FIZIKA, 7 (1975) 13—21

TIME DELAY IN ELECTROMAGNETIC BARRIER PENETRATION

J. STRNAD and A. KODRE

Department of Physics and Institute „J. Stefan", University of Ljubljana, Ljubljana

Received 10 october 1974

Abstract: The time delay of an electromagnetic wave reflected or transmitted by a layer of optically less-dense medium is studied. The delay of a wave packet is calculated by means of a minimum-marked wave and the results are verified in the scope of scattering theory. The effective group velocity in the optically less-dense medium for the transmitted wave packet can exceed the velocity of light in vacuum c_0, while the signal velocity does not.

1. *Introduction*

Recently, tachyonic properties of the evanescent wave in the optically less-dense medium at total reflection were conjectured[1]. On the other hand the longitudinal shift of a totally reflected beam, or the Goos-Hänchen effect, was studied experimentally as well as theoretically[2-6]. In this connection more information can be gathered from the study of penetration of a wave across a barrier than from a mere consideration of reflection on a semiinfinite medium. Consequently we extended our recent work on this subject[7] to include the time delay.

In Sect. 2 the delay time is calculated by a generalized minimum-marked-wave method. Results obtained are identical with the results of the stationary phase method for wave packets. In Sect. 3 the same problem is considered in the scope of scattering theory. In Sect. 4 it is demonstrated that in a certain region of angle of incidence and layer thickness the effective group velocity inside the layer for the transmitted wave packet is superluminal. Arguments are given that the signal velocity even in this case does not exceed the velocity of light in vacuum.

2. A generalized minimum-marked-wave method

Let a monochromatic plane wave be incident on a thin layer of a medium with index of refraction n' and thickness Z. On both sides the medium is surrounded by a medium with index of refraction $n > n'$. The boundaries of the layer are the planes $z = 0$ and $z = Z$. The plane xz is chosen as the plane of incidence. The wave vector of the incident or transmitted wave is $\vec{k}_I = k\,(\sin\Theta, 0, \cos\Theta)$, the wave vector of the reflected wave is $\vec{k}_R = k\,(\sin\Theta, 0, -\cos\Theta)$, the wave vectors inside the layer are $k_S = k'\,(\sin\Theta', 0, \cos\Theta')$ and $k_P = k'\,(\sin\Theta', 0, -\cos\Theta')$. Here $k = nk_0$ and $k' = n'\,k_0$ where $k_0 = 2\pi/\lambda_0$ and λ_0 is the vacuum wavelength. Θ is the angle of incidence and the angle of refraction Θ' is given by $n\sin\Theta = n'\sin\Theta'$.

Two basic polarization states are considered: the transverse electric polarization (TE) with the electric field $\vec{E}$ perpendicular to the plane of incidence and the transverse magnetic polarization (TM) with the magnetic field $\vec{B}$ perpendicular to the plane of incidence. The relevant field component of the wave, i.e. E_y for TE or B_y for TM, can be written as

$$
\begin{aligned}
&z < 0 \qquad && A_1 = A_I + A_R, \quad A_I = e^{i(\vec{k}_I\vec{r} - \omega t)}, \quad A_R = Re^{i(\vec{k}_R\vec{r} - \omega t)},\\
&0 < z < Z \qquad && A_2 = Se^{i(\vec{k}_S\vec{r} - \omega t)} + Pe^{i(\vec{k}_P\vec{r} - \omega t)}, \\
&z > Z \qquad && A_3 = Te^{i(\vec{k}_I\vec{r} - \omega t)}.
\end{aligned}
\tag{1}
$$

