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COHERENT PRODUCTION OF CHARGED PIONS

N. BILIĆ and M. MARTINIS

Institute »Ruđer Bošković«, Zagreb

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Abstract: A particular model of coherent production of charged pions is suggested and restrictions due to momentum conservation, charge conservation and isospin conservation are dealt with in detail. Multiplicities $< n_0 >$, $< n_+ >$ and $< n_{ch} >$ in $\pi^+ p$ scattering as well as the total cross sections $\sigma_{\pi^+ p} (n_0 n_+ n_-)$, $\sigma (\pi^+ p)$, dispersion D_{ch} and the correlation functions f_2^{--}, f_2^{0-} and f_2^{ch} have been calculated. The theoretical curves D_{ch} and f_2^{--} as function of $< n_- >$ are compared with existing experimental data. The results are consistent with presently available data, although further refinements of the model are necessary.

1. Introduction

The fact that there are only a few particles produced in a high-energy experiment (about 5 to 10 at presently available energies) does not allow us to use a simple description in terms of statistical methods. Experimentally, the average number of produced particles grows very slowly with energy, the trend of the growth is either $A + B \ln s$ (multiperipheral models[1]) or Cs^a (statistical type models[2]). It is, therefore, difficult to prove or disprove some of the models of particle production. Many models differ only in their description of »asymptotic« phenomena.

Most particles produced in high-energy collisions are pions. Typical distributions of the cross sections for production of pions fall on a curve similar to a Poisson distribution.

The production amplitude should in general be subjected to three categories of constraints:

a) conservation of four momenta,

b) conservation of additive and multiplicative quantum numbers, such as charge, hypercharge, etc., and

c) conservation of isospin.

Although there are several theoretical models dealing with this problem[3], it is not yet clear what the best way of introducing constraints b) and c) would be. The simplest model that might be able to incorporate these constraints is the uncorrelated jet model[4] first proposed by Van Hove[5]. This model assumes the factorization of the production matrix element in the momentum space.

The present paper is an attempt to deal with constraint c) in the $N\pi \to N + n\pi's$ process. To introduce isospin in this model, we use the concept of a coherent state which offers a simple physical interpretation and mathematical structure. In other words, pion emission is regarded as a process similar to the classical radiation of a pionic field extended by isospin conservation.

The paper is divided into four sections. In sect. 2. we give a detailed description of the model using the concept of a coherent state. We also derive the general form of the n-pion production amplitude. The $\pi^+ p \to N + n\pi's$ process is analyzed in detail. In sect. 3. we give expressions for multiplicities $< n_{0,\pm} >$ and $< n_{ch} >$, $\sigma_{\pi+p}(n_0 n_+ n_-)$, $\sigma(\pi^+ p)$, dispersion D_{ch} and the correlation functions f_2^{--} and f_2^{0-} in the region where s is very large. Comparison with available experimental data is discussed. In sect. 4. we discuss some of the results the model is able to offer and also suggest improvements in order to obtain the satisfactory behaviour of $\sigma(\mathrm{el}) / \sigma(\mathrm{tot})$. We also make comments on the f_2^{0-} correlation function.

2. *Description of the model*

Let us consider the $\pi^+ p \to \pi_a X$ process, where X consists only of pions and one nucleon. This is not a very restrictive assumption, since pions are known to dominate secondaries at high energy. In πp reactions, only 16 % of the produced channels include strange particles.

To find the isospin structure of the $(n\pi's)\, N$ system, we introduce the concept of a coherent state of a system of identical pions. A coherent state $|\eta>$ of charged pions is given by the equation

$$|\eta> = \eta(\pi_a)\,|\,0>, \tag{1}$$

where

$$\eta(\pi_a) = c : \exp(\lambda \gamma_5 \vec{\tau}\, \vec{\pi}): \quad \lambda > 0, \tag{2}$$

$$\pi_a = \int \mathrm{d}\hat{q}\,[f^\star(q)\,a_a(q) + \text{h. c.}]\;;\;\mathrm{d}\hat{q} = [2E_q(2\pi)^3]^{-1}\,\mathrm{d}^3 q, \tag{3}$$

and $E_q = (\vec{q}^{\,2} + m_\pi^2)^{1/2}$. $f(q)$ is a relativistic invariant function of q, a pion wave function, which may also depend on some external momenta that will be specified later. The coefficient c is chosen in such a way that $|\eta>$, when acting ona single--nucleon state, satisfies the normalization condition $<\eta\,|\,\eta> = 1$.

