FIZIKA 6 (1974) 135—143

AN ALTERNATIVE WAY TO THE GOOS-HÄNCHEN SHIFT ON A LAYER

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Received 29 April 1974

Abstract: The shifts of a beam of electromagnetic waves reflected and transmitted by a thin layer are derived for the case of total reflection. The solution for the layer is based on the solution of the wave equation for a semiinfinite medium and on geometrical optics. The results agree with results obtained by solving the wave equation for the layer directly and some insight into the process of electromagnetic barrier penetration is gained.

1. *Introduction*

The longitudinal shift of a light beam totally reflected at the boundary of a semiinfinite medium, the Goos-Hänchen effect, has been studied experimentally and theoretically[1–9]. Recently, the corresponding shifts of beams reflected and transmitted by a thin layer have been calculated [10] in two ways. First a superposition of two plane waves with infinitesimally different angles of incidence and opposite phase was exploited. Such a minimum-marked wave[8] allowed direct deduction of shifts for the reflected and transmitted beam, D_r and D_t respectively. Afterwards from the energy flux within the layer a relation between D_r and D_t was derived. This relation alone, however, is not sufficient to determine both D_r and D_t. An alternative derivation of both shifts was found based on multiple reflection of the wave inside the layer. Though it conforms with previous results it appears to give additional insight into the process of electromagnetic barrier penetration.

2. *Reflection and transmission coefficients of a layer*

Let a plane monochromatic electromagnetic wave be incident on a thin layer with index of refraction n_r imbedded in a medium with index of refraction n_i. The boundaries of the layer are placed in the planes $z = Z$ and $z = 0$. The incident wave vector $n_i k_0$ (sin Θ_i, 0, cos Θ_i) lies in the plane of incidence xz. Θ_i is the angle of incidence and $k_0 = 2\pi/\lambda_0$ where λ_0 denotes the vacuum wavelength. The angle of refraction Θ_r is given by the refraction law $n_i \sin \Theta_i = n_r \sin \Theta_r$.

Two basic polarization states are considered: the transverse electric (TE) polarization with the electric field perpendicular to the plane of incidence and the transverse magnetic (TM) polarization with the magnetic field perpendicular to the plane of incidence.

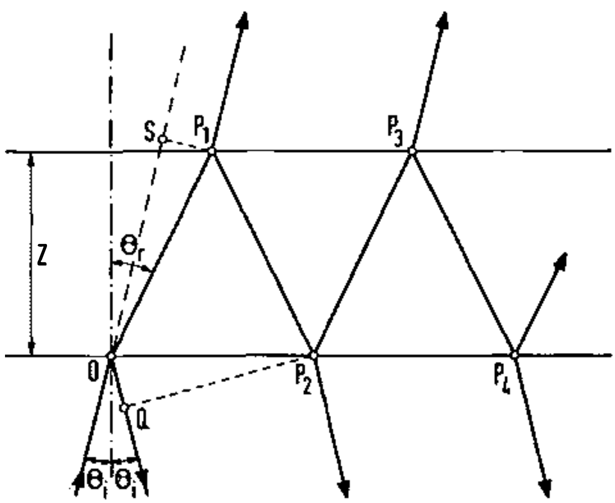

Fig. 1. A schematic presentation of partial waves. The geometrical shifts $\overline{QP_2} = 2g$ and $\overline{SP_1} = g_0$ are shown. The optical path differences $n_r\overline{OP_2} - n_i\overline{OQ}$ and $n_r\overline{OP_1} - n_i\overline{OS}$ are $2\delta_r/k_0$ and $(\delta_r - \delta_i)/k_0$, respectively.

In the case of partial reflection, $\Theta_i < \Theta_c = \arcsin (n_r/n_i)$, it is possible to construct the reflected and transmitted wave by means of elementary single-boundary processes. Thus, the passage of the wave across the layer is decomposed in a sequence of elementary transitions across a single boundary plane. Thereby two families of partial waves are formed with wave vectors $k_0 n_r$ (sin Θ_r, 0, cos Θ_r) and $k_0 n_r$ (sin Θ_r, 0, $-$cos Θ_r). Each of this partial waves is partly reflected and partly transmitted in the encounter at the boudary plane to which it is travelling. With each elementary transition across the boundary plane a vertex is associated. At vertex P_m the partial wave is split into the reflected wave with amplitude $A_{m+1} = r'A_m$ and the transmitted wave with amplitude $B_{m+1} = t'A_m$. Here $A_m$denotes the amplitude of the m-times reflected partial wave inside the layer and r' and t' are the Fresnel coefficients for reflection and transmission by a single boundary[11]. The vertex 0 for $m = 0$ is not equivalent to the other ones. Provided the amplitude of the incident wave equals unity, the amplitudes at this vertex are $A_0 = t$ and $B_0 = r$. Here r and t are the Fresnel coefficients for inverse incidence. Hence

$$A_m = tr'^m \text{ and } B_m = tt'r'^{m-1}, \qquad m = 1, 2, 3 \ldots \tag{2.1}$$

