has been solved. For very light nuclei like the trinucleon system or ^{4}He, the ground state, without tensor interaction, is nearly described (98 — 99%) by the first partial wave of grand orbital $L = 0$ only[3].

This wave is isotropic in the D space.

An I.P model description of the tail in this case gives an anisotropic wave in the D space. In order to show the tail contributes in the H.E to the wave function we have drawn on Fig. 1 the first partial wave $u_0(r) = r^{5/2}\psi_0(r)$ for the trinucleon system extracted from the converged solution of the Schrödinger equation. We have chosen potentials giving the good r. m. s. radius and a binding energy E_B of about 8.4 MeV for ^{3}H.

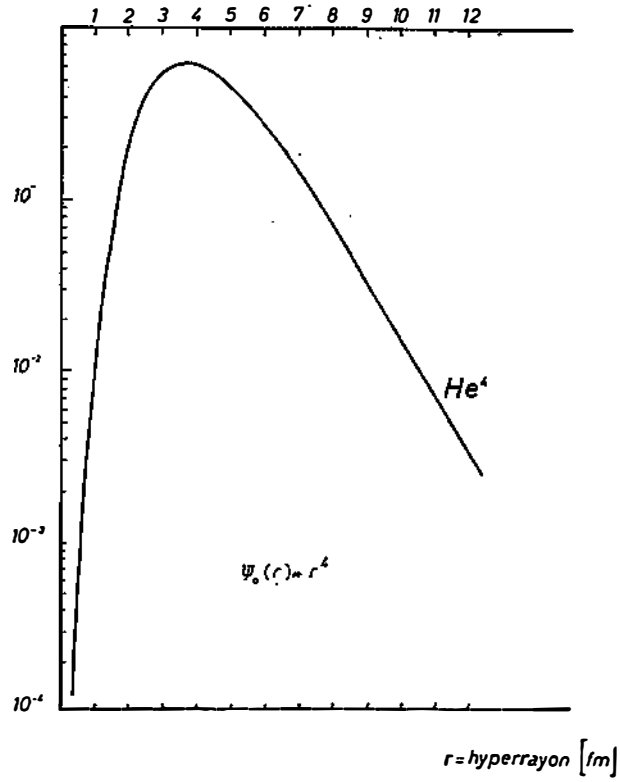

Fig. 2.

The upper curve refers to the very soft core V^x potential[3] (about 50 MeV of repulsion near the origine). The full line proceeds from the G2 potential[3]

which has a very strong repulsive core amounting to 2.5 Gev at the origin. In logaritmic scale the $u_0(r)$ partial waves are practically straight line for $r > 6$ f. m. Their slope corresponds exactly to the binding energy E_B. In the physical 3 dimensional space a configuration in which all the nucleons are outside a sphere of radius 2.5 f.m corresponds to the value $r > 6$ f.m of the hyperradius. As $r = \left[2 \sum_{i=1}^{3} (\vec{x_i} - \vec{\chi})^2\right]^{1/2}$ is a collective variable, the exponential behaviour is also reached when the distance of one particle only from the center of mass $\vec{X}$ exceeds 4 f.m. This shows that an accurate description of the tail of the wave function, at least for the very light nuclei, is not negligible at all.

It can affect such calculations as the dipole photoeffect in which the overlapping of the final state with the tail of the initial state is important at low energy.

Fig. 2 shows the first partial wave $u_0(r) = r^4 \psi_0(r)$ od ^{4}He for the V^x potential. The same conclusion holds: the exponential tail is reached for all the particles out of a sphere of radius 2 fm. or for one nucleon only farther than 4 fm. from the center of mass. The coefficient of the exponential is still determined by the total binding energy E.

3. *Short range correlations*

The short range correlations arise essentially from that part of the two body potential which cannot be simulated by an interaction around the center of mass. For a nuclear interaction involving a strong repulsive core, the density of matter will appear like bubbles into soda. Such a shape cannot be taken into account by a Slater determinant used in the Hartree-Fock approximation.