The basic Equ. (1) is a natural extension of the usual scalar-boson coherent state in which no reference to charge isospin or parity is made[6]. The Pauli matrix $\vec{\tau}$ in (2) takes into account the correct isospin coupling of pions to a nucleon. γ_5 takes care of parity conservation. Parity conservation has some severe consequences; it leads to a distinction between even and odd number of emitted pions[6].

The coherent-state expectation values of the pion-number operators $N_0, \frac{1}{2}(N_+ + N_-)$ and $N = N_0 + N_+ + N_-$ satisfy the simple relation

$$<\eta\,|N_0|\,\eta> = \frac{1}{2} <\eta\,|\,(N_+ + N_-)\,|\,\eta> = \frac{1}{3} <\eta\,|N|\,\eta>. \tag{4}$$

Note that

$$\frac{1}{c^2} = \operatorname{sh} z + (1 + z)\,\mathrm{e}^z, \tag{5}$$

where

$$z = \lambda^2 \int \mathrm{d}\hat{q}\,|f(q)|^2. \tag{6}$$

The production amplitude for the process $\pi^+ p \to N + n_-\pi^- + n_+\pi^+ + n_0\pi^\circ$, $(n_0 + n_- + n_+ = n)$ in which two incoming particles (a pion with momenta q_+ and a proton with momenta p) produce an outgoing leading particle (a nucleon with momenta p') and n π-mesons of momenta $q_1 \,..\, q_n$ which are part of a coherent state, is described by the following factorized T-matrix structure[6]

$$T_n(\pi^+ p) = \bar{u}(p')\,F_n(q_+\,;q_1 \ldots q_n)\,u(p)\,G(p, p'), \qquad n > 1, \tag{7}$$

where

$$F_n(\ldots\ldots) = <q_1 \ldots q_n\,|\,\eta(\pi_a)\,|\,q_+>. \tag{8}$$

Equ. (7) may be viewed as a version of a model of diffractive production of particles (Fig. 1). Thus $G(p, p')$, which is often called the „skeleton" of the process, depends on the invariant variables

$$s = (p + q_+)^2,$$

$$t = (p - p')^2, \tag{9}$$

$$M^2 = (q_1 + q_2 + \dots q_n)^2 = Q^2.$$

In the framework of the uncorrelated jet model $f(q)$ is interpreted as the form factor for emission of one pion, i. e.,

$$f(q) \to f(s, q). \tag{10}$$

Isospin is usually introduced[7] by means of $_n$ factor taken from the statistical model[8], so that isospin is conserved only on the average not exactly.

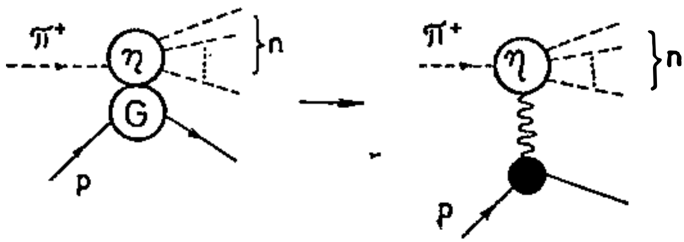

Fig. 1. Diffractive dissociation of pion.

The determination of f and G is beyond the possibility of the model, so that various parametrizations may be tried to account for the data.

To be more quantitative, we assume the following simple parametrization of f and G often used in the statistical-type models[2]

$$|f(s, q)|^2 \to \exp[-(aq)], \tag{11}$$

where a is a time-like vector ($a^2 = a_0^2 - \hat{a}^2 > 0$) composed of incident momenta, so that $f(s, q)$ is an invariant function of q,

$$|G(p, p')|^2 \to g(s) \exp[-(ap')]. \tag{12}$$

The phase-space integration is performed in the zero pion-mass limit.

Before writing down the n-particle total cross section $\sigma(n_0 n_+ n_-)$, let us calculate the n-particle phase space.