The wave reflected by the layer is constructed as a superposition of all transmitted partial waves from even vertices. There is a phase difference $2\delta_r = 2Zn_rk_0 \cos \Theta_r$ between adjacent outgoing partial waves, corresponding to the additional optical path (Fig. 1). The reflection coefficient R of the layer is defined as the net amplitude

$$R = r + \sum_{m=1}^{\infty} B_{2m} \, e^{2im\delta_r} = r + \frac{tt'r' e^{2i\delta_r}}{1 - r'^2 e^{2i\delta_r}} = \frac{r\,(1 - e^{2i\delta_r})}{1 - r^2 e^{2i\delta_r}}. \tag{2.2}$$

In the same sense the wave transmitted by the layer is constructed as the superpostition of all transmitted partial waves from odd vertices. The phase difference between adjacent partial waves is the same as above. In this case, however, an overall phase difference $\delta_r - \delta_l$ with $\delta_l = Zn_lk_0 \cos \Theta_l$ has to be introduced as phases are defined with regard to the incident wave (Fig. 1). The transmission coefficient T of the layer is defined as the net amplitude

$$T = e^{i(\delta_r - \delta_l)} \sum_{m=1}^{\infty} B_{2m-1} \, e^{2i(m-1)\delta_r} = \frac{e^{i(\delta_r - \delta_l)} \, tt'}{1 - r'^2 e^{2i\delta_r}} = \frac{e^{i(\delta_r - \delta_l)}(1 - r^2)}{1 - r^2 e^{2i\delta_r}}. \tag{2.3}$$

With the Fresnel coefficients

$$r = -r' = \frac{a \cos \Theta_l - b \cos \Theta_r}{a \cos \Theta_l + b \cos \Theta_r},$$

$$t = \frac{2n_l \cos \Theta_l}{a \cos \Theta_l + b \cos \Theta_r}, \tag{2.4}$$

$$t' = \frac{2n_r \cos \Theta_r}{a \cos \Theta_l + b \cos \Theta_r},$$

the known reflection and transmission coefficients

$$R = \frac{\varrho \sin \delta_r}{\nu \sin \delta_r + i\sigma \cos \delta_r}, \tag{2.5}$$

and
$$T = \frac{i\sigma e^{-i\delta_t}}{\nu \sin \delta_r + i\sigma \cos \delta_r}, \tag{2.6}$$

are obtained. Here the abbreviations

$$\varrho = a^2 \cos^2 \Theta_t - b^2 \cos^2 \Theta_r,$$
$$\nu = a^2 \cos^2 \Theta_t + b^2 \cos^2 \Theta_r, \tag{2.7}$$
and
$$\sigma = 2n_t n_r \cos \Theta_t \cos \Theta_r,$$

were introduced. In the case of TE polarization

$$a = n_t \text{ and } b = n_r, \tag{2.8a}$$

whereas in the case of TM polarization

$$a = n_r \text{ and } b = n_t. \tag{2.8b}$$

The coefficients (2.5) and (2.6) are usually derived by taking into account boundary conditions for electric and magnetic field at both boundary planes simultaneously[11]*.

The derivation is valid if the beam width is much greater than the layer thickness. The results retain their validity in the case of total reflection for angles of incidence $\Theta_t > \Theta_c$. In this case there is no limitation with respect to the layer thickness. Some elements of the derivation loose their physical meaning beyond the critical angle as long as the concerned quantities are considered in the real domain. In the complex domain, however, the same formalism applies on both sides of the critical angle.