The amplitude of the field in the incident plane wave has been put equal to unity. The well-known boundary conditions for the field at both boundary planes lead to the following expressions

$$
\begin{aligned}
&R = a\,\varrho\sin\delta' \qquad && T = i\,a\,\sigma\,e^{-i\delta} \\
&S = i\,a\,a(a\cos\Theta + b\cos\Theta')e^{-i\delta'} \qquad && P = i\,a\,a(-a\cos\Theta + b\cos\Theta')\,e^{i\delta'}
\end{aligned}
\tag{2}
$$

with the abbreviations

$$
\begin{aligned}
&a = 1/(v\sin\delta' + i\,\sigma\cos\Theta'), \qquad && \sigma = 2\,n\,n'\cos\Theta\cos\Theta', \\
&v = a^2\cos^2\Theta + b^2\cos^2\Theta', \qquad && \varrho = a^2\cos^2\Theta - b^2\cos^2\Theta', \\
&\delta = k\,Z\cos\Theta, \qquad && \delta' = k'\,Z\cos\Theta'.
\end{aligned}
$$

For TE polarization $a = n$ and $b = n'$ whereas for TM polarization $a = n'$ and $b = n$. In the region of total reflection which is of interest to us, i.e. for $\Theta > \Theta_c = \arcsin(n'/n)$, $\cos\Theta'$ becomes imaginary and so does δ'. In this case we employ real parameters $K = n'\cos\Theta'/i = (n^2\sin^2\Theta - n'^2)^{1/2}$ and $\gamma = k_0\,KZ = \delta'/i$.

In calculating and measuring the longitudinal or Goos-Hänchen shift of a light beam totally reflected on a semiinfinite medium Wolter[2)] exploited a minimum--marked wave. He combined two incident plane waves of opposite phase with equal wavelength and amplitude but slightly different angles of incidence. Such a wave is characterized by a stationary zero-amplitude ray in the plane of incidence which can be traced up as a minimum-amplitude ray in the reflected wave. This procedure allows the determination of the spatial shift. In a generalization of this method the progression of a point on this ray can be followed. This device provides an information of both the spatial shift and the delay time.

Let us combine analogously two plane waves with wave vectors slightly different in magnitude and direction. The frequency $\omega = c_0 k_0$ is used as the second independent variable instead of the magnitude of the wave vector, c_0 being the velocity of light in vacuum. We obtain

$$A(\omega + \delta\omega, \Theta + \delta\Theta) - A(\omega, \Theta) =$$

$$= (\partial A/\partial\omega + \partial A/c_g \partial k)\,\delta\omega + (\partial A/\partial\Theta)\,\delta\Theta. \tag{3}$$

Here A represents any of the field components A_I, A_R, or A_3 (1) and $c_g = d\omega/dk$ is the group velocity in the medium with index of refraction n. The amplitude of the incident wave compositum (3) is then explicitly

$$iA_I \{(x \sin\Theta + z \cos\Theta - c_g t)\,\delta\omega/c_g + (x \cos\Theta - z \sin\Theta)\,k\delta\Theta_k\}. \tag{4}$$

Since ω and Θ are varied independently there is a point in the plane of incidence with zero-field amplitude given by

$$x = c_g t \sin\Theta \qquad z = c_g t \cos\Theta. \tag{5}$$

This point reaches the layer boundary at the origin ($x = 0$, $z = 0$) at time $t = 0$.

The amplitude of the reflected wave compositum is given by

$$iA_R \{(c_g \partial \ln R/i\partial\omega + x \sin\Theta - z \cos\Theta - c_g t)\,\delta\omega/c_g +$$

$$+ (\partial \ln R/i\,k\,\partial\Theta + x \cos\Theta + z \sin\Theta)\,k\,\delta\Theta\}. \tag{6}$$

This compositum does not exhibit a zero-amplitude point, owing to the slight difference in phase shifts of both constituent plane waves at reflection. However, a well defined minimum-amplitude point is given by

$$x = (t - \partial \arg R/\partial\omega)\,c_g \sin\Theta - (\partial \arg R/k\,\partial\Theta) \cos\Theta,$$

$$z = (-t + \partial \arg R/\partial\omega)\,c_g \cos\Theta - (\partial \arg R/k\,\partial\Theta) \sin\Theta, \tag{7}$$

where the relation $\ln R = \ln |R| + i \arg R$ has been exploited. Evidently, this point travels along a ray in the reflected wave with velocity c_g, emerging from the layer at the point

$$X_R = -(k\cos\Theta)^{-1}\,\partial \arg R/\partial\Theta, \qquad z = 0 \tag{7a}$$

a the time

$$t_R = \partial \arg R/\partial\omega - (\partial \arg R/\omega\partial\Theta)\tan\Theta. \tag{7b}$$

This time of reappearance can be regarded as the reflection delay time, while X_R stands in a simple connection with the formerly defined shift $D_R = X_R \cos\Theta$, measured normally to the ray[7].