Using (11) and (12), we find

$$z_a(x) = \lambda^2 \int d\hat{q}\, |f(q)|^2 e^{-iqx} \underset{m_\pi \to 0}{\sim} \left(\frac{\lambda}{2\pi}\right)^2 \frac{1}{(a + ix)^2}, \tag{13}$$

so that

$$I_{n+1}(P) = \frac{1}{(n+1)!} \int \prod_{i=1}^{n} d\hat{q}_i\, |\lambda f(q_i)|^2\, d p'\, |\lambda f(p')|^2\, (pp') \cdot$$

$$\cdot\ (2\pi)^4\,\delta^{(4)}(P - p' - \Sigma q_i) = \frac{(-p\,\partial_a)}{(n+1)(n+1)!}\int d^4x\, e^{iP.x}[z_a(x)]^{n+1} \sim$$
$$\sim 4I_2(P)\frac{\left(\frac{\lambda^2 P^2}{16\pi^2}\right)^{n-1}}{(n-1)!\,[(n+1)!]^2}, \tag{14}$$
$$I_2(P) = \frac{\lambda^4}{32\pi}(p\cdot P)e^{-aP},\ P^2 = (p+q_+)^2 = s.$$

The n-particle cross section is now

$$\sigma(n_0 n_+ n_-) = 2\sigma_{el}(\pi^+p)\frac{\left(\frac{\lambda^2 P^2}{16\pi^2}\right)^{m-1}}{(m-1)!\,[(m+1)!]^2}\begin{cases} W_n(n_0n_+n_-), & m = 2n-1 \\ \widetilde{W}_n(n_0n_+n_-), & m = 2n \end{cases}, \tag{15}$$

where

$$W_n(n_0n_+n_-) = \delta_{n,i+j}\frac{(2n)!!}{(2n-1)!!}(n-i)\frac{(2i-1)!!}{(2i)!!},\quad \begin{matrix} n_0 = 2i \\ n_+ = n_- + 1 = j \end{matrix} \tag{16}$$

$$\widetilde{W}_n(n_0n_+n_-) = \delta_{n,i+j}\frac{(2n)!!}{(2n+1)!!}(n-i)\frac{(2i-1)!!}{(2i)!!}\begin{cases} 2i+1; & \begin{matrix} n_0 = 2i+1 \\ n_+ = n_- + 1 = j \end{matrix} \\ 2(n-i+1); & \begin{matrix} n_0 = 2i \\ n_+ - 1 = n_- + 1 = j \end{matrix} \end{cases}.$$

Summing W_n and $\widetilde{W}_n$ over $n_0n_+n_-$ gives

$$\sum_{n_0+n_++n_-=2n-1} W_n(n_0\,n_+n_-) = \frac{1}{3}(2n)(n+1),$$
$$\sum_{n_0+n_++n_-=2n} \widetilde{W}_n(n_0\,n_+\,n_-) = \frac{1}{3}(2n)(2n+3). \tag{17}$$

The total cross section can be written in the form

$$\sigma_{tot}(\pi^+p) = \sum_{n_0n_+n_-}\sigma(n_0\,n_+\,n_-) =$$
$$= \text{const.}\ \sigma_{el}(\pi^+p)\left(\frac{\lambda^2 s}{16\pi^2}\right)^{-1}\exp\left[3\left(\frac{\lambda^2 s}{16\pi^2}\right)^{1/3}\right]\cdot \tag{18}$$
$$\cdot\left\{1 + \frac{7}{9}\left(\frac{\lambda^2 s}{16\pi^2}\right)^{-1/2} + \dots\right\},$$

where the summation over n is performed using the aproximate formula $(n!)^3 \simeq$
$\simeq \frac{(3n+1)!}{3^{3n+1}} \frac{2\pi}{\sqrt{3}} \left(1 + \frac{1}{9n} + \ldots\right)$.

Note the embarrasingly small ratio for

$$\frac{\sigma\,(\mathrm{el})}{\sigma\,(\mathrm{tot})} \sim \exp\left[-3\left(\frac{\lambda^2 s}{16\pi^2}\right)^{1/3}\right]. \tag{19}$$

This feature is encountered in most of the statistical models mainly due to the parametrization of the type (12).