3. *The shifts of the reflected and transmitted beam*

We extend the above procedure to the determination of the shifts D_r and D_t. Again the procedure is developed in the region of partial reflection and then the results extended beyond the critical angle. The idea is supported by the existence of a geometrical shift

$$2g = 2Z \operatorname{tg} \Theta_r \cos \Theta_t \tag{3.1}$$

between adjacent outgoing partial waves (Fig. 1). The reflected and transmitted beam, reconstructed out of partial waves, are evidently shifted. This picture can be clarified in the following model: let us assume that the incident wave is sharply

* A. Kodre and J. Strnad, Inst. J. Stefan Report R-618 (unpubl.) and to Am. J. Phys. (in press).

cut off by a plane through origin 0, normal to the plane of incidence and containing the incident wave vector. In this case the rays in Fig. 1 represent the cut-off planes of the incident wave and of the consecutive partial waves.

The field in the reconstructed reflected and transmitted beam is not constant across the beam. It changes in a stepwise way with every partial wave and tends asymptotically to its constant value R or T with increasing distance from the origin. Thus, effective cut-off planes can be defined for both outgoing beams by comparing them with idealized beams of constant field. The position of the effective cut-off plane is given by the demand that the integral of the field across the idealized beam be asymptotically equal to the corresponding integral for the actual beam. The distance of the effective cut-off plane from the origin is the beam shift. As can easily be shown, the shifts thus defined are obtained as averages of shifts d_m of the partial waves

$$RD_r = \sum_{m=0}^{\infty} B_{2m} d_{2m} e^{2im\delta_r}, \tag{3.2}$$

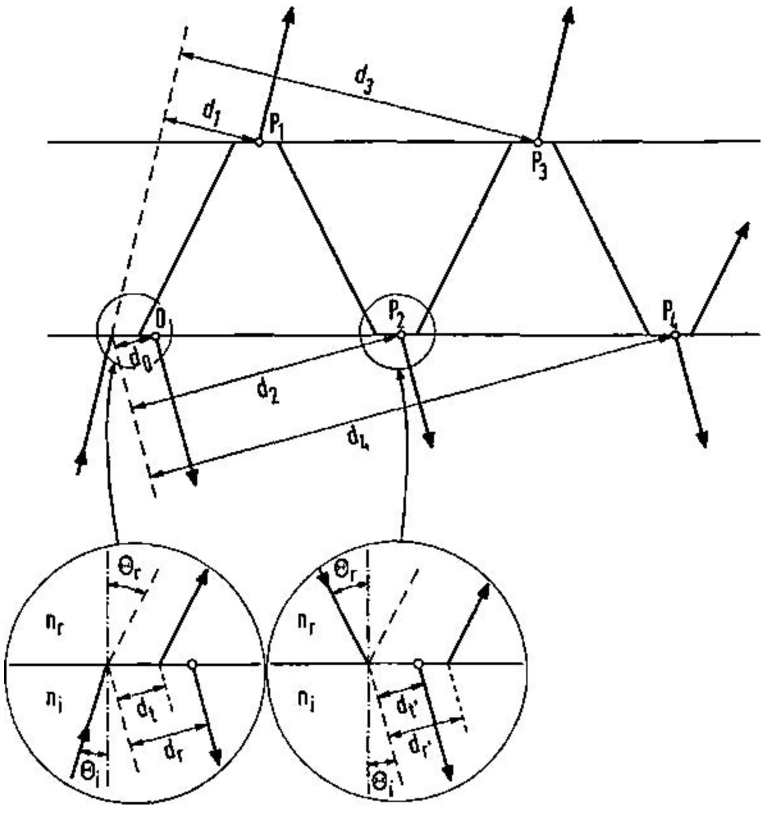

Fig. 2. A schematic presentation of the shifts of partial waves. The elementary shifts d_r, d_t, d_r' and d_t', are shown in the enlarged insertions.

and
$$TD_t = \sum_{m=1}^{\infty} B_{2m-1}\, d_{2m-1}\, e^{2i(m-1)\delta_r}. \tag{3.3}$$

However, the shifts d_m are not merely the accumulated geometrical shifts. As is known from single-boundary experiments and calculations, partial waves are additionally shifted at the reflection and transmission at each vertex. These elementar-shift are denoted as d_r, d_t, $d_{r'}$, and $d_{t'}$, according to the corresponding Fresnel coefficients. All these shifts are, similarly to the geometrical shift g (3.1) defined with respect to the outgoing waves (Fig. 2).