By a similar procedure, studying the appearance of the minimum-amplitude point in the transmitted wave, analogous quantities are obtained

$$X_T = Z\tan\Theta - (k\cos\Theta)^{-1}\,\partial \arg T/\partial\Theta, \qquad z = Z \tag{8a}$$

$$t_T = z/c_g\cos\Theta + \partial \arg T/\partial\omega - (\partial \arg T/\omega\partial\Theta)\tan\Theta. \tag{8b}$$

By inspecting expressions (2) it can be shown that

$$X_R = X_T = X \tag{9a}$$

and

$$t_R = t_T = t_0. \tag{9b}$$

Thus the minimum-amplitude points in the reflected and in the transmitted wave appear at the same moment and shifted by the same amount from the origin in x-direction (Fig. 1).

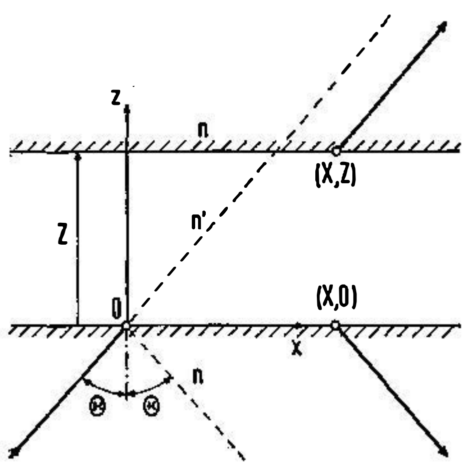

Fig. 1. The propagation of the minimum-amplitude points. $(X, 0)$ and (X, Z) are the points of reappearance

After a rather lengthy manipulation explicit results are obtained

$$X = 2 \sin\Theta \{n^2 - n'^2) \varrho \sinh\gamma \cosh\gamma - n^2 \nu \gamma \cos^2\Theta\}/$$

$$/k_0 K (\varrho^2 \sinh^2\gamma - \sigma^2) \cos\Theta \tag{10a}$$

and

$$t_0 = 2n \{(n^2 - n'^2) \varrho \sin^2\Theta \sinh\gamma \cosh\gamma - n'^2 \nu \gamma \cos^2\Theta\}/$$

$$/\omega K (\varrho^2 \sinh^2\gamma - \sigma^2) \cos\Theta. \tag{10b}$$

Generally, n and n' depend on frequency ω, introducing thus terms with $\omega\, \mathrm{d}n/n\, \mathrm{d}\omega$ and $\omega\, \mathrm{d}n'/n'\, \mathrm{d}\omega$ into t_0. These small terms complicate the results without invoking any essential change. So it is reasonable to consider in a first approximation, as was done in the last steps of the present calculation, the media as nondispersive, i.e. to neglect the frequency dependence of n and n'. This amounts also to replacing the group velocity c_g by c_0/n.

The use of a compositum of plane wave evokes the idea that the above results are connected with the effective group velocity of propagation of waves across the barrier. Indeed, our model is equivalent to the picture based on wave packets. Let us construct a wave packet travelling to the layer boundary as

$$A_I = \int\int f(\omega,\Theta)\, e^{i(\vec{k}_I \vec{r} - \nu t)}\, \mathrm{d}\omega\, \mathrm{d}\Theta, \tag{11a}$$

where $f(\omega,\Theta)$ is a suitable weighting function. The reflected part of the packet is then

$$A_R = \int\int f(\omega,\Theta)\, |R|\, e^{i(\vec{k}_R \vec{r} - \omega t + \arg R)} \mathrm{d}\omega\, \mathrm{d}\Theta, \tag{11b}$$

and the transmitted part

$$A_3 = \int\int f(\omega,\Theta)\, |T|\, e^{i(\vec{k}_I \vec{r} - \omega t + \arg T)}\, \mathrm{d}\omega\, \mathrm{d}\Theta. \tag{11c}$$