3. *Application*

Using Equs. (15), (16) and the definition

$$\sigma\,(\pi^+ p) < n_a > = \sum_{n_0\, n_+\, n_-} n_a\, \sigma\,(n_0\, n_+\, n_-), \tag{20}$$

we find the following expressions for multiplicities

$$\begin{aligned} < n \quad > &= \left(\frac{\lambda^2 s}{16\pi^2}\right)^{1/3}, \\ < n_p \quad > &= 0.86 + 0\,(1\,/ < n >), \\ < n_+ \quad > &= 0.38 < n > + 0.65 + 0\,(1\,/ < n >), \\ < n_- \quad > &= 0.38 < n > - 0.49 + 0\,(1\,/ < n >), \\ < n_0 \quad > &= 0.24 < n > - 0.16 + 0\,(1\,/ < n >), \\ < n_{ch} > &= 0.76 < n > + 1.02 + 0\,(1\,/ < n >). \end{aligned} \tag{21}$$

Note the validity of the model-independent relation $< n_{ch} > = Q + 2 < n_- >$ where Q is the charge of the initial state. In our case $Q = 2$.

$D_{ch} = (< n_{ch}^2 > - < n_{ch} >^2)^{1/2}$ is given by

$$D_{ch} = 0.28 < n > + 0.21 + 0\,(1\,/ < n >) \sim 0.37 < n_{ch} > - 0.17. \tag{22}$$

The functional dependence of D_{ch} on $< n_{ch} >$ is shown in Fig. 2, where it is compared with the existing data on π^+p scattering.

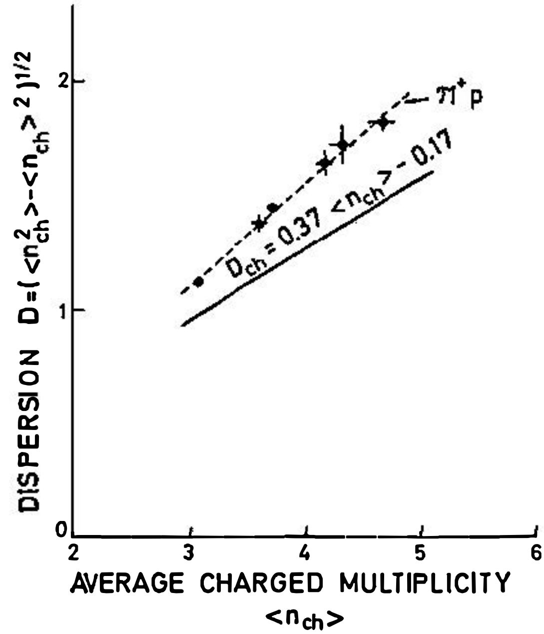

Fig. 2. Functional dependence of D_{ch} on $< n_{ch} >$ for π^+p scattering (solid line).

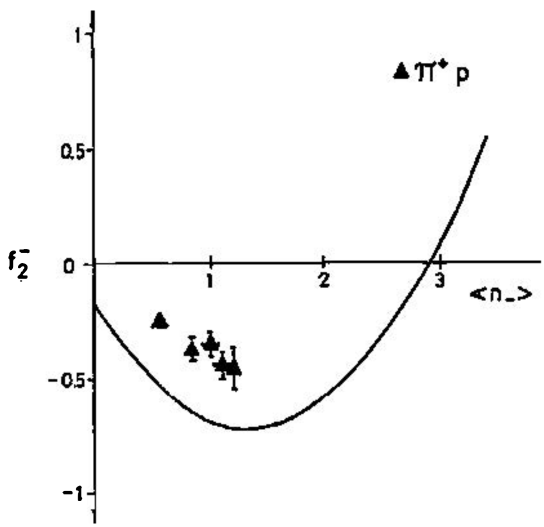

Fig. 3. Functional dependence of f_2^- on $< n_- >$ for π^+p scattering (solid line).