Thus, the shift of the outgoing partial wave at the even vertex P_{2m} is given by

$$d_{2m} = d_t + d_{t'} + (2m - 1)\, d_{r'} + 2mg. \tag{3.4}$$

According to the definition, the shift d_0 of the directly reflected partial wave at the vertex 0 is d_r. The shift of the outgoing partial wave at the odd vertex P_{2m-1} is

$$d_{2m-1} = d_t + d_{t'} + 2\,(m - 1)\, d_{r'} + g_0 + 2\,(m - 1)\, g. \tag{3.5}$$

Here

$$g_0 = \frac{Z \sin(\Theta_r - \Theta_t)}{\cos \Theta_r} = g - Z \sin \Theta_t, \tag{3.6}$$

denotes the trivial refraction shift of the transmitted beam.

By means of the above expressions Equs. (3.2) and (3.3) become

$$RD_r = rd_r + r'tt'\, e^{2i\delta_r} \left[\frac{[(d_t + d_{t'} - d_{r'})}{1 - r'^2 e^{2i\delta_r}} + \frac{2\,(d_{r'} + g)}{(1 - r'^2 e^{2i\delta_r})^2}\right] \tag{3.7}$$

and

$$TD_t = tt'\, e^{i(\delta_r - \delta_t)} \left[\frac{d_t + d_{t'} + g_0}{1 - r'^2 e^{2i\delta_r}} + \frac{2\,(d_{r'} + g)\, r'^2 e^{2i\delta_r}}{(1 - r'^2 e^{2i\delta_r})^2}\right]. \tag{3.8}$$

The shifts of the reflected and transmitted beam are then obtained with R and T from (2.2) and (2.3) and applying the relations between Fresnel coefficients as

$$D_r = \left\{(1 - r^2 e^{2i\delta_r})\, d_r - (1 - r^2)\, e^{2i\delta_r} \left[d_t + d_{t'} - d_{r'} + \right.\right.$$
$$\left.\left. + \frac{2\,(d_{r'} + g)}{1 - r^2 e^{2i\delta_r}}\right]\right\} : (1 - e^{2i\delta_r}) \tag{3.9}$$

and
$$D_t = d_t + d_{t'} + g_0 + \frac{2\,(d_{r'} + g)\, r^2 e^{2i\delta_r}}{1 - r^2 e^{2i\delta_t}}. \tag{3.10}$$

4. *Results and discussion*

To determine the elementary shifts d_r, d_t, $d_{r'}$, and $d_{t'}$ the passage of the wave across a single boundary has to be studied. The results slightly depend on the model used, i. e. on the form of the incident wave. The energy flux argument, employing an incident plane wave, seems in this sense the most unambiguous. However, only the shift d_r can be deduced in that way and, furthermore, only for angles beyond the critical angle.

All four shifts can formally be obtained by the method of the minimum-marked wave[8', 10]. We introduce the incident wave of the form

$$e^{-i\omega t}\{e^{ik_0 n_l[x\sin(\Theta_l + d\Theta_l) + z\cos(\Theta_l + d\Theta_l)]} - e^{ik_0 n_l(x\sin\Theta_l + z\cos\Theta_l)}\} =$$

$$= e^{i[k_0 n_l(x\sin\Theta_l + z\cos\Theta_l) - \omega t]} \cdot i\,k_0\,n_l\,(x\cos\Theta_l - z\sin\Theta_l)\,d\,\Theta_l. \qquad (4.1)$$

The zero of the amplitude in the plane $x\cos\Theta_l - z\sin\Theta_l = 0$ marks an incident ray through origin 0. Let such a wave approach the boundary from the medium n_l. It can be shown that the amplitude in the reflected wave is proportional to $x\cos\Theta_l + z\sin\Theta_l + (i\,k_0\,n_l\,r)^{-1}\,\dfrac{\partial r}{\partial\Theta_l}$.

The last term is generally complex, so we cannot find a ray of zero amplitude in the reflected wave. However, a ray of minimum amplitude exists along the plane $x\cos\Theta_l + z\sin\Theta_l + \mathrm{Re}\left\{(ik_0 n_l r)^{-1}\dfrac{\partial r}{\partial\Theta_l}\right\} = 0$. Hence we see that the reflected ray is shifted with respect to the mirrored incident zero amplitude ray by $-\,\mathrm{Re}\left\{(ik_0\,n_l\,r)^{-1}\dfrac{\partial r}{\partial\Theta_l}\right\}$. This expression is usually defined as the shift and we can see that in the region of partial reflection this shift is zero.