The short range correlations in the ground state of nuclei are mainly sensitive to the scattering of the nucleons in a relative s state because on the average the kinetic energy of each particle is rather small. They can be taken into account by a Jastrow correlation function[4, 5]. This corresponds in an I. P model to the introduction of an expansion in which each term contains pairs of particles coupled into relative s waves.

The other terms are negligible as long as the $l \neq 0$ phase shifts are.

In using this approximation Afan and Tang[6] treated the short range correlations in writing the wave function of a few nucleon system as a product of two body wave functions, extracted from an exact solution of the two body scattering for negative energy. A smooth function was chosen in order to describe properly the properties of the wave at medium and long range.

Variational calculations have been done for the same few particle problems[7] in using Jastrow functions. The results obtained have nearly the same accuracy

as the best obtained by H. E. method[3, 8]. But such a treatment of the correlations, becomes very tedious as soon as one increases the number of particles involved or if one introduces complicated potential like the realistic two body nuclear potential including tensor, $\vec{l}\vec{s}$, $\vec{l}^2$ terms.

In the H.E method the correlations appear as the byproduct of the expansion in hyperspherical harmonics of the investigated state.

The main terms of this expansion proceed from the so called optimal subset.

This optimal subset is constructed by starting from an angular function which describes the main properties of the investigated state in combining it with the »potential basis«[3]. This potential basis is constituted by the hyperspherical harmonics appearing into the expansion of the potential $\sum V(\vec{x}_i - \vec{x}_j)$. Each element of the potential basis is an even function of »grand orbital« $L = 2K$ which contains s orbitals between any pair of particles only[9] in the case of two body central interaction.

Therefore the correlations described by a Jastrow function correspond to the contribution of the various elements of the optimal subset.

4. *Medium range correlations*

The most important physical parameters like the size, the binding energy, the density of matter... are mainly sensitive to the medium range part of the wave function.

The problem is now to investigate to what extent the truncation which consists of keeping the dominant part of an exact solution of the Schrödinger equation only, may affect the physical properties deduced from this approximation.

In a Hartree-Fock calculation the ground state is described by a single Slater determinant. In the H.E method a similar approximation is obtained when the ground state is the product of a properly symmetrised hyperspherical harmonic including eventually the spin and isospin variables, multiplied by a hyperradial function. The grand orbital of the angular function is the lowest allowed by the Pauli principle. This is the approximation we will refer to as »the hypercentral approximation« because it assumes that in the expansion one value of the grand orbital is significant only. One single radial function will contribute only as if the potential were »central« in the D dimensional space.

A Slater determinant $D(\vec{x})$ describing the ground state of a system of fermions inside a harmonic oscillator well becomes, when converted into hyperspherical coordinates, the product of an angular function $D_{L_0}(\Omega)$ by a hyperradial function.

The angular function $D_{L_0}(\Omega)$ is a hyperspherical harmonic in the D dimensional space. $L_0 = \sum_1^A (2n_i + l_i)$, where n_i and l_i are the radial and orbital quantum

numbers of the oscillator, L_0 is the grand orbital. The hyperradial function is proportional to $r^{L_0} e^{-\alpha r^2}$. The parameter α is related to the strength of the oscillator

$$D(\vec{x}) = D_{L_0}(\Omega)\, r^{L_0} e^{-\alpha r^2}. \tag{6}$$

In the oscillator shell model a variational calculation gives the binding energy

$$E_{os} = \langle D(\vec{x}) \,|\, H \,|\, D(\vec{x}) \rangle \tag{7}$$

obtained by the best fit of the oscillator parameter.