The total pion multiplicity distribution is approximately given by

$$P_n = \sigma_n / \sigma_{\text{tot}} \simeq 3 \frac{\langle n \rangle^{3n}}{(3n)!} \exp(-3 \langle n \rangle). \tag{23}$$

It differs slightly from the standard Poisson distribution that would have been obtained if the isospin conservation constraint had not been required. The value of the correlation coefficient f_2^{--} as a function of $\langle n_- \rangle$ is shown in Fig. 3. and compared with the corresponding experimental data on $\pi^+ p$ scattering. $f_2^{\text{ch}} = D_{\text{ch}}^2 - \langle n_{\text{ch}} \rangle$ as a function of $\langle n_{\text{ch}} \rangle$ is negative up to $\langle n_{\text{ch}} \rangle > 14$. At this point f_2^{ch} becomes positive.

4. *Discussion*

The comparison of the results of the model with data seems encouraging, the general trend of the curves is correct. However, different parametrizations for $f(q)$ and $G(p, p')$ should be tried to avoid the embarrassingly small ratio for $\sigma(\text{el}) / \sigma(\text{tot})$ given in Equ. (19). An alternative approach would be to use the transverse momentum cut-off, i. e.

$$\begin{gathered} |f(s, q)|^2 \to \exp[-a q_T^2], \\ 2(pp' + m^2) \, | G(p, p') |^2 \to p(s) \exp[-a p_{T'}^2], \end{gathered} \tag{24}$$

instead of (11) and (12).

The corresponding phase-space integral given by Equ. (14) is evaluated using the longitudinal phase-space method[9]. Choosing only the dominant term, we find

$$\begin{gathered} I_{n+1}(s) = I_2(s) \frac{2\left(\frac{\lambda^2 \ln s}{16\pi^2 a}\right)^{n-1}}{(n+1)(n-1)!}, \\ I_2(s) = \frac{\lambda^4}{16\pi a} \cdot \frac{1}{s}. \end{gathered} \tag{25}$$

For the n-particle cross section we can write

$$\sigma(n_0 n_+ n_-) = \sigma_{\text{el}}(\pi^+ p) \frac{\left(\frac{\lambda^2 \ln s}{16\pi^2 a}\right)^{m-1}}{(m+1)(m-1)!}.$$

$$
\cdot \begin{cases} W_n(n_0 n_+ n_-). & m = 2n - 1, \\ \widetilde{W}_n(n_0 n_+ n_-). & m = 2n, \end{cases} \tag{26}
$$

and also

$$
\sigma_{\text{tot}}(\pi^+ p) = \sigma_{\text{el}}(\pi^+ p) \frac{1}{3} (2a \ln s + a^2 \ln^2 s) s^a. \tag{27}
$$

where

$$
a = \frac{\lambda^2}{16\pi^2 a}.
$$

This simple example shows how the model of particle production with exact conservation of isospin can be constructed to obtain the much better ratio

$$
\frac{\sigma(\text{el})}{\sigma(\text{tot})} \sim s^{-a}; \quad a > 0. \tag{28}
$$

The correlation function $f_2^{0-} = \langle n_0 n_- \rangle - \langle n_0 \rangle \langle n_- \rangle$ is found to be negative in both examples treated here. The same result is found in all cases in which pions are produced in an independent manner. Only the conservation of four-momenta and isospin is required. Experimentally, f_2^{0-} is positive[10], thus indicating that there must be some other dynamical mechanism responsible for it. If, however, pions are produced in resonances or bunches, the positive correlation function f_2^{0-} is found[10].

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KOHERENTNA PRODUKCIJA NABIJENIH PIONA

N. BILIĆ i M. MARTINIS

Institut »Ruđer Bošković«, Zagreb

Sadržaj

Sugeriran je model koherentne produkcije nabijenih piona. Ograničenja koja dolaze od sačuvanja energije i impulsa, naboja i izospina obrađena su detaljno. Izračunati su multipliciteti $< n_0 >$ $< n_{\pm} >$ i $< n_{ch} >$ u $\pi^+ p$ raspršenju kao i totalni udarni presjeci $\sigma_{\pi^+ p}(n_0\, n_+\, n_-)$, $\sigma(\pi^+ p)$, disperzija D_{ch} i korelacione funkcije f_2^{--}, f_2^{0-} i f_2^{ch}. Teoretske krivulje D_{ch} i f_2^{--} kao funkcije $< n_- >$ uspoređene su sa eksperimentalnim podacima. Rezultati su u saglasnosti sa dostupnim eksperimentalnim podacima iako je potrebno daljnje poboljšanje modela.