According to the well-known formalism[8,9] the maximum of the wave packet occurs at a space-time point where the phase is stationary. This leads immediately to Equs. (5), (7), and (8) for wave packets (11a), (11b), and (11c), respectively. By this reasoning the minimum-marked wave, a seemingly artificial device, is given a firm physical meaning.

3. Scattering theory

Similar results are obtained in the scope of scattering theory following Wigner, Smith and Froissart et al.[10]. These ideas were used by Agudin to calculate the delay time at total reflection on a semiinfinite medium[11]. The difference in propagation times for two processes is calculated as the difference of the corresponding integrated energy densities normalized to unit incident flux.

Since we are interested in the time of reappearance, i.e. the total time of barrier penetration, we compare solution (1) with the solution of a corresponding thought experiment in which the barrier is traversed instantaneously. Thereby we suppose that the field inside the layer is zero and that outside the layer it conforms with the field (1) for an idealized semitransparent mirror, placed at the boundary plane $z = 0$, invoking solely the change of amplitudes

$$\begin{aligned} z < 0 \qquad & \widetilde{A}_1 = e^{i(kx\sin\Theta - \omega t)}\left(e^{ikz\cos\Theta} \pm |R|\, e^{-ikz\cos\Theta}\right), \\ 0 < z < Z \qquad & \widetilde{A}_2 = 0, \\ z > Z \qquad & \widetilde{A}_3 = e^{i(kx\sin\Theta - \omega t)}\, |T|\, e^{ikz\cos\Theta}. \end{aligned} \tag{12}$$

For TE polarization, taking into account the energy density $\frac{1}{2}\,\varepsilon_0\, n^2 A^\star A$ and the normal component of the incident flux $\frac{1}{2}\,\varepsilon_0\, c_0\, n\cos\Theta$, we obtain

$$t_0 = \left\{ n^2 \int_{-\infty}^{0} (A_1{}^\star A_1 - \widetilde{A}_1{}^\star \widetilde{A}_1)\, dz + n'^2 \int_{0}^{Z} A_2{}^\star A_2\, dz \right\} / \, n c_0 \cos\Theta. \tag{13}$$

A third term $n^2 \int_{Z}^{\infty} (A_3{}^\star A_3 - \widetilde{A}_3{}^\star \widetilde{A}_3)\, dz$ gives no contribution. Inserting the fields from Equs. (1) and (12) and averaging out the oscillating terms we get the result (10b). For TM polarization the energy density and the normal component of the incident flux are altered to $A^\star A/2\mu_0$ and $c_0 \cos\Theta/2n\mu_0$, respectively, but the final result is again brought into the form (10b).

4. Discussion

It can be inferred from Equ. (10b) that the time of reappearance tends to a finite value for increasing layer thickness[13]

$$t_0\,(Z \to \infty) = 2n\,(n^2 - n'^2)\,\tan\Theta\,\sin\Theta/\omega K \varrho.$$

For a sufficiently thick layer this entails the possibility of superluminal propagation of the maximum of the transmitted wave packet across the layer. We consider the propagation as superluminal if*

$$t_0 \leqq (X^2 + Z^2)^{1/2}/c_0. \tag{14}$$

Here $(X^2 + Z^2)^{1/2}$ is the apparent distance travelled by the transmitted wave inside the layer and $(X^2 + Z^2)^{1/2}/c_0$ the time light in vacuum would travel this distance. The expression (14) with the equality sign defines the boundary of the superluminal region. In Fig. 2 this region is shown for a vacuum gap ($n' = 1$) in glass ($n = 1, 5$).