In the H.E method, the hyperradial wave $\psi_0(r) = r^{\frac{D-1}{2}} u_0(r)$ is a solution of the radial equation

$$\left\{ -\frac{\hbar^2}{m}\left[\frac{d^2}{dr^2} + \frac{\nu^2 - 1/4}{r^2}\right] + \langle D_{L_0} \left| \sum_{i,j<i} V(\vec{x}_i - \vec{x}_j) \right| D_{L_0} \rangle - E_{H.E} \right\} u_0(r) = 0, \tag{8}$$

$$\nu = L_0 + \frac{D}{2} - 1.$$

The exact solution u_{L_0} will always give a binding energy $E_{H.E}$ better than E_{os} because E_{os} is just obtained by applying to (8) a variational calculation with respect to the parameter α of the ground state harmonic oscillator wave $u_{os} = r^{\nu+1/2} e^{-\alpha r^2}$.

Nevertheless it has been shown[10,11] that the difference between $E_{H.E}$ and E_{os} is of the order of magnitude of a MeV only. An improvement is obtained by a Hartree-Fock calculation which determines the set of radial functions $\{\varphi(r_i)\}$ giving the minimal binding energy. It corresponds in an H.E to an expansion of the wave function in terms of hyperspherical harmonics differing from $D_{L_0}(\Omega)$ by the hyperspherical quantum number n_i only. The $\{n_i\}$ proceed from the quantification in the space of the hyperspherical angles $\{\Phi_i\}$ related to the radial coordinates r_i in the Zernike-Brinkman[12] representation by

$$r_i = r \sin\Phi_N \ldots \sin\Phi_{i+1} \cos\Phi_i, \quad \Phi_1 \equiv 0.$$

It therefore ignores completely the correlations introduced by the orbitals other than those contained into $D_{L_0}(\Omega)$. These neglected orbitals correspond to particle--hole configurations associated mainly with the short range correlations. In the H.E method the wave function is expanded into hyperspherical harmonics of grand orbital $L_0 + 2K$ of parity $(-1)^{L_0}$

$$\Psi(r, \Omega) = \sum_{K=0}^{+\infty} D_{L_0+2K}(\Omega)\, r^{-\frac{D-1}{2}} u_K(r). \tag{9}$$

Even a small amount of partial waves $u_K(r)$, $K \neq 0$, can change very much the binding energy and the shape of the system. This arises from the contribution of the dominant partial wave $u_0(r)$ into the coupled equations

$$-\left[\frac{\hbar^2}{m}\left(\frac{d^2}{dr^2} - \frac{\nu_K^2 - \frac{1}{4}}{r^2}\right) + E\right] u_K(r) + \sum_{K'} U_K^{K'}(r)\, u_{K'}(r) = 0, \tag{10}$$

$$\nu_K = L_0 + 2K + \frac{D}{2} - 1.$$

$U_K^{K'}(r)$ is the potential matrix

$$U_K^{K'}(r) = \langle D_{L_0+2K}(\Omega) \Big| \sum_{i,j>i} V(x_i - x_j) \Big| D_{L_0+2K'}(\Omega) \rangle.$$

The partial waves indeed appear in (10) by their amplitude and the norm of $u_0(r)$ is of the order of unity.

One can guess the shape of $u_0(r)$ when the coupled equations (10) have been solved.

The main equation relative to $u_0(r)$ is

$$-\left(\frac{\hbar^2}{m}\frac{d^2}{dr^2} + E\right) u_0(r) + W_0(r)\, u_0(r) = 0, \tag{11}$$

$$W_0(r) = \frac{\hbar^2}{m}\frac{\nu_0 - 1/4}{r^2} + \sum_{K=0}^{+\infty} U_0^K(r)\, u_K(r)/u_0(r). \tag{12}$$

$W_0(r)$ is the effective central potential for the main partial wave $u_0(r)$ of the $D_{L_0}(\Omega)$ state. The wave function of the ground state extends around the position r_m of the minimum of $W_0(r)$.