$-\,\mathrm{Im}\left\{(i\,k_0\,n_l\,r)^{-1}\dfrac{\partial r}{\partial\Theta_l}\right\}$ denotes the residual amplitude in the reflected minimum-amplitude ray, it corresponds to an admixture of an off-phase wave. Thus, we can regard the term $-\,(i\,k_0\,n_l\,r)^{-1}\dfrac{\partial r}{\partial\Theta_l}$ as a generalized shift, its real part representing the spatial shift and the imaginary part the shift of the phase.

As the definition of the shift D_r of the reflected beam in Equ. (3.2) takes into account phases of partial waves, the elementary shifts are accepted as complex quantities, e. g.

$$d_r = i\,(k_0\,n_l\,r)^{-1}\,\frac{\partial r}{\partial\Theta_l} = \frac{2\,i\sin\Theta_l\,(n_l^2 - n_r^2)}{\varrho\,k_0\,n_r\cos\Theta_r}. \qquad (4.2)$$

By a similar procedure the remaining three shifts are obtained as

$$d_t = i\,(k_0 n_r t)^{-1} \frac{\cos\Theta_t}{\cos\Theta_r} \frac{\partial t}{\partial \Theta_r} = i\,(k_0 n_t t)^{-1} \frac{\partial t}{\partial \Theta_t} =$$

$$= ib\,(n_t^2 - n_r^2) \sin\Theta_t \frac{a\cos\Theta_t - b\cos\Theta_r}{\varrho k_0 n_t n_r^2 \cos\Theta_t \cos\Theta_r}, \tag{4.3}$$

$$d_{r'} = i\,(k_0 n_r r')^{-1} \frac{\cos\Theta_t}{\cos\Theta_r} \frac{\partial r'}{\partial \Theta_r} = d_r. \tag{4.4}$$

and

$$d_{t'} = i\,(k_0 n_t t') \frac{\partial t'}{\partial \Theta_t} = \frac{-ia\,(n_t^2 - n_r^2) \sin\Theta_t\,(a\cos\Theta_t - b\cos\Theta_r)}{\varrho k_0 n_t n_r^2 \cos^2\Theta_r}. \tag{4.5}$$

Cosines in d_t and $d_{r'}$ are introduced in order to define the shifts with respect to the outgoing waves.

We note that

$$d_t + d_{t'} = d_r \left(1 - \frac{\nu}{\sigma}\right). \tag{4.6}$$

The shifts D_r (3.9) and D_t (3.10) are evidently complex and by a similar argument as above the measurable spatial shifts are obtained as their real parts. In the region of total reflection, where the main interest lies, $\cos\Theta_r$ becomes imaginary and so does $\delta_r = k_0 n_r Z \cos\Theta_r = i k_0 Z (n_t^2 \sin^2\Theta_t - n_r^2)^{1/2} = i\gamma$. An elementary yet rather lengthy calculation gives

$$\mathrm{Re} D_r = \frac{d_r \varrho^2 \,\mathrm{sh}\,\gamma \,\mathrm{ch}\,\gamma - 2 Z n_t^2 \nu \sin\Theta_t \cos^2\Theta_t}{\varrho^2 \,\mathrm{sh}^2 \gamma - \sigma^2}, \tag{4.7}$$

and

$$\mathrm{Re} D_t = \mathrm{Re} D_r - Z \sin\Theta_t. \tag{4.8}$$

It should be mentioned that nonzero shifts are obtained also in the region of partial reflection for layers that are much narrower than the beam width.

The form of the result (4.7) combines the features of the results based on the energy flux argument and those of the minimum-marked-wave method. The ideas developed in Sect. 3 are, in essence, equivalent to the energy flux argument employing plane waves, regardless of the cut-off introduced. The minimum-marked-wave method enters only through the elementary shifts. Both results can be brought into accordance replacing factors $n_t^2 - n_r^2$ in Equs. (4.2) — (4.5) by $n_t^2 \cos^2\Theta_t$, transforming thus d_r into the form obtained by the energy flux argument [7]. Consequently, the results (4.7) and (4.8) become identical with those of the energy flux argument[10].

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ALTERNATIVNA POT DO PREMIKA GOOS-HÄNCHEN NA PLASTI

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Vsebina

Izpeljan je premik curka elektromagnetnih valov, ki se odbije na tanki plasti ali ga plast prepusti, za primer totalnega odboja. Pot do rezultatov omogoči vpogled v dogajanje pri prehodu curka skozi plast. Rešitev za plast sloni na rešitvi valovne enačbe za polneskončno sredstvo in na geometrijski optiki. Rezultati se skladajo z rezultati, ki sva jih dobila z neposrednim reševanjem valovne enačbe za plast.