There is no superluminal behaviour in the reflected wave as the shift X also tends to a constant value

$$X(Z \to \infty) = 2c_0 (n^2 - n'^2) \tan\Theta/\omega K_\varrho$$

and the ratio

$$(X/c_0 t_0)_{Z \to \infty} = 1/n \sin\Theta \leqq 1.$$

At first sight it may appear that superluminal behaviour in barrier penetration implies macrocausality violation, since information can be relayed by a minimum--amplitude point or maximum of a wave packet. However, this point or the maximum result only from a special construction of waves[12]. Besides, it should not be over-

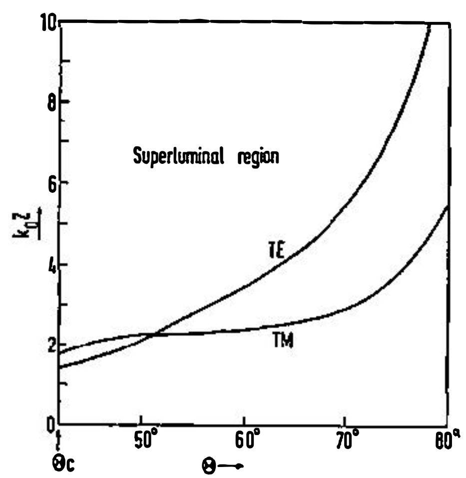

Fig. 2. The superluminal region for a transmitted wave packet in a vacuum gap ($n' = 1$) surrounded by glass ($n = 1.5$) for TE and TM polarization. The curves were calculated by means of Equs. (10) and (14).

*An effective group velocity inside the barrier for the transmitted wave can be defined as $(X^2 + Z^2)^{1/2}/t_0$.

looked that the transmitted wave is strongly attenuated with increasing barrier thickness, the relative transmitted flux being asymptotically $(-4\sigma^2/\varrho^2)$ exp $(-4\pi KZ/\lambda_0)$. Yet the transmitted flux has easily been followed in experiments with gaps of widths up to eight wavelengths[13].

Thus, the actual velocity of the wavefront is by no means greater than c_0. This can be demonstrated by a modification of the classical argument of Sommerfeld[14], where the microscopic picture of matter as an aggregate of charged particles — scatterers in the void space is considered. The evanescent wave is a diffraction phenomenon; the connection between the wave vector and the frequency of the incident wave in the gap boundary is such, that the interference is destructive for all directions pointing into the gap. Over distances of the order of the wavelength there is still some remaining field which naturally propagates with the velocity c_0, and may, for narrow gaps, excite the field in the material beyond.

What remains interesting is the fact that the group velocity of the wave inside the barrier exceeds c_0 even for vacuum gaps. It may seem incorrect to speak of group velocity in the absence of ordinary dispersion. Yet the devices with which we studied the propagation of waves distinctly imply the concept of wave groups.

The experimental determination of the delay time in barrier penetration seems for the moment impossible with visible light, except in an indirect way via longitudinal shifts. Up to now the shifts were determined only at total reflection on a semiinfinite medium near the critical angle. The direct determination of the delay time seems, however, feasible for microwaves[15]. At low frequencies one could hope to measure directly the phase differences between the transmitted and the unperturbed incident wave.

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ZAKASNITEV ELEKTROMAGNETNEGA VALOVANJA NA OVIRI

J. STRNAD in A. KODRE

Odsek za fiziko in Institut „J. Stefan", Univerza v Ljubljani, Ljubljana

Vsebina

Članek obravnava čas zakasnitve pri prehodu elektromagnetnega valovanja skozi tanko plast optično redkejšega sredstva in pri odboju na plasti, če je vpadni kot večji kot mejni kot totalnega odboja. Najprej so narejeni računi za val, ki ga označuje minimum. Enak rezultat da tudi teorija sipanja. Zanimivo je, da je efektivna skupinska hitrost v optično redkejšem sredstvu za prepuščeno valovanje lahko večja kot hitrost svetlobe v vakuumu c_0, medtem ko signalna hitrost ne more preseči c_0.