Assuming that $W_0(r)$ is a smooth function around its minimum, one uses the limited expansion

$$W_0(r) = -W_0 + \frac{\hbar^2}{m\varrho^4}(r - r_m)^2, \quad (\varrho \text{ constant}). \tag{13}$$

The round state is a harmonic oscillator wave around r_m

$$u_0(r) = C\, e^{-\frac{1}{2}\left(\frac{r-r_m}{\varrho}\right)^2}, \quad (C \text{ normalisation constant}). \tag{14}$$

The ground state binding energy is

$$E = -W_0 + \frac{\hbar^2}{m\varrho^2}. \tag{15}$$

The parameter ϱ is roughly related to the first excitation energy $h^2/m\varrho^2$ of the hyperspherical breathing mode in which the expansion set $\{D_{L_0+2K}(\Omega)\}$ is conserved but the radial function $u_0(r)$ is replaced by $(r - r_m)\, e^{-\frac{1}{2}\left(\frac{r-r_m}{\varrho}\right)^2}$.

For nuclei an excitation energy of 10 MeV corresponds to $\varrho = 2$ fm. r_m is related to the r.m.s. nuclear radius a by

$$r_m = \sqrt{2A}; \quad a = \sqrt{\frac{6}{5}}\, A^{5/6} r_0; \; r_0 = 1.2 \text{ fm}. \tag{16}$$

The states of the breathing mode are similar to the ground state. They differ by the shape of the radial partial waves. An example is given by the collective excited 0^+ states of the even-even nuclei (e. g. the 20.2 MeV excited 0^+ state of ^{4}He). In using the effective potential $W_0(r)$ one still introduces some correlations into $u_0(r)$ because one takes the partial waves u_K together with the non diagonal elements of the potential matrix into account in the first of the coupled equations. This has the effect of modifying:

— the value of the total binding energy E, and

— the position of the minimum r_m of $W_0(r)$, i.e. the size of the nucleus, but not the shape of the wave function around r_m which is nearly gaussian.

Therefore in the framework of the hypercentral approximation a gaussian partial wave (14) may give a fair description of the round state.

5. *Correlations in the trinucleon and ^{4}He*

We intend now to investigate how the correlations appear in the simple case of the trinucleon. One eliminates the center of mass by using the standard Jacobi coordinates

$$\vec{x}_1 - \vec{x}_2 = \vec{\xi}_1, \; \sqrt{3}(\vec{x}_3 - \vec{X}) = \vec{\xi}_2. \tag{17}$$

The internal kinetic energy operator

$$T = -\frac{\hbar^2}{m}(\nabla^2_{\xi_1} + \nabla^2_{\xi_2}) \tag{18}$$

is symmetric for an exchange of $\vec{\xi}_1$ and $\vec{\xi}_2$.

In terms of $\vec{\xi}_1$ and $\vec{\xi}_2$ the coordinates become

$$\begin{aligned}
&\vec{x}_1 - \vec{x}_2 = \vec{\xi}_1, && \sqrt{3}\,(\vec{x}_3 - \vec{X}) = \vec{\xi}_2,\\
&\vec{x}_2 - \vec{x}_3 = -\frac{1}{2}(\vec{\xi}_1 + \sqrt{3}\,\vec{\xi}_2), && \sqrt{3}\,(\vec{x}_1 - \vec{X}) = -\frac{1}{2}(\vec{\xi}_2 - \sqrt{3}\,\vec{\xi}_1),\\
&\vec{x}_3 - \vec{x}_1 = -\frac{1}{2}(\vec{\xi}_1 - \sqrt{3}\,\vec{\xi}_2), && 3\,(\vec{x}_2 - \vec{X}) = -\frac{1}{2}(\vec{\xi}_2 + \sqrt{3}\,\vec{\xi}_1).
\end{aligned} \tag{19}$$

The permutation of $\vec{\xi}_1$ and $\vec{\xi}_2$ followed by the change of $\vec{\xi}_1$ into $-\vec{\xi}_1$, exchanges the sets $\{\vec{x}_i - \vec{x}_j\}$ and $\{\sqrt{3}\,(\vec{x}_k - \vec{X})\}$.

The trinucleon wave function $\psi(\vec{\xi}_1, \vec{\xi}_2)$ is solution of the Schrödinger equation

$$(T + \sum_{i,j>i} V(x_i - x_j) - E)\,\psi(\vec{\xi}_1, \vec{\xi}_2) = 0. \tag{20}$$

Permuting $\vec{\xi}_1$ and $\vec{\xi}_2$ one obtains another equation

$$(T + \sum_{k=1}^{3} V[\sqrt{3}\,(\vec{x}_k - \vec{X})] - E)\,\psi(\vec{\xi}_2, \vec{\xi}_1) = 0. \tag{21}$$

It is the Schrödinger equation of a system of three particles, each submitted to a central interaction around the center of mass X. The two body potential $V(x_i - x_j)$ and the central potential $V[\sqrt{3}\,(\vec{x}_k - \vec{X})]$ provide the same energy spectrum but not the same density of matter.

In the first case the density of matter is given by

$$\varrho(x) = \int |\psi(\vec{\xi}_1, \sqrt{3}\,\vec{x}|^2\, \mathrm{d}^3\,\xi_1. \tag{22}$$

In the central case it becomes

$$\varrho_C(x) = \int |\psi(\sqrt{3}\,\vec{x}, \vec{\xi}_1)|^2\, \mathrm{d}^3\,\xi_1.$$

But $\varrho_C(\vec{x})$ is related to the correlation density $R(x)$ which gives the probability for two particles to be at a distance $\vec{x}$

$$R(\vec{x}) = \int |\psi(\vec{x}, \vec{\xi}_2)|^2\, \mathrm{d}^3\,\xi_2, \tag{23}$$

by

$$\varrho_c(\vec{x}) = 3^{3/2} R(\sqrt{3}\,\vec{x}).$$

The correlations in this case proceed from the part of the wave function $\psi(\vec{\xi}_1,\vec{\xi}_2)$ which is antisymmetric in the permutation of $\vec{\xi}_1$ and $\vec{\xi}_2$.

This induces the difference, except a scale factor ($\sqrt{3}$), between the density of matter and the correlation density. This difference is clearly related to the short range behaviour of the potential. For very strong repulsive core or hard core potentials the correlation density is canceled at the origin, but nothing can prevent the configuration in which the three particles are in a straight line, therefore the density of matter does not vanish for $x = 0$. The density of matter in this case is the probability of finding one particle at the center of mass. In order to simplify the discussion in the H.E method we will confine ourselves to the dominant completely symmetrical state. The wave function is the product of the completely antisymmetrical spinisospin state A by the completely symmetrical wave $\psi_s(\vec{\xi}_1, \vec{\xi}_2)$.

$\psi_s(\vec{\xi}_1, \vec{\xi}_2)$ is expanded on the optimal subset $\{(\mathscr{P}_{2K}(\Omega)\}$

$$\psi_s(\xi_1, \xi_2) = \sum_{K=0}^{+\infty} \mathscr{P}_{2K}(\Omega)\,\psi_K(r),\ r^2 = \xi_1^2 + \xi_2^2. \tag{24}$$

$\mathscr{P}_2$ does not appear in the sum because it exhibits a mixed symmetry only. $\mathscr{P}^2{}_K(\Omega)$ has the parity $(-1)^K$ in an exchange of the two sets (19).

The two first partial waves ($K = 0,2$), contribute around 99% to ψ_s[13]. The correlations account for about one percent only to the state. If one ignores this last contribution the relation

$$\varrho(x) = 3^{3/2} R(\sqrt{3}\,x), \tag{25}$$

between the density of matter and the correlation density is strictly valid.

It explains why a repulsive core in the two body potential generates a density of matter which in turn produces a zero to the charge form factor. The stronger is the core the smaller is the value of the momentum transfer which cancels the form factor.

In Fig. 1 the full line refers to the $u_0(r)$ partial wave obtained by solving the Schrödinger equation for the strong repulsive core G2 potential[3].

This potential has been chosen in order to obtain the wave function giving the experimental binding energy and a good charge form factor for the trinucleon system. The broken line refers to the very soft core V^x potential which enables to fit the binding energy and the r.m.s. radius of the trinucleon system only[3]. The form factor is not correctly reproduced because the core is not strong enough.

The study of $u_0(r)$ for the G2 potential shows that a hole of about 1.5 fm in the partial wave is needed in order to fit the zero of the charge form factor. It corresponds in the physical space to a hole of about 0.5 fm which nearly empties the center of the correlation density. This hole can be obtained either by a strongly repulsive core two body potential or by a strongly repulsive core 3 body potential which will exactly simulate the effect of a strong two body repulsion.

The tensor force which introduces a D state into the ground state improves the situation because the coupling of the D state to $u_0(r)$ has the effect of pushing this partial wave out of the origin. It seems nevertheless that the tensor force alone is not able to explain the position of the zero of the form factor.

The G2 potential is too much repulsive to give the correct two nucleon phase shifts[3]. Therefore a 3 body strong repulsive core potential may be the best candidate for explaining the occurence of a large hole inside the density of correlation if one uses a realistic $N-N$ potential.

The same results hold for ^{4}He. The treatment of the α particle is the following: one eliminates the center of mass by the use of the Jacobi coordinates

$$\xi_i = \sqrt{\frac{2(i+1)}{i}}\,(\vec{x}_{i+1} - \vec{X}_{i+1}),\ i = 1,\ 2,\ 3. \tag{26}$$

$\vec{X}_i = \frac{1}{i}\sum_{j=1}^{i}\vec{x}_j$ is the center of mass of the i first nucleons. The kinetic energy operator

$$T = -\frac{\hbar^2}{m}\sum_{i=1}^{3}\nabla^2_{\xi_i} \tag{27}$$

is symmetric.

The wave function is solution of the Schrödinger equation

$$[T + \Sigma\, V(x_i - x_j) - E]\,\psi(\vec{\xi}_1,\ \vec{\xi}_2,\ \vec{\xi}_3) = 0. \tag{28}$$

The density of matter ϱ and the correlation density R are respectively defined by

$$\varrho(\vec{x}) = \int |\psi(\vec{\xi}_1,\ \vec{\xi}_2,\ 2\sqrt{\frac{2}{3}}\,\vec{x})|^2\ \mathrm{d}^3\xi_1\ \mathrm{d}^3\xi_2, \tag{29}$$

$$R(\vec{x}) = \int |\psi(\vec{x}, \vec{\xi}_2, \vec{\xi}_3)|^2\ \mathrm{d}^3\xi_2\ \mathrm{d}^2\xi_3. \tag{30}$$

An H.E treatment of the Schrödinger equation (28) has shown[3] that the completely symmetrical wave function of the ground state of ^{4}He contains 98—99% of the first partial wave $u_0(r)$ which is a function of $r = [\xi_1^2 + \xi_2^2 + \xi_3^2]^{1/2}$ only. This function is obviously invariant for any change of the $\vec{\xi}_i$.

Neglecting the other partial waves one obtains again a relation between the correlation density and the density of matter

$$\varrho(\vec{x}) = \left(2\sqrt{\frac{2}{3}}\right)^3 R\left[2\sqrt{\frac{2}{3}}\vec{x}\right]. \tag{31}$$

The zero of the charge form factor of ^{4}He proceeds, like for the trinucleon, from the shape of the density $\varrho(\vec{x})$ similar to the one induced into the correlation density by a two or 3 body repulsive core potential.

6. *Conclusion*

The investigation of the correlation in the framework of the hyperspherical expansion method has shown that:

- the asymptotic behaviour of the wave function is determined by the total binding energy only, the way in which the energy is shared among the particles being described by an angular function in the many dimensional space,
- the short range correlations are strongly related to the anisotropic shape of the total potential in this space, and
- a fair description of the medium range wave function can be obtained for the round state using the hypercentral approximation.

The H.E method leads also to some symmetry relations useful for the interpretation of the form factors of the few nucleons systems